Year 6 MAA HL Preliminary Examination 2024 Paper 1 (Mark Scheme)

Qn	Suggested Solutions	Marks
1	**SL 2.5 Composite Function** **SL 2.2 Informal Concept of Inverse Function**	**[Maximum mark: 5]**
(a)	$(g \circ f)(x) = g(f(x)) = 18(f(x))^2 - 5 = 18\left(\frac{x+1}{3}\right)^2 - 5$	**M1**
	$(g \circ f)(x) = 2(x+1)^2 - 5 = 2(x^2 + 2x + 1) - 5$	**A1**
	$(g \circ f)(x) = 2x^2 + 4x - 3$	**AG**
(b)	**Method 1 Using domain of f^{-1} equals range of f**	
	$(g \circ f)^{-1}(a) = 2$ is equivalent to $a = (g \circ f)(2)$	**M1**
	$a = 2(2)^2 + 4(2) - 3 = 8 + 8 - 3 = 13$	**M1 A1**
	Method 2 Finding the inverse of $g \circ f$	
	$y = 2x^2 + 4x - 3 = 2(x+1)^2 - 5$	
	$(g \circ f)^{-1}(x) = \sqrt{\frac{x+5}{2}} - 1,\ x \geq -5$	**M1**
	$(g \circ f)^{-1}(a) = \sqrt{\frac{a+5}{2}} - 1 = 2$	
	$a = 2(2+1)^2 - 5 = 18 - 5 = 13$	**M1 A1**
	Method 3 Finding f^{-1} and g^{-1} to find the inverse of $g \circ f$	
	$f^{-1}(x) = 3x - 1,\ x \geq 0$ and $g^{-1}(x) = \sqrt{\frac{x+5}{18}},\ x \geq -5$	
	$(g \circ f)^{-1}(x) = f^{-1}(g^{-1}(x)) = 3\sqrt{\frac{x+5}{18}} - 1 = \sqrt{\frac{x+5}{2}} - 1,\ x \geq -5$	**M1**
	$(g \circ f)^{-1}(a) = \sqrt{\frac{a+5}{2}} - 1 = 2$	
	$a = 2(2+1)^2 - 5 = 18 - 5 = 13$	**M1 A1**

2	**SL 1.9 The Binomial Theorem** **AHL 1.10 Extension of Binomial Theorem to Fractional and Negative Indices**	**[Maximum mark: 5]**
(a)	$(1-x)^6 = 1-6x+15x^2-20x^3+\ldots+x^6$	**A1 for the first 4 terms**
	$\frac{1}{\sqrt{1+4x}} = (1+4x)^{-\frac{1}{2}}$ $= 1+\left(-\frac{1}{2}\right)(4x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(4x)^2+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}(4x)^3+\ldots$ $= 1-2x+6x^2-20x^3+\ldots$	**A1 for the first 4 terms**
	$\frac{(1-x)^6}{\sqrt{1+4x}} = \left(1-6x+15x^2-20x^3+\ldots+x^6\right)\left(1-2x+6x^2-20x^3+\ldots\right)$ $= 1-6x+15x^2-20x^3-2x+12x^2-30x^3+6x^2-36x^3-20x^3+\ldots$ $= 1-8x+33x^2-106x^3+\ldots$	**M1** **A1**
(b)	$\lvert 4x\rvert<1$ or $\lvert x\rvert<\frac{1}{4}$ or $-\frac{1}{4}<x<\frac{1}{4}$	**A1**
3	**SL 4.5 Complementary Events** **SL 4.6 Use of Venn Diagram, Combined Events and Conditional Probability**	**[Maximum mark: 6]**
	Method 1 $\mathrm{P}(B) = 1-\mathrm{P}(B') = 1-0.4 = 0.6$ $\mathrm{P}(A\cup B) = 1-\mathrm{P}(A'\cap B') = 1-0.1 = 0.9$	**(M1) A1 for either** $\mathrm{P}(B)$ **or** $\mathrm{P}(A\cup B)$
	$\mathrm{P}(A\cap B) = \mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cup B) = 0.5+0.6-0.9 = 0.2$	**(M1) A1**
	Method 2 $\mathrm{P}(A\cap B') = \mathrm{P}(B')-\mathrm{P}(A'\cap B') = 0.4-0.1 = 0.3$ $\mathrm{P}(B\cap A') = \mathrm{P}(A')-\mathrm{P}(A'\cap B') = 0.5-0.1 = 0.4$	**(M1) A1 for either** $\mathrm{P}(A\cap B')$ **or** $\mathrm{P}(B\cap A')$
	$\mathrm{P}(A\cap B) = 1-\mathrm{P}(A'\cap B')-\mathrm{P}(A\cap B')-\mathrm{P}(B\cap A')$ $\mathrm{P}(A\cap B) = 1-0.1-0.3-0.4 = 0.2$ or $\mathrm{P}(A\cap B) = \mathrm{P}(A)-\mathrm{P}(A\cap B') = 0.5-0.3 = 0.2$	**(M1)** **A1**
	$\mathrm{P}(A\mid B) = \frac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)} = \frac{0.2}{0.6} = \frac{1}{3}$	**M1 A1**

4	**SL 1.7 Laws of Logarithms** **SL 3.5 Exact Values of Trigonometric Ratios** **SL 3.6 Double Angle Identities for Sine and Cosine** **SL 3.8 Solving Trigonometric Equations in a Finite Interval**	**[Maximum mark: 7]**
(a)	$\log_4(1+\sin 2x) = \dfrac{\log_2(1+\sin 2x)}{\log_2 4} = \dfrac{1}{2}\log_2(1+\sin 2x)$ $\log_4(1+\sin 2x) = \log_2\sqrt{1+\sin 2x}$ $\log_4(1+\sin 2x) = \log_2\left(\sqrt{2}\cos x\right)$	**M1 A1** **AG**
(b)	$\log_2\left(\sqrt{1+\sin 2x}\right) = \log_2\left(\sqrt{2}\cos x\right)$ $\sqrt{1+\sin 2x} = \sqrt{2}\cos x$ $1+\sin 2x = 2\cos^2 x$ $\sin 2x = 2\cos^2 x - 1$ $\sin 2x = \cos 2x$ $\tan 2x = 1$ $\text{Basic Angle} = \arctan 1 = \dfrac{\pi}{4}$ $2x = \dfrac{\pi}{4}$ $x = \dfrac{\pi}{8}$	**M1 for using part (a)** **M1 for double angle formula for cosine** **A1 for tan 2*x* = 1** **A1 for basic angle** **A1**

5	**AHL 1.15 Proof by Mathematical Induction involving Inequalities and Factorial**	**[Maximum mark: 7]**
	Let P(n) be the statement that $(2n)! \geq 2^n (n!)^2$ for $n \in \mathbb{N}$. When $n = 0$, LHS = 0! = 1, RHS = 2^0 $(0!)^2$ = 1(1) = 1 Thus, LHS ≥ RHS. Therefore, P(0) is true.	**A1**
	Assume that P(k) is true for some value of k for $k \in \mathbb{N}$, i.e. $(2k)! \geq 2^k (k!)^2$ for $k \in \mathbb{N}$.	**M1**
	When $n = k+1$, $(2(k+1))! = (2k+2)! = (2k+2)(2k+1)[(2k)!]$	**A1**
	$\geq (2k+2)(2k+1)2^k (k!)^2$	**M1 for inductive step**
	$> (2k+2)(k+1)2^k (k!)^2$ $= 2(k+1)(k+1)2^k (k!)^2$ $= 2(2^k)(k+1)k!(k+1)k!$	**A1 for $2k+1 \geq k+1$ or $2k^2 + 3k + 1 \geq 2k^2 + 2k + 1$ or $4k^2 + 6k + 2 \geq 2k^2 + 4k + 2$**
	$= 2^{k+1}((k+1)!)^2$ Therefore, P(k +1) is true.	**A1**
	Since P(0) is true and P(k) is true implies that P(k+1) is true for some $k \in \mathbb{N}$, by mathematical induction, $(2n)! \geq 2^n (n!)^2$ for $n \in \mathbb{N}$.	**A1 awarded only if at least 5 of preceding marks are obtained**
6	**SL 5.5 Integration with Boundary Condition** **SL 5.10 Integration by Inspection and Integral of 1/x**	**[Maximum mark: 8]**
	$y = f(x) = \int \frac{a(2x-1)}{2x^2-2x+1}\,dx = \frac{a}{2}\int \frac{4x-2}{2x^2-2x+1}\,dx$ $y = f(x) = \frac{a}{2}\ln\left\|2x^2-2x+1\right\| + C$	**M1** **A1 for ln with modulus** **A1 for + C**
	When $x = 0$ or $x = 1$, $y = \ln 2$ gives $C = \ln 2$	**M1 A1**
	When $x = 0.5$, $y = 0$ gives $a = 2$	**M1 A1**
	$f(x) = \ln\left\|2x^2-2x+1\right\| + \ln 2$ or $\ln(2x^2-2x+1) + \ln 2$ or $f(x) = \ln\left\|4x^2-4x+2\right\|$ or $\ln(4x^2-4x+2)$ or equivalent	**A1**
	Note: Award maximum of M1 A1 A1 M1 A1 M0 A0 A0 to candidates who assume that a = 2	

7	**AHL 4.14 Mean and Median of Continuous Random Variable** **AHL 5.16 Integration by Parts**	**[Maximum mark: 8]**
	Let m be the median of X. $\mathrm{P}(0 \le X \le m) = \frac{1}{2}$ gives $\int_0^m \frac{\pi}{6}\sin\left(\frac{\pi}{6}x\right)\mathrm{d}x = \frac{1}{2}$, $0 \le m \le 3$	**M1 with correct limits**
	$\left[-\cos\left(\frac{\pi}{6}x\right)\right]_0^m = \frac{1}{2}$ $1-\cos\left(\frac{\pi m}{6}\right) = \frac{1}{2}$	**A1**
	$\cos\left(\frac{\pi m}{6}\right) = \frac{1}{2}$, $0 \le m \le 3$ $\frac{\pi m}{6} = \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$ $m = 2$	**A1**
	Mean of X = $\mathrm{E}(X) = \int_0^3 x\,f(x)\,\mathrm{d}x = \int_0^3 \frac{\pi}{6}x\sin\left(\frac{\pi}{6}x\right)\mathrm{d}x$	**M1 for E(*X*) with correct limits**
	$\mathrm{E}(X) = \left[-x\cos\left(\frac{\pi}{6}x\right)\right]_0^3 + \int_0^3 \cos\left(\frac{\pi}{6}x\right)\mathrm{d}x$	**M1 A1 for integrating by parts**
	$\mathrm{E}(X) = \left[\frac{6}{\pi}\sin\left(\frac{\pi}{6}x\right)\right]_0^3 = \frac{6}{\pi}$	**A1**
	Since $\pi > 3$, therefore $\frac{6}{\pi} < \frac{6}{3}$, i.e. $\frac{6}{\pi} < 2$	**R1 for stating $\pi > 3$**
	Hence the mean of X is less than the median of X.	**AG**
8	**SL 2.2 Inverse Function** **SL 5.11 Area Between Curves**	**[Maximum mark: 9]**
(a)	**Method 1 Using $f(x) = f^{-1}(x) = x$** Solving $f(x) = f^{-1}(x)$ is equivalent to solving $f(x) = x$	**M1**
	$\frac{1}{4}x^2 - x + 3 = x$, $x \ge 2$ $x^2 - 4x + 12 = 4x$ $x^2 - 8x + 12 = 0$ $(x-2)(x-6) = 0$	**M1 for solving quadratic equation**
	$x = 2$ or $x = 6$	**A1 A1**

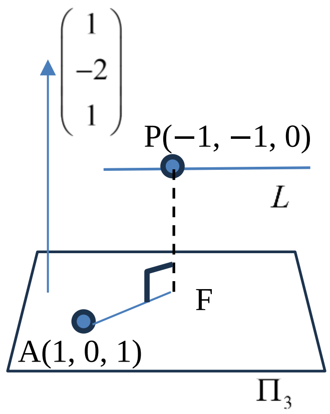
	Method 2 Finding f^{-1} $y=f(x)=\frac{1}{4}x^2-x+3=\frac{1}{4}(x-2)^2+2$ $f^{-1}(x)=2+2\sqrt{x-2}$, $x\geq 2$ Solving $f(x)=f^{-1}(x)$ $\frac{1}{4}(x-2)^2+2=2+2\sqrt{x-2}$, $x\geq 2$ $\frac{1}{4}(x-2)^2=2(x-2)^{\frac{1}{2}}$ $(x-2)^2=8(x-2)^{\frac{1}{2}}$ $(x-2)^{\frac{1}{2}}\left((x-2)^{\frac{3}{2}}-8\right)=0$ Since $x\geq 2$, $x=2$ or $x=6$	**M1 for finding f^{-1}** **M1 for solving equation** **A1 A1**
(b)	**Method 1** $\text{Area}=2\int_2^6 (x-f(x))\,\mathrm{d}x$ or equivalent $=2\int_2^6\left(x-\left(\frac{1}{4}x^2-x+3\right)\right)\mathrm{d}x$ $=2\int_2^6\left(-3+2x-\frac{1}{4}x^2\right)\mathrm{d}x$ $=2\left[-3x+x^2-\frac{1}{12}x^3\right]_2^6$ $=2\left[(-18+36-18)-\left(-6+4-\frac{2}{3}\right)\right]$ $=5\frac{1}{3}$ or $\frac{16}{3}$ units^2	**M1 for twice area or integral** **A1 FT for integral with correct limits** **A1 for correct integration** **M1 for substitution of limits** **A1 FT from their limits**
	Method 2 $\text{Area}=\int_2^6\left(f^{-1}(x)-f(x)\right)\mathrm{d}x$ or equivalent $=\int_2^6\left(2+2\sqrt{x-2}-\left(\frac{1}{4}x^2-x+3\right)\right)\mathrm{d}x$ $=\int_2^6\left(2(x-2)^{\frac{1}{2}}-1+x-\frac{1}{4}x^2\right)\mathrm{d}x$ $=\left[\frac{4}{3}(x-2)^{\frac{3}{2}}-x+\frac{1}{2}x^2-\frac{1}{12}x^3\right]_2^6$ $\text{Area}=\left[\left(\frac{32}{3}-6+18-18\right)-\left(-2+2-\frac{2}{3}\right)\right]$ $=5\frac{1}{3}$ or $\frac{16}{3}$ units^2	**M1 for difference of functions** **A1 FT for integral with correct limits** **A1 for correct integration** **M1 for substitution of limits** **A1 FT from their limits**

9	**Integration; Integrating Factor and DE; Maclaurin Series**	**[Maximum mark: 18]**
(ai)	$\dfrac{x}{(2-x)^2}=\dfrac{A}{(2-x)^2}+\dfrac{B}{(2-x)}$ $x=A+B(2-x)$ $B=-1,\ \ A=2$	A1
(aii)	$\displaystyle\int\frac{x}{(2-x)^2}\,\mathrm{d}x$ $\displaystyle=\left(\int\frac{2}{(2-x)^2}-\frac{1}{(2-x)}\right)\mathrm{d}x$ $\displaystyle=\frac{2}{(2-x)}+\ln(2-x)+c \qquad (\because x<2)$	M1 $\displaystyle\int\frac{1}{2-x}\,\mathrm{d}x=\alpha\ln(2-x)$ M1 $\displaystyle\int\frac{1}{(2-x)^2}\,\mathrm{d}x=\frac{\beta}{2-x}$ A1 accept $+\ln\lvert 2-x\rvert$
(b)	At (x,y), gradient $=\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{x-y}{2-x}$ **<u>OR:</u>** $y-y_1=m(x-x_1)$ $y-x=\dfrac{\mathrm{d}y}{\mathrm{d}x}(x-2)$ $x-y=\dfrac{\mathrm{d}y}{\mathrm{d}x}(2-x)$ (shown)	A1 using gradient A1 using gradient AG
(c)	$(2-x)\dfrac{\mathrm{d}y}{\mathrm{d}x}=x-y$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}+\dfrac{y}{2-x}=\dfrac{x}{2-x}$ [linear form] $\quad -(1)$ Integrating factor $=\mathrm{e}^{\int\frac{1}{2-x}\mathrm{d}x}$ $=\mathrm{e}^{-\ln(2-x)} \qquad (\because x<2)$ $=\dfrac{1}{2-x}$	M1 – DE change to linear form M1 A1
(d)	Multiply (1) in (c) by integrating factor: $\dfrac{1}{2-x}\dfrac{\mathrm{d}y}{\mathrm{d}x}+\dfrac{y}{(2-x)^2}=\dfrac{x}{(2-x)^2}$ $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{y}{2-x}\right)=\dfrac{x}{(2-x)^2}$ $\displaystyle\frac{y}{2-x}=\int\frac{x}{(2-x)^2}\,\mathrm{d}x$ $=\dfrac{2}{2-x}+\ln(2-x)+c$ [from a(ii)] When $x=0$, $y=0$ $\quad\therefore c=-1-\ln 2$	M1 multiply (A1) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{y}{2-x}\right)=\dfrac{x}{(2-x)^2}$ A1 M1 Use (0, 0) to find c

	$\frac{y}{2-x}=\frac{2}{2-x}+\ln(2-x)-1-\ln 2$ $=\frac{2-2+x}{2-x}+\ln\frac{2-x}{2}$ $y=x+(2-x)\ln\frac{2-x}{2}$ (shown)	A1 AG
(e)	$(2-x)\frac{\mathrm{d}y}{\mathrm{d}x}=x-y$ $\frac{\mathrm{d}}{\mathrm{d}x}$: $(2-x)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-\frac{\mathrm{d}y}{\mathrm{d}x}=1-\frac{\mathrm{d}y}{\mathrm{d}x}$ $(2-x)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=1$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{1}{2-x}$ (shown) $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{1}{(2-x)^2}$ When $x=0$, $y=0$ (given), $\frac{\mathrm{d}y}{\mathrm{d}x}=0$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{1}{2}$, $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{1}{4}$ $y=0+0x+\frac{\frac{1}{2}}{2!}x^2+\frac{\frac{1}{4}}{3!}x^3+\dots$ $=\frac{1}{4}x^2+\frac{1}{24}x^3+\dots$	 A1 AG A1 M1 M1 Maclaurin series formula A1
10	**Arithmetic & Geometric Sequences**	**[Maximum mark: 17]**
(ai)	$x_7=x_1+6d \quad \Rightarrow \quad d=\frac{x_7-x_1}{6}$ $x_3=x_1+2d$ $=x_1+2\left(\frac{x_7-x_1}{6}\right)$ $=\frac{2}{3}x_1+\frac{1}{3}x_7$	(M1) AP formula seen here or later, A1 M1 eliminating d A1
(aii)	$x_2+x_4+x_6=(x_1+d)+(x_1+3d)+(x_1+5d)$ $=3x_1+9d$ $=3x_1+9\left(\frac{x_7-x_1}{6}\right)$ $=3x_1-\frac{3}{2}x_1+\frac{3}{2}x_7$ $=\frac{3}{2}(x_1+x_7)$ (shown)	M1 for expressing each term in terms of x_1, and d or x_7, A1 $3x_1+9d$ A1 AG

	OR: $x_2+x_4+x_6=(x_1+d)+(x_1+3d)+(x_1+5d)$ $=3x_1+9d$ $=\frac{3}{2}(2x_1+6d)$ $=\frac{3}{2}(x_1+x_1+6d)$ $=\frac{3}{2}(x_1+x_7)$ (shown)	M1 for expressing each term in terms of x_1, and d or x_7, A1 $3x_1+9d$ A1 AG
(aiii)	$\frac{x_3}{x_1}=\frac{x_7}{x_3} \Rightarrow x_3^2=x_1x_7$ $\left(\frac{2x_1+x_7}{3}\right)^2=x_1x_7$ $4x_1^2+4x_1x_7+x_7^2=9x_1x_7$ $4x_1^2-5x_1x_7+x_7^2=0$ $(4x_1-x_7)(x_1-x_7)=0$ $x_7=4x_1$ or $x_7=x_1$(reject)	M1 GP common ratio M1 using (i) A1 quadratic exp M1 factorise or solving quad A1 accept if $x_7=x_1$ is not rejected
(bi)	$y_1+y_2+y_3 = y_1(1+r+r^2)$ or $\frac{y_1(1-r^3)}{1-r}$	A1
(bii)	$y_1+y_2+y_3=y_1(1+r+r^2)$ $=y_1\left[\left(r+\frac{1}{2}\right)^2-\frac{1}{4}+1\right]$ $=y_1\left(r+\frac{1}{2}\right)^2+\frac{3}{4}y_1$ The minimum value is $\frac{3}{4}y_1$ when $r=-\frac{1}{2}$ **OR:** $\frac{\mathrm{d}}{\mathrm{d}r}\left[y_1(1+r+r^2)\right]=0 \Rightarrow y_1(1+2r)=0$ $r=-\frac{1}{2}$ $\therefore$ min $y_1(1+r+r^2)=y_1\left[1-\frac{1}{2}+\left(-\frac{1}{2}\right)^2\right]=\frac{3}{4}y_1$ (coeff of $r^2=y_1>0$, thus minimum)	 M1 complete the square A1 A1, A1 M1 solving first derivate = 0 A1 $1+2r$ A1 A1

11	**Vectors**	**[Maximum mark: 20]**
(ai)	$2(-1)-5(-1)+3(0)=-2+5=3=\text{RHS}$ $3(-1)+2(-1)-5(0)=-3-2=-5=\text{RHS}$ $\therefore(-1,-1,0)$ satisfies both equations and lies in Π_1 , Π_2 .	M1 A1 AG
(aii)	$\begin{pmatrix}2\\-5\\3\end{pmatrix}\times\begin{pmatrix}3\\2\\-5\end{pmatrix}=\begin{pmatrix}19\\-(-19)\\19\end{pmatrix}=19\begin{pmatrix}1\\1\\1\end{pmatrix}$ $\therefore L$ has equation $\underset{\sim}{r}=\begin{pmatrix}-1\\-1\\0\end{pmatrix}+\alpha\begin{pmatrix}1\\1\\1\end{pmatrix},\quad \alpha\in\mathbb{R}.$	M1 cross product of 2 direction vectors A1 A1
(aiii)	$\begin{pmatrix}1\\-1\\1\end{pmatrix}.\begin{pmatrix}1\\1\\1\end{pmatrix}=\left\|\begin{pmatrix}1\\-1\\1\end{pmatrix}\right\|\left\|\begin{pmatrix}1\\1\\1\end{pmatrix}\right\|\cos(90^\circ-\theta)$ $1-1+1=\sqrt{3}\sqrt{3}\sin\theta$ $\sin\theta=\frac{1}{3}$	M1 dot product A1 A1
(b)	If the 3 planes meet in L, then L must lie in Π_3 . $\Rightarrow L\parallel\Pi_3$ and a point on L is in Π_3 $L\parallel\Pi_3\Rightarrow L\perp$ normal of Π_3 $\Rightarrow\begin{pmatrix}1\\\lambda\\1\end{pmatrix}.\begin{pmatrix}1\\1\\1\end{pmatrix}=0$ $\Rightarrow 1+\lambda+1=0$ $\Rightarrow\lambda=-2$ L is in $\Pi_3\Rightarrow$ P lies in Π_3: $\Rightarrow\begin{pmatrix}-1\\-1\\0\end{pmatrix}.\begin{pmatrix}1\\-2\\1\end{pmatrix}=\mu$ $\Rightarrow(-1)+(-2)(-1)+0=\mu$ $\Rightarrow\mu=1$	M1 dot product = 0 A1 M1 P satisfies π_3 equation A1
(ci)	$\lambda=-2$ and $\mu\neq1$ ($L\parallel\Pi_3$ and a point on L is <u>not</u> in Π_3)	A1, A1
(cii)	**Method 1** Let A(1, 0, 1) be a point in Π_3: $x-2y+z=2$. $\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{OA}=\begin{pmatrix}-2\\-1\\-1\end{pmatrix}$	A1 for point A A1

	The shortest distance is PF $= \left\| \overrightarrow{AP} . \frac{1}{\sqrt{1+4+1}} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \right\|$ $= \frac{1}{\sqrt{6}} \left\| \begin{pmatrix} -2 \\ -1 \\ -1 \end{pmatrix} . \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \right\|$ $= \frac{1}{\sqrt{6}} \left\| -2+2-1 \right\|$ $= \frac{1}{\sqrt{6}}$  	M1 using projection M1 unit vector of normal M1 evaluating dot product A1
(cii)	**Method 2** Refer to above diagram $\quad l_{PF} : \underset{\sim}{r} = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} + s \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, s \in \mathbb{R}$ When l_{PF} intersects π_3, $\quad \left[\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} + s \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \right] . \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = 2$ $-1+2+s(1+4+2) = 2$ $s = \frac{1}{6}$ $\overrightarrow{PF} = \frac{1}{6} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ $\|\overrightarrow{PF}\| = \frac{1}{6}\sqrt{6}$ or $\frac{1}{\sqrt{6}}$	A1 equation of line M1 simultaneous equations M1 evaluating dot product A1 M1 finding $\|\overrightarrow{PF}\|$ A1
(cii)	**Method 3** Let plane $\pi_4 \parallel \pi_3$ and π_4 contain L $\therefore \pi_4$ equation: $\quad \underset{\sim}{r} \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$, ie. $\underset{\sim}{r} \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = 1$ Distance between π_4 & π_3 $= \frac{2}{\left\| \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \right\|} - \frac{1}{\left\| \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \right\|} = \frac{1}{\sqrt{6}}$ (Recall shortest distance from O to $\underset{\sim}{r} \cdot \underset{\sim}{n} = p$ is $\frac{\|p\|}{\|\underset{\sim}{n}\|}$.)	M1 $\overrightarrow{OP} \cdot \underset{\sim}{n}$ M1 evaluate dot product A1 M1 A1 magnitude $= \sqrt{6}$ A1