



Name: Wan Ren Han (21) Class: 3E1 Date: 2nd Aug

MA303.2.2 – Experimental Data

At the end of this unit, you should be able to

- use linear law to draw straight line graphs using experimental data
- solve for unknown parameters
- identify and correct outliers in experimental data

In experiments, recordings or measurements of any variable usually results in inaccurate data collection. For data points which lie roughly on a straight line, we can quite easily predict the relationship between the variables by determining the equation of the straight line.

E1. In an experiment it is known that y varies directly as x^2 , and when measurements of y are taken with values of x , the recordings are shown below:

x	0.5	1.0	1.5	2.0	2.5	3
y	2.0	8.0	12.0	32.0	50.0	72.0

Since y varies directly as x^2 , a straight line will be revealed if y is plotted against x^2 . Thus we can create a new row of data showing the values of x^2 :

independent
dependent

x	0.5	1.0	1.5	2.0	2.5	3
y	2.0	8.0	12.0	32.0	50.0	72.0
x^2	0.25	1.00	2.25	4.00	6.25	9.00

And by plotting the values of y against the values of x^2 correspondingly, a straight line can be drawn such that it passes through 5 out of the 6 points. The point that does not lie on the straight line is known as an **outlier** (see next page).

Hence, there is an ^{dependent} incorrect reading or measurement in the experiment, which happens when $x = 1.5$, and the incorrect reading is $y = 12.0$. What should be the correct reading? By reading the point on the straight line where $x = 1.5$, we can see that y should be 18.0 and not 12.0.

Can the equation between x and y be found? The following steps are to be done:

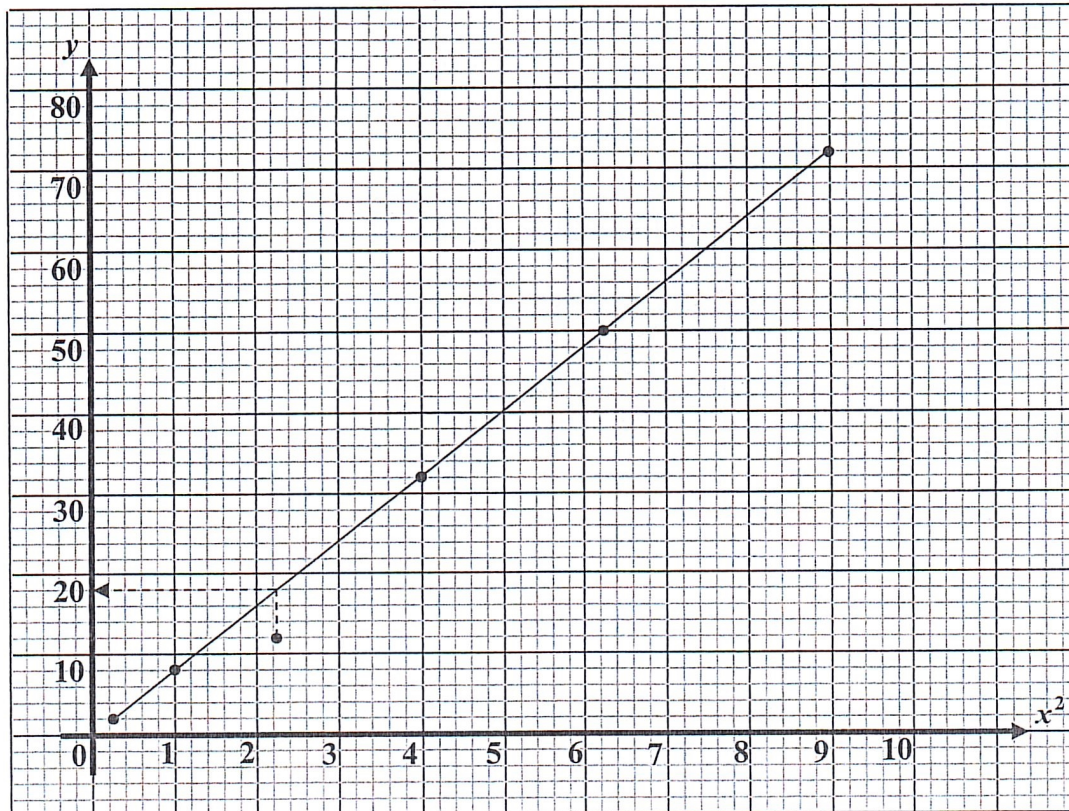
Step 1: Let $Y = y$ and $X = x^2$ to represent the vertical and horizontal axes respectively.

Step 2: The equation of the straight line can be found by taking two points that **lie on the line**. In this case since 5 of the 6 points lie on the line itself, by choosing (1, 8) and (9, 72) the equation is given by

$$\frac{Y - 8}{X - 1} = \frac{72 - 8}{9 - 1} \Rightarrow Y - 8 = 8(X - 1) \Rightarrow Y = 8X$$

Step 3: By replacing back $Y = y$ and $X = x^2$ we can easily see that the equation linking x and y is given by

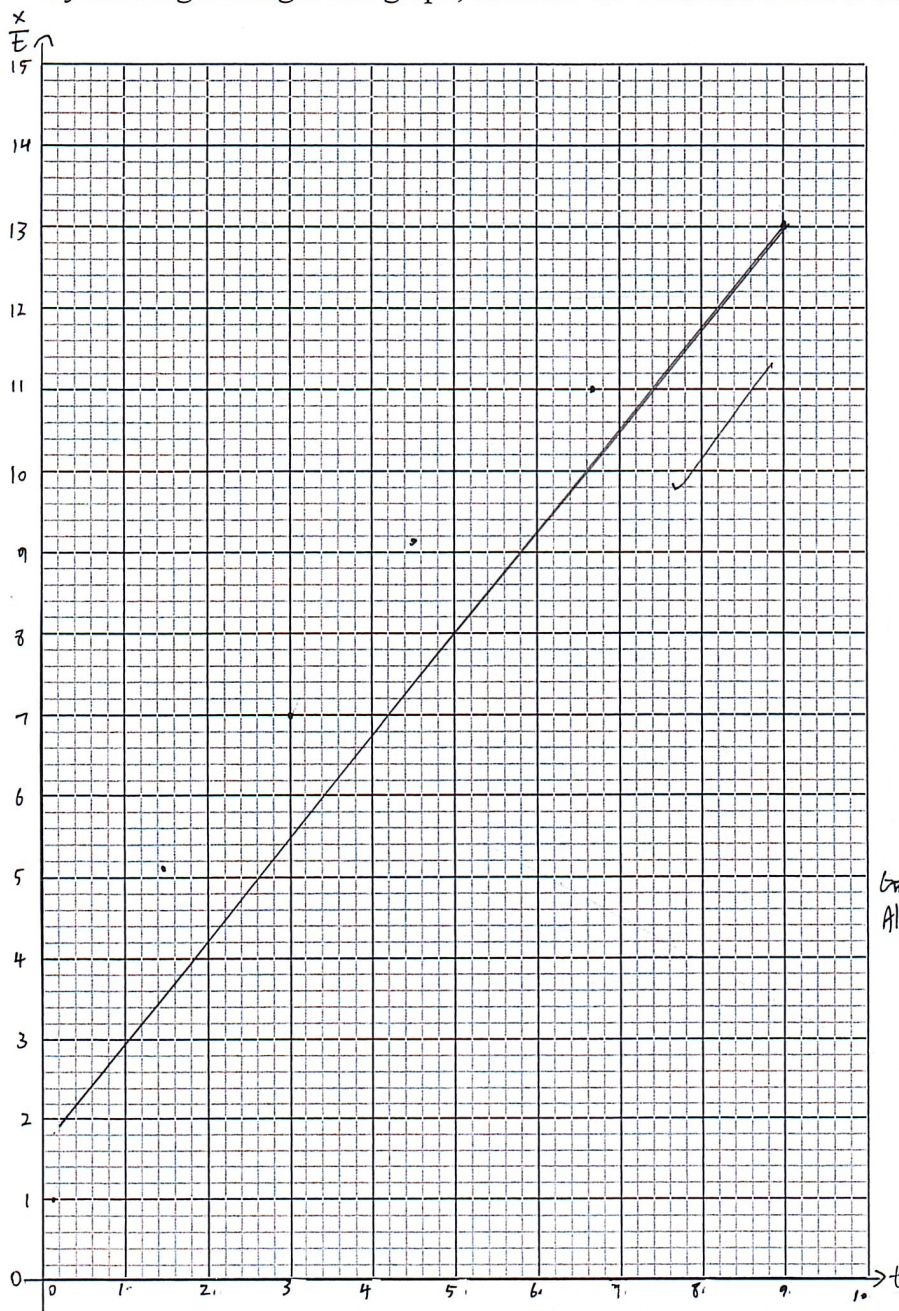
$$y = 8x^2.$$



E2. The table below shows the experimental values of two quantities x and t in which they are related by the equation $x = at^2 - bt$, where a and b are real constants.

t	1	2	3	4	5	6	7
x	1.0	6.1	15.2	27.8	45.6	66.1	90.5
$\frac{x}{t}$	1.00	3.05	5.07	6.95	9.12	11.0	12.9

By drawing a straight line graph, estimate the constants a and b . Show your working clearly.



$$a = \frac{12.9 - 6.95}{7 - 4}$$

$$\approx \frac{11.9}{6.0}$$

Sub $\frac{x}{t} = 6.95$, $t = 4$ into equation

$$\frac{x}{t} = \frac{11.9}{6.0} t - b$$

$$b \approx \frac{5.9}{6.0}$$

Grid = accuracy of data
Always start from 0.

P1. It is given that $y = ax^b$ where a and b are real constant. An experiment was conducted to measure the quantities x and y and the following table is obtained.

x	2	3	4	5	6	7	8
y	6.5	12.5	19.3	26.7	35.3	44.1	54.4

By drawing a straight line graph, estimate to one decimal place, the values of a and b . Hence, estimate the value of y when $x = 5.6$

$$y = ax^b$$

$$\lg y = \lg a + b \lg x$$

x	2	3	4	5	6	7	8
y	6.5	12.5	19.3	26.7	35.3	44.1	54.4
$\lg y$	0.81	1.1	1.3	1.4	1.5	1.6	1.7
$\lg x$	0.30	0.48	0.60	0.70	0.78	0.85	0.90

$$\lg a = 0.45$$

$$\therefore a = 10^{0.45}$$

$$\approx 2.8$$

$$\text{Gradient} = \frac{1.55 - 0.50}{0.80 - 0.30}$$

$$= \frac{1.05}{0.50}$$

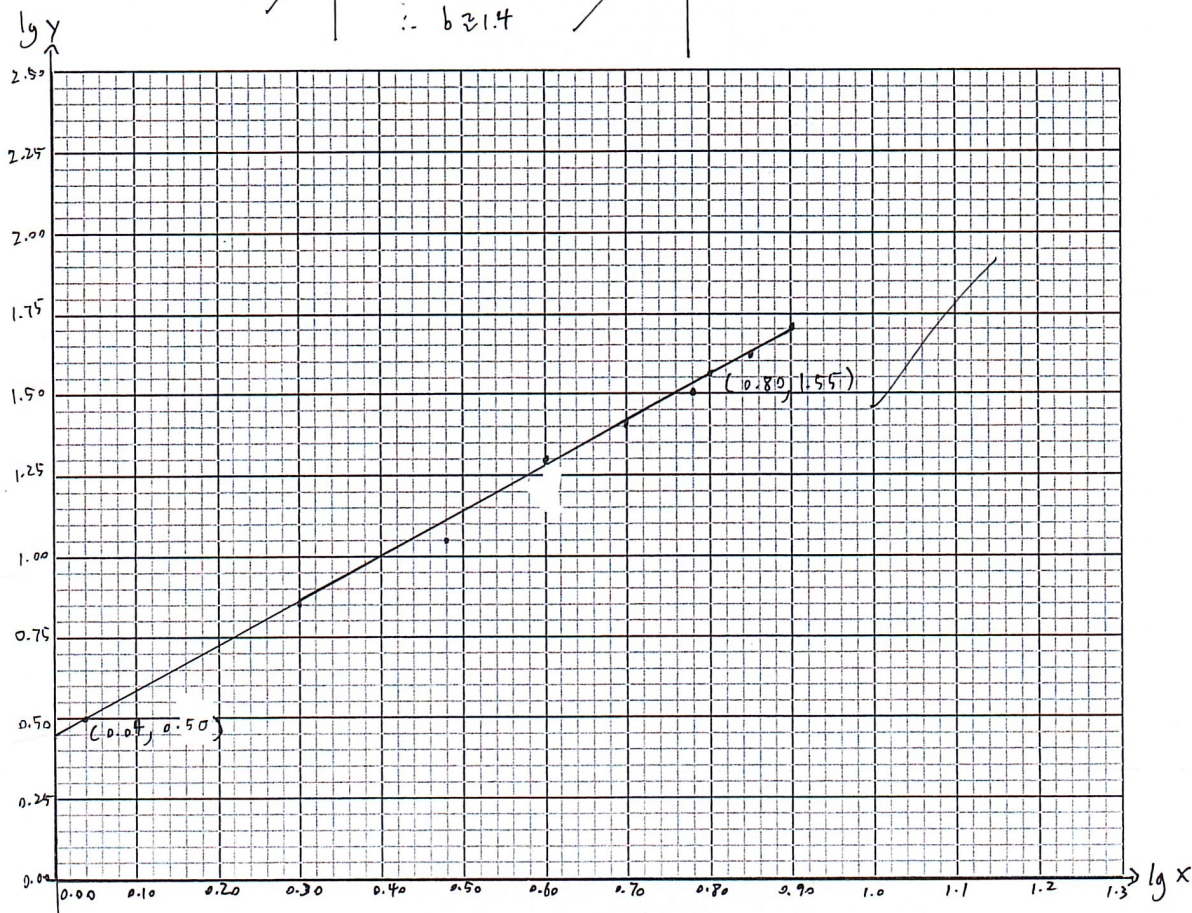
$$\approx 2.1$$

$$\therefore b \approx 2.1$$

$$\text{sub } x = 5.6$$

$$\therefore \lg y = 0.45 + 2.1 \lg 5.6$$

$$y \approx 1.45$$



P2. The table below shows experimental readings of p for given values of q such that $pq = ap - q^2 + b\sqrt{q}$ where a and b are real constants. One of the readings is incorrect.

q	0	1	2	3	4	5	6
p	0	2.9	85.6	-23.5	-13.8	-12.2	-12.0

- (a) By using a scale of 4 cm to 1 unit on the vertical axis and 2 cm to 0.1 unit on the horizontal axis, draw a straight line graph of $\frac{pq+q^2}{p}$ against $\frac{\sqrt{q}}{p}$, and identify the incorrect reading.
- (b) Estimate the values of a and b , and hence estimate the correct reading from (a).

q	0	1	2	3	4	5	6
p	0	2.9	85.6	-23.5	-13.8	-12.2	-12.0
$\frac{pq+q^2}{p}$	undefined	1.34	2.05	2.61	2.84	2.95	3.00
$\frac{\sqrt{q}}{p}$	undefined	0.345	0.017	-0.074	-0.145	-0.183	-0.204

Wrong reading: $p = 85.6$

$$pq = ap - q^2 + b\sqrt{q}$$

$$\frac{pq+q^2}{p} = \frac{b\sqrt{q}}{p} + a$$

$$\text{Gradient} = \frac{2.85 - 1.5}{-0.16 - 0.31}$$

$$0.147Y = -1.35X + 1.12$$

$$Y \approx -2.87X + 2.38$$

$$\therefore b \approx -2.9$$

$$a \approx 2.4$$

$$\text{Sub } q=2$$

$$2p = 2.4p - 4 - 4.1$$

$$-0.4p = -8.1$$

$$p = 20.3$$

The correct reading is 20.3 .

