

Class	Register Number	Name
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南洋女子中学校
NANYANG GIRLS' HIGH SCHOOL

End-of-Year Examinations 2017
Secondary 3

INTEGRATED MATHEMATICS 1

2 hours

Thursday

5 October 2017

0845 - 1045

READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Write your name, register number and class on all the work you hand in.
2. Answer questions 1 – 11 before attempting question 12 (Bonus Question).
3. Write your answers and working on the separate writing paper provided, unless otherwise stated.
4. Write in dark blue or black pen on both sides of the paper.
5. You may use an HB pencil for any diagrams or graphs.
6. Do not use staples, paper clips, glue or correction fluid.
7. Omission of essential steps will result in loss of marks.
8. The use of an electronic calculator is expected, where appropriate.
9. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degree to one decimal place.
For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

INFORMATION FOR CANDIDATES

1. At the end of the examination, fasten all your work securely together.
2. The number of marks is given in brackets [] at the end of each question or part question.
3. The total number of marks for this paper is 80.

Mathematical Formulae**1. TRIGONOMETRY**

Formulae for $\triangle ABC$

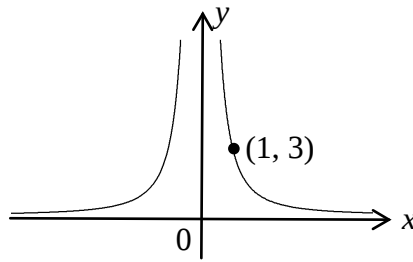
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

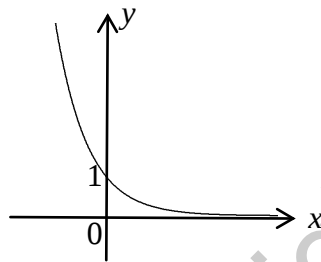
$$\text{Area of triangle} = \frac{1}{2} ab \sin C$$

Nanyang Girls' High School

- 1 (a) The diagram represents the graph of $y = kx^n$. Write down the value of k and a possible value of n . [2]



- (b) Write down a possible equation for the graph below, where $x \in \mathbb{R}$. [1]



- 2 In separate diagrams, sketch the graphs that represent each of the following statements respectively.
- (a) The total amount of a utility bill, y , consists of a fixed charge of \$10 plus a variable charge which is proportional to the volume of water, x , used. [2]
- (b) The strength of the current of an electric circuit, y , is inversely proportional to the amount of resistance, x . [2]
- 3 The number of days required to build a house depends on the number of workers available. When 12 workers are doing the job, each working 8 hours daily, the construction takes 60 days.
- (a) If the owner wants to receive the completed house earlier, how many workers, each working 8 hours daily, are required to complete the job in 36 days? [2]
- (b) As the construction company is unable to find more workers to work on the job, how many more hours must each of the 12 workers put in so as to complete the job in 36 days? [2]

- 4 The equation of a circle C_1 is given as $(x + 2)^2 + (y - 5)^2 = 9$.
- (i) A circle C_2 is formed when C_1 is reflected in the line $y = -x$. State the centre of C_2 and hence, write down the equation of C_2 . [2]
- (ii) The line $y = -x$ intersects the circle C_1 at two points. Find the coordinates of these two points. [4]

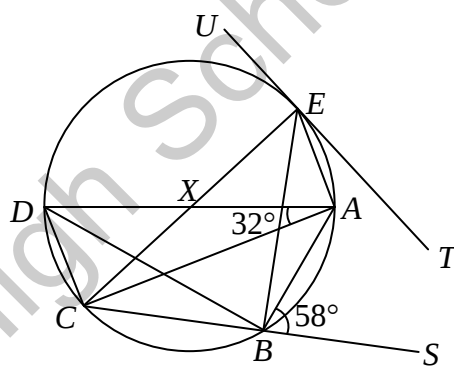
- 5 The points A, B, C, D and E lie on a circle. The chords AD and CE intersect at the point X . TU is a tangent to the circle at E and SBC is a straight line. $\angle SBA = 58^\circ$, $\angle CAD = 32^\circ$ and CE is the diameter.

(a) Find

- (i) $\angle CBD$, [1]
 (ii) $\angle AEC$, [1]
 (iii) $\angle AET$, [1]
 (iv) $\angle EBA$. [1]

(b) State the geometrical property you used in (a)(i). [1]

(c) Prove that X is the centre of the circle. [2]



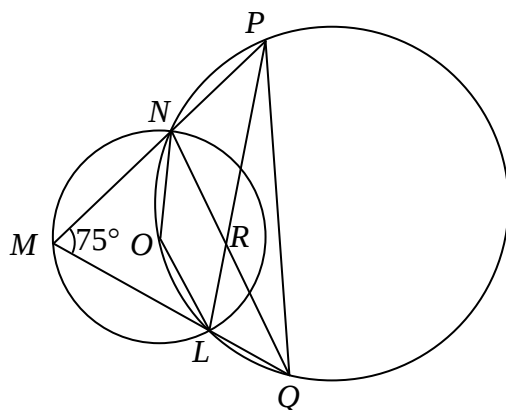
- 6 The diagram shows two circles intersecting at points L and N . O is the centre of the smaller circle. MNP , MLQ , NRQ and PRL are straight lines.

(a) Given that $\angle PMQ = 75^\circ$, find

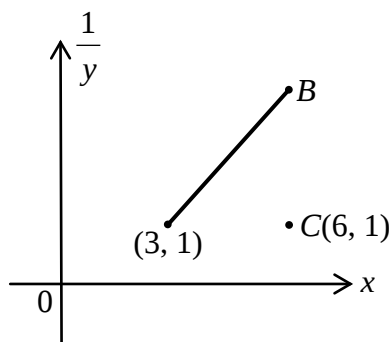
- (i) $\angle LON$, [1]
 (ii) $\angle NPL$. [1]

(b) What is the special name for $\triangle LMP$? Explain your answer. [2]

(c) Calculate $\angle PRQ$. [3]



7



The diagram shows part of a straight line graph obtained by plotting $\frac{1}{y}$ against x . The graph passes through (3, 1) and point B , making an angle of 45° with the line $\frac{1}{y} = 1$. C is at (6, 1) such that BC is parallel to the vertical axis.

- (i) Find the coordinates of B . [3]
- (ii) Find the non-linear equation by expressing y in terms of x . [2]
- (iii) When y is plotted against x , the non-linear graph obtained passes through $(p, \frac{1}{4})$. Find the value of p . [2]

- 8 The table below shows the amount of beverages sold at 2 different drinks stalls within an hour on a particular day. Stall K operates 10 hours that day and stall L operates 12 hours that day.

	Coffee	Tea	Milo	Barley
Stall K	25	21	18	12
Stall L	27	20	18	10

The prices of the different beverages (per cup) are as follows:

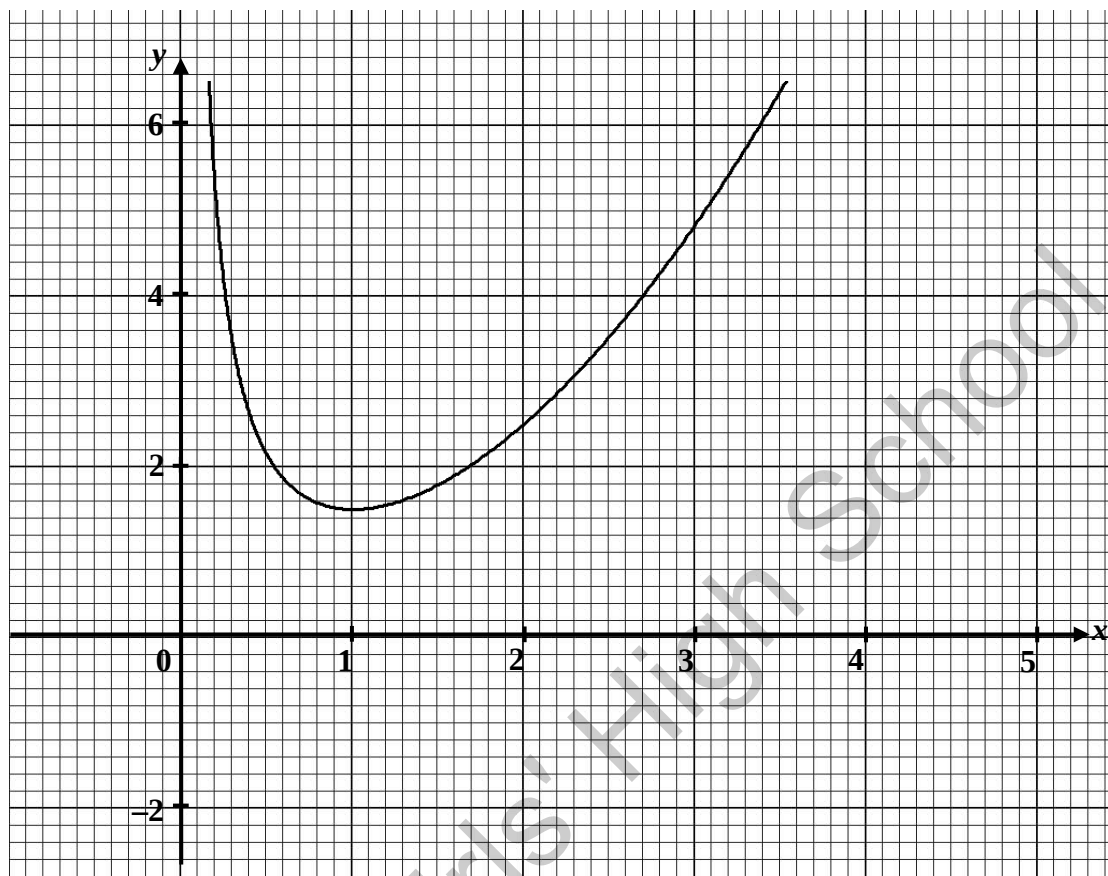
Coffee at \$1.10, Tea at \$0.90, Milo at \$1.20, Barley at \$1.50.

- (a) Write down two matrices whose product gives the total amount of money received by each drinks stall in that one hour. Find this matrix product. [3]
- (b) Assuming that the amount of beverages sold per hour remains the same throughout the entire day, find, by matrix multiplication, the total amount of money received by both drink stalls combined for that day. [2]
- (c) The net profit for stall K is 60% of the total amount it receives while that for stall L is 50%. By expressing this information as a 2×2 matrix and using matrix multiplication, find the net profit for each stall for that day. [3]

[Turn over

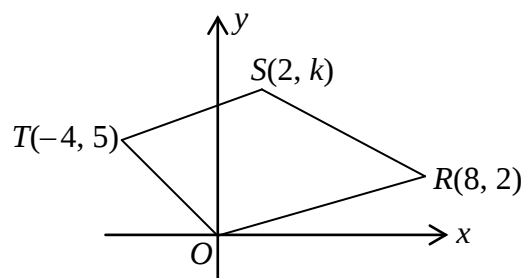
9 Answer the whole of this question on the INSERT provided.

The diagram shows part of the curve $y = \frac{1}{2}x^2 + \frac{1}{x}$.



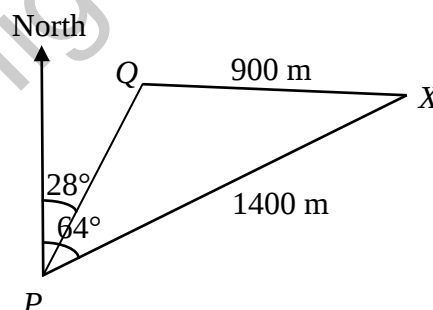
- (i) Use the graph to estimate the gradient of the curve at the point $x = 0.5$. [2]
- (ii) By drawing a suitable tangent on the graph, find the x -coordinate of the point on the curve at which the gradient of the tangent is 2. [2]
- (iii) Find the range of values of x that satisfies the inequality $\frac{1}{2}x^2 + \frac{1}{x} - 5 < 0$. [2]
- (iv) By adding a suitable straight line on the same axes, use the graph to estimate the values of x for which $8 - 4x - \frac{2}{x} - x^2 = 0$. [3]

- 10 In the diagram, $ORST$ is a trapezium where O is the origin and $OR \parallel TS$. The coordinates of R , S and T are $(8, 2)$, $(2, k)$ and $(-4, 5)$ respectively.



- Find the equation of the line RT . [3]
- Given that P is $(4, 3)$, prove that P , R and T are collinear. [2]
- Find the length of PT , express your answer in the form of $a\sqrt{b}$, where $a \neq 1$. [2]
- Find the value of k if the area of the trapezium is 42 square units. [2]
- Given that $ORTU$ is a parallelogram, find the coordinates of U . [2]
- The point V lies on the line that passes through R and T such that the ratio of $RV : TV$ is $3 : 1$. Find the two possible positions of V . [4]

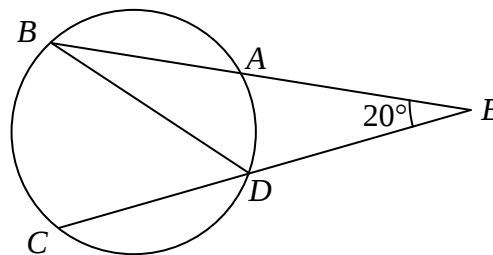
- 11 Two students A and B are canoeing towards an islet X , starting at points P and Q respectively. $PX = 1400$ m, $QX = 900$ m, the bearing of X from P is 064° and the bearing of Q from P is 028° . At 0900, A and B start paddling in a straight line towards X at constant speeds of 1.1 m/s and 0.8 m/s respectively.



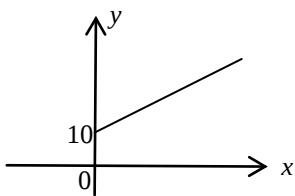
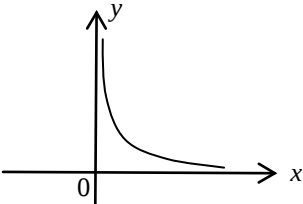
- Calculate the obtuse angle PQX , correct to the nearest degree. [2]
- Find the distance between the two students at 0910. [4]
- At 0910, a drone is seen hovering h m vertically above X . Find the range of the possible values of h such that the angles of elevation of the drone from both A and B are between 10° and 22° . Give your answer to the nearest metre. [4]

Bonus

- 12 In the diagram on the right, the arcs AB , BC and CD have the same length, and $\angle BEC = 20^\circ$. Find $\angle ABD$. [2]



Answers:

1a	$k = 3, n = -2$ or -4 or -6 or ...	8a	$\begin{pmatrix} 25 & 21 & 18 & 12 \\ 27 & 20 & 18 & 10 \end{pmatrix} \begin{pmatrix} 1.10 \\ 0.90 \\ 1.20 \\ 1.50 \end{pmatrix} \text{ or } (1.1 \ 0.9 \ 1.2 \ 1.5) \begin{pmatrix} 25 & 27 \\ 21 & 20 \\ 18 & 18 \\ 12 & 10 \end{pmatrix}$ $= \begin{pmatrix} 86.00 \\ 84.30 \end{pmatrix} = (86 \ 84.3)$
b	$y = 3^{-x}, y = 2^{-x}, y = 4^{-2x}, \dots$ (Any eqn in the form $y = a^{-nx}$ where $a > 1, n > 0$)	b	$(10 \ 12) \begin{pmatrix} 86.00 \\ 84.30 \end{pmatrix} = (860 + 1011.60)$ $= (1871.60)$ $\therefore$ The total amount of money received by both drink stalls is \$1871.60.
2a		c	$\begin{pmatrix} 0.6 & 0 \\ 0 & 0.5 \end{pmatrix} \begin{pmatrix} 860 \\ 1011.60 \end{pmatrix} = \begin{pmatrix} 516 \\ 505.80 \end{pmatrix}$ $\therefore$ The net profit for stall K and L is \$516 and \$505.80 respectively.
b		9i	Gradient = -3.5
3a	20 workers are required	ii	$x = 2.2$
b	Each worker has to work $5\frac{1}{3}$ hours more daily.	iii	$0.2 < x < 3.05$
4i	Centre of C_2 is $(-5, 2)$ $(x + 5)^2 + (y - 2)^2 = 9$	iv	$x = 0.3$ or 1.2
ii	The two points are $(-2, 2)$ and $(-5, 5)$	10a	$y = -\frac{1}{4}x + 4$ or $x + 4y - 16 = 0$
5ai	$\angle CBD = 32^\circ$	c	$2\sqrt{17}$ units
aii	$\angle AEC = 58^\circ$	d	$k = 6.5$
aiii	$\angle AET = 32^\circ$	e	$U(-12, 3)$
aiv	$\angle EBA = 32^\circ$	f	$V\left(-1, 4\frac{1}{4}\right)$ or $V\left(-10, 6\frac{1}{2}\right)$
b	Angles in the same segment	11a	Obtuse $\angle PQX = 114^\circ$ (nearest degree)
6ai	$\angle LON = 150^\circ$	b	$AB = 432$ m
aii	$\angle NPL = 30^\circ$	c	$131 \leq h \leq 169$
c	$\angle MQN = 30^\circ$ (6b) Isosceles triangle	12	Bonus $x = 30^\circ$
7i	$B(6, 4)$		
ii	$y = \frac{1}{x-2}$		
iii	$p = 6$		