

Revision for Common Test

- 1 A polynomial $p(x)$ is of degree 3 such that $x = -1$ and $x = 1$ are two of the roots for $p(x) = 0$. If given further that $p(3) = 4p(2)$, show that x is a factor of $p(x)$.
- 2 The function f is defined such that $f(x) = |x - 1| + 2$.
 - (a) Sketch the graph of $y = f(x)$ for $-1 \leq x \leq 4$.
 - (b) Hence, or otherwise, solve for x when $f(x) < g(x)$, where $g(x) = \frac{11 - x}{3}$.
 - (c) Sketch on a separate graph, the function $h(x) = |x - 1|^{\frac{5}{2}} + x$.
- 3 It is given that $px^3 - x^2 + 18x - 7 \equiv (2x - 1)(x + 3)(k - x) + mx - 1$ for some real constants p , k , and m .
 - (a) Explain why p must be -2 .
 - (b) Find the values of k and m .
- 4 Show that both $x - \sqrt{2}$ and $x + \sqrt{2}$ are factors of $g(x) = 2x^3 - 8x^2 - 4x + 16$.
 - (a) Factorise $g(x)$ completely and sketch the graph of $y = g(x)$, showing clearly the intersections with the axes.
 - (b) Solve the inequality $x^3 + 8 > 2x(2x + 1)$.
- 5 A polynomial $P(x)$ is such that $P(x) = px^4 + 4x^3 - 3x^2 + qx - 5$. When $P(x)$ is divided by $x^2 - 1$ the remainder is $5x - 6$.
 - (a) Find the values of p and q .
 - (b) Solve for x when $P(x) = 2x^4 + 7x - 10$.
- 6 Solve the simultaneous equations $e^x(e^2) = (e^2)^y$ and $\lg(x^2 - 1) = \lg y + \lg(2x - y)$.

- 7 Solve for x when:
- (i) $6^x = 5^{x+1}$
 - (ii) $\log_2(x+6) + \log_2(x-1) = 1 + \log_2(3x)$
- 8 It is given that $22x^3 + 15x^2 - 4x - 6$ and $2 + 14x - 12x^2 - 5x^3$ have the same remainder when divided by $x - a$.
- (a) Show that $27a^3 + 27a^2 - 18a - 8 = 0$.
 - (b) By substituting $3a$ by b , or otherwise, solve the above equation in a .
- 9 It is given that $f(x) = ax^3 - (a + 3b)x^2 + 2bx + c$ is exactly divisible by $x^2 - 2x$. When $f(x)$ is divided by $x - 1$, the remainder is 8 more than when it is divided by $x + 1$.
- (a) Find the values of a , b and c . Hence factorise $f(x)$ completely.
 - (b) Solve for x , when $f(x) > 0$.
- 10 A function f is defined by $f : x \mapsto \frac{kx+1}{x}$, $x \neq 0$, where k is a positive real constant.
- (a) Sketch the graph of $y = f(x)$.
 - (b) State the range of f .
 - (c) Find an expression for $f^{-1}(x)$, in terms of k , and state the domain of f^{-1} .
- 11 A function f is defined by $f : x \mapsto e^{-2x} + k$, where k is a positive real constant.
- (a) Sketch the graph of $y = f(x)$.
 - (b) State the range of f .
 - (c) Find an expression for $f^{-1}(x)$, in terms of k , and state the domain of f^{-1} .

12 Solve for x for each of the following:

(i) $\ln(\lg x) = -0.4$

(ii) $\log_3(x-2) = \log_9 729 - \log_3(x+4)$

(iii) $\log_9[2 + \log_2(x-4)] = \log_{25} 5$

13 The mass, m grams, of a radioactive substance X produced as a result of an experiment, present at time t hours after first being observed, is given by the formula $m = 16e^{-0.4t}$.

(a) Sketch the graph of m against t .

(b) Determine the amount of radioactive substance left 5 hours after first being observed.

(c) Determine, also, the time taken, to the nearest 0.1 hour, for the radioactive substance to be reduced to 5% of the original amount.

14 Express $\ln(3e^2) - \frac{1}{2}\ln\left(\frac{9}{e}\right) - \ln\left(\frac{\sqrt{e}}{5}\right)$ in the form $r + \ln s$ where r and s are rational numbers.