Chapter 5

Work, Energy and Power

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"There is a fact, or if you wish, a law governing all natural phenomena that are known to date. There is no known exception to this law – it is exact so far as we know. The law is called the *__conservation of energy__*.

It states that there is a certain quantity, which we call "energy" that does not change in the manifold changes that nature undergoes. That is a most abstract idea, because it is a mathematical principle; it says there is a numerical quantity which does not change when something happens.

It is not a description of a mechanism, or anything concrete; it is a strange fact that when we calculate some number and when we finish watching nature go through her tricks and calculate the number again, it is the same.

It is important to realize that in physics today, we have no knowledge of what energy "is." We do not have a picture that energy comes in little blobs of a definite amount. It is not that way. It is an abstract thing in that it does not tell us the mechanism or the reason for the various formulas."

Richard Feynman
The Feynman Lectures on Physics (Volume 1)

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Videos of Lecture Examples can be found at
https://youtube.com/playlist?list=PL_b5cjrUKDlYIoMzYi383KRNFmeWtS5Mh

H2 Physics Syllabus 9749

Topic 5: Work, Energy and Power

Content

- Work
- Energy conversion and conservation
- Potential energy and kinetic energy
- Power

Learning Outcomes

Candidates should be able to:

(a) define and use work done by a force as the product of the force and displacement in the direction of the force.

(b) calculate the work done in a number of situations including the work done by a gas which is expanding against a constant external pressure: $W = p\Delta V$ **(*will be covered in Thermal Physics)**

(c) give examples of energy in different forms, its conversion and conservation, and apply the principle of energy conservation.

(d) show an appreciation for the implications of energy losses in practical devices and use the concept of efficiency to solve problems.

(e) derive, from the equations for uniformly accelerated motion in a straight line, the equation $E_k = ½\, mv^2$.

(f) recall and use the equation $E_k = ½\, mv^2$.

(g) distinguish between gravitational potential energy, electric potential energy and elastic potential energy.

(h) deduce that the elastic potential energy in a deformed material is related to the area under the force-extension graph.

(i) show an understanding of and use the relationship between force and potential energy in a uniform field to solve problems.

(j) derive, from the definition of work done by a force, the equation $E_p = mgh$ for gravitational potential energy changes near the Earth's surface.

(k) recall and use the equation $E_p = mgh$ for gravitational potential energy changes near the Earth's surface.

(l) define power as work done per unit time and derive power as the product of a force and velocity in the direction of the force.

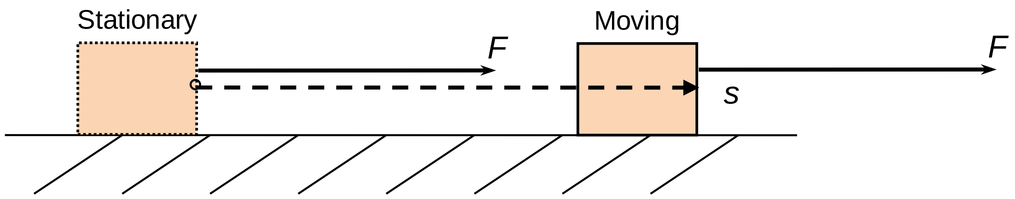
5.1 Work

The meaning of ‘work’ in physics differs from everyday usage. Work done by a force acting on a body refers to the amount of energy that was transferred to the body by the agent that exerts the force.

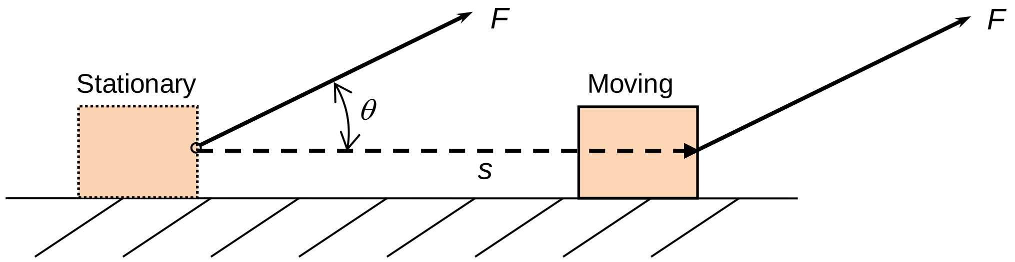
A person holding a heavy book at rest may feel that he is doing work as he feels tired after a while but, although he is exerting a force on the book, there is no energy transferred to the book as a result and hence the work done on the book by him is zero.

5.1.1 Work done by a constant force

Consider a constant force F acting on a body over a period of time, during which the object has a displacement s in the direction of the force.



The work done BY the force ON the body is defined as the **product of the magnitude of the force *F* and the displacement *s* in the direction of the force.**



For the general case when the displacement is not in the direction of the constant force, the work done by the force is the product of the displacement and the component of the force in the direction of the displacement, or, equivalently, the product of the force and the component of the displacement that is in the direction of the force. As can be seen below, both have the same mathematical expression.

$$W = F\,s_{/\!/} = F\,(s \cos \theta)$$

Alternatively $$W = F_{/\!/}\,s = (F \cos \theta)\,s$$

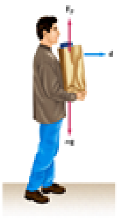
where F is the magnitude of constant force
s is the displacement
θ is the angle between the force and displacement
**subscript $_{/\!/}$ refers to its vector component parallel with respect to some other vectors*

Notes:

- Work is a **scalar** quantity. The unit of work is the **joule** (J).
- It is assumed that the object is *not rotating*, or *changing shape* (i.e. it is rigid), so all parts of the object move with the same displacement. In this case, the work done by a force can be calculated by using a point to represent the object, with the force acting on that point
- Where there are more than one force acting on a body, work done by the individual forces can be calculated separately.

Concept Check 1

As the man is walking around at a constant speed and holding up the load, is any work being done on the load? Explain why.



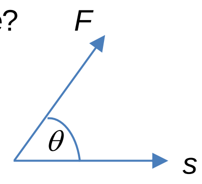
Since F and s are perpendicular.

Work done by Man exerting force F on the load **= F s cos 90° = 0 J**

Similarly work done by gravitational force is also zero.

Concept Check 2

Under what circumstance is the work done by a force acting on a body negative?



$W = Fs\cos\theta$

Work W is negative

when $180^{\circ} > \theta > 90^{\circ}$, $\cos\theta$ *is negative.*

e.g. When $\theta = 180^{\circ}$, $\cos\theta = -1$

Qn: What does this mean physically? The force is resisting the motion and doing negative work to reduce the kinetic energy of the body.

Example 1

A lawn mower is pushed by a force P of 20.0 N at 30° to the horizontal, against a horizontal resistive force D of 8.0 N. It moves a horizontal distance of 10.0 m. Calculate

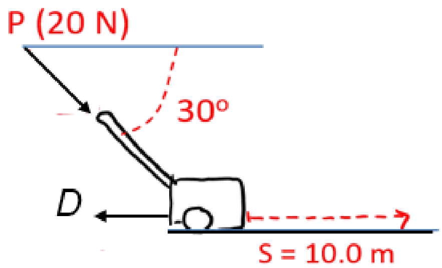
(i) the work done by the force P on the lawn mower.

(ii) the work done by the resistive force D.

(iii) the work done by the normal contact force from the ground and the weight of the lawn mower.

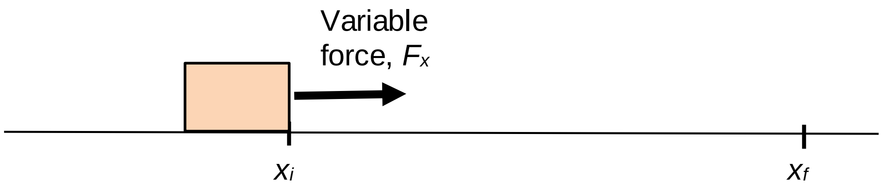
(iv) the change in kinetic energy of the mower.

[173 J, - 80.0 J, 0 J, 93 J]

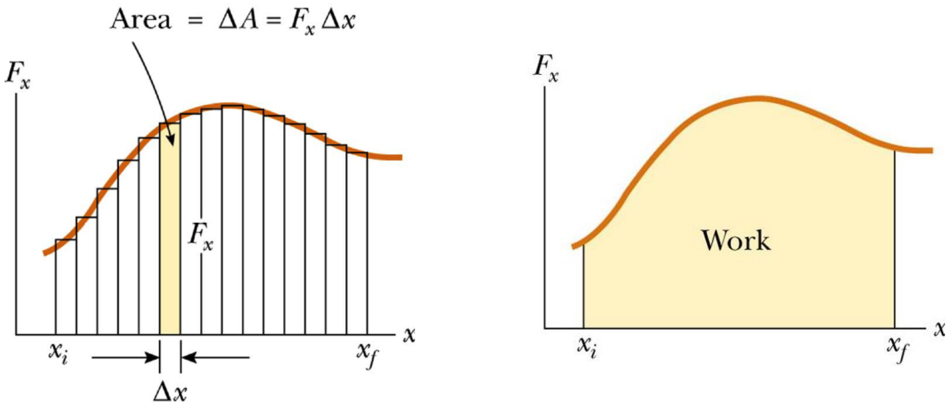


5.1.2 Work done by a variable force

Suppose an object is being displaced along the x-axis under the action of a variable force F_x that acts in the x direction. The object is displaced in the direction of increasing x from $x = x_i$ to $x = x_f$.



In order to calculate work done by the ***variable force*** F_x, we first need to know how the force F_x vary with displacement. Let's imagine we were able to use a force sensor and a data logger to measure this variation and that we were able to get the plot shown below.



The total displacement x can be regarded as the sum of many very small steps Δx. During each small step, the force is practically constant.

The total work done = sum of work done for each step

$$W = \Delta W_1 + \Delta W_2 + \Delta W_3 + \ldots\ldots$$

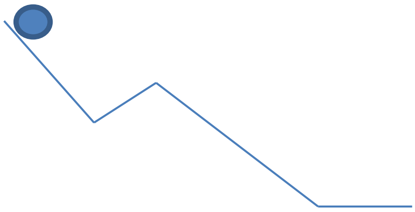
$$= F_1\Delta x + F_2\Delta x + F_3\Delta x + \ldots..$$

As the width of the small steps, Δx, gets really small, the sum of area of these strips will just be the area under the graph of F_x against x.

$$W = \int_{x_i}^{x_f} F_x \, dx$$

In other words, **the work done by variable force acting on an object that undergoes a displacement is equal to the area under the force–displacement graph**.

5.2 Energy



Suppose a ball is allowed to roll down a hill and we would like to calculate its speed by the time it reaches the bottom of the hill. The ball does not follow a straight path as it is not a straight slope. It will be very tiresome to try and calculate the speed of the ball using what we learnt in kinematics.

An alternative and fast approach to solving this problem is to apply the principle of conservation of energy. If we ignore friction, there are only two forms of energy we need to concern ourselves with, the kinetic energy of the ball by virtue of its motion and the gravitational potential energy stored in the ball-earth system.

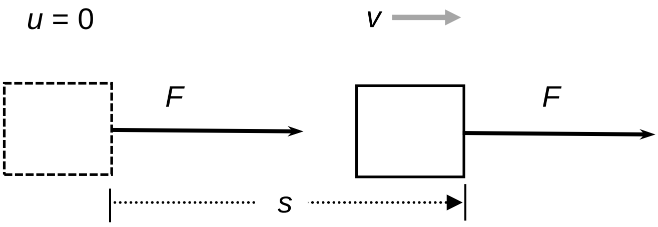
The ball might gain or lose some of its kinetic energy as it rolls down the slope but if we realise that all its kinetic energy gained must be balanced by a corresponding decrease in the gravitational potential energy of the ball-earth system (assuming the earth is stationary), we could easily find the kinetic energy it had at the bottom of the slope by simply comparing the change in the gravitational potential energy of the ball-earth system.

5.2.1 Kinetic energy, E_k

Kinetic energy of a body is a measure of the energy possessed by the body **by virtue of its motion.**

Derivation of $E_k = \frac{1}{2}mv^2$

Consider an object, initially at rest, is being displaced along the *x*-axis under the action of a constant force,



We can infer the formula for kinetic energy from the amount of work that is done by an external force to bring a body from rest to its state of motion.

$$E_k = F s$$

By Newton's second law: $E_k = (ma)s$

Using the equation of motion for uniform acceleration: $v^2 = u^2 + 2as$,

Re-arrange: $as = \frac{1}{2}\left(v^2 - u^2\right)$

Since $u = 0$, $as = \frac{1}{2}v^2$

Thus, the kinetic energy of the object gained through work done by the constant force:

$$E_k = m(as)$$
$$= m\left(\frac{1}{2}v^2\right)$$
$$\therefore\ E_k = \frac{1}{2}mv^2$$

Work- Energy Theorem (good to know)

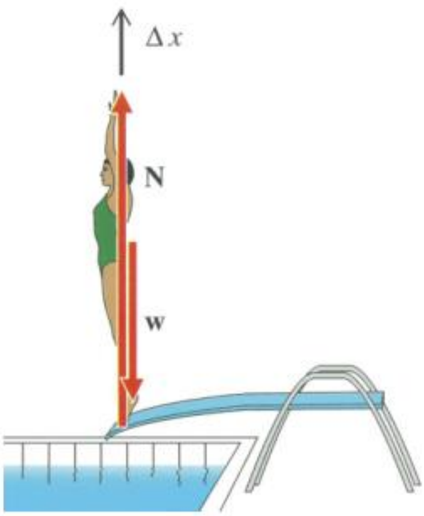
More generally, if the object does not start from rest ($u \neq 0$), and is acted by several forces, then the net work done on the object (i.e. the work done by the net force),

$$W_{net} = mas = m \times \frac{1}{2}\left(v^2 - u^2\right) = E_{k\ final} - E_{k\ initial}$$

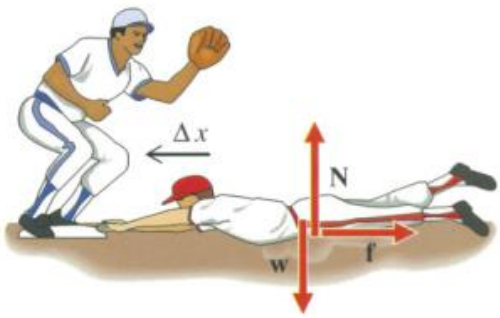
$$\boxed{\therefore W_{net} = \Delta E_k}$$

When there is no net work done on an object (i.e. work done by net force is zero), the object's change in kinetic energy is zero. Referring back to Concept Check 1 (p.5), no work is done on the load, so its kinetic energy remains constant. In Fig.(a), A diver's kinetic energy increases as she springs upward because positive work is done on her, since normal contact force **N** is greater than her weight **w**. In Fig. (b), a baseball player's kinetic energy decreases as he slides because friction does negative work on him.

Work-energy theorem is valid even for varying net force. One useful example is when an object at rest is pushed from one position to another position, back to rest, we conclude that the net work done is zero since there is no change in kinetic energy in the process.



(a) $\Delta K = W_{net} > 0$



(b) $\Delta K = W_{net} < 0$

5.3 Potential energy

We know that two bodies will tend to attract each other due to gravity. This means that if we want to separate the two bodies, we need to exert forces on them. The energy we use to do this is stored in the two-body system as gravitational potential energy. If we now release the two bodies, they will begin to move towards each other. The gain in the kinetic energies of the two bodies comes from the gravitational potential energy that was stored earlier.

Similarly, we know that two positively charged bodies tend to repel each other due to electrostatic forces. We can try to force them together, and in so doing, we need to exert forces on them and the energy we expend is stored in the form of electrical potential energy of the two-body system. If we now release the two bodies, they will repel each other and move apart at increasing speed. The gain in their kinetic energies comes from the electrical potential energy that is stored earlier.

The same principle applies to a spring. When loading a spring gun, we expend energy in compressing the spring. This energy is stored in the form of elastic potential energy and when fired, the spring exerts a force on the bullet and the work done by this force, which is the gain in the bullet's kinetic energy, came from the stored elastic potential energy.
In all these examples, the amount of stored potential energy in the system came from the work done by an external agent who set up the system in the first place.

There are three kinds of potential energy (PE) you will commonly encounter. They are summarized in the table below.

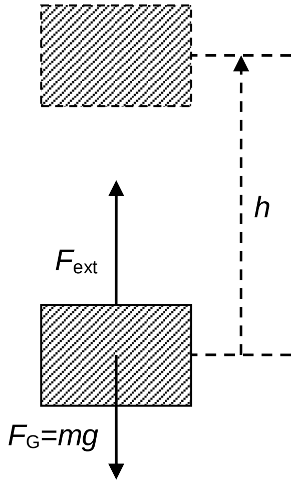
Elastic PE	Gravitational PE (GPE)	Electrical PE
The amount of elastic PE stored in a spring depends on the extent to which the spring is stretched or compressed.	The amount of GPE stored in a system depends on the relative position of the masses in the system.	The amount of EPE stored in a system depends on the relative position of electrical charges in the system.
When a spring is stretched, attractive forces act to pull the parts of the spring together and when the spring is compressed, repulsive forces act to push the parts of the spring apart. Elastic PE increases when the spring is stretched or compressed.	Attractive forces act between any two masses. In a two-mass system, GPE increases when two masses are separated.	Attractive forces act on two bodies of opposite charges and repulsive forces act on two bodies of like charges. In a two charged body system, EPE increases when two opposite charges are separated or when two like charges are brought closer together

5.3.1 Gravitational Potential Energy (GPE), E_p or U

Derivation of $E_p = mgh$

Consider a person lifting a box of mass m at constant speed to a height h.

Since the box is moving at constant speed, there is no change in its kinetic energy. Net work done on the box must be zero. In the process, however, the person exerting a constant upward force on the box does positive work on the box while the gravitational force acting downwards does negative work. The work done by the person on the box is stored in the form of gravitational potential energy (GPE), E_p.



Since velocity is constant,

there is no acceleration $F_{net} = ma = 0$, thus $F_{ext} = F_G = mg$.

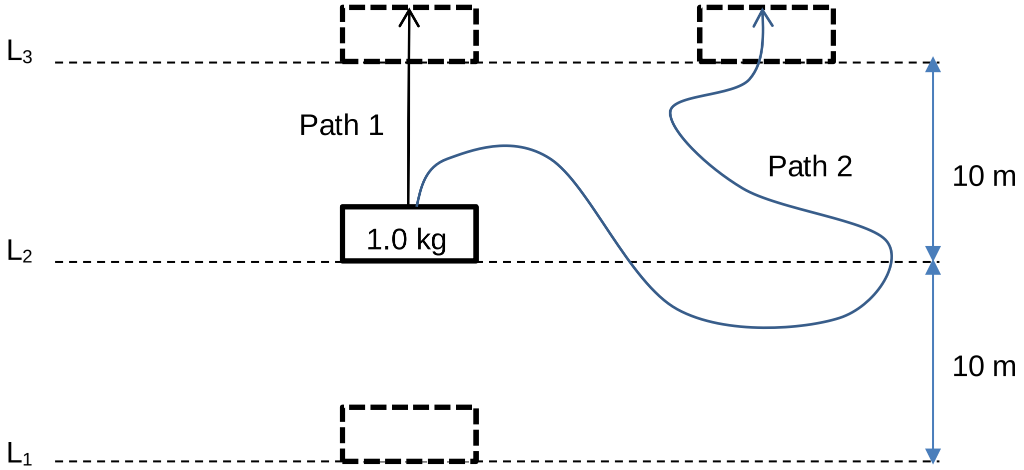
We can form an equation as follows.

Increase in E_p = Work done by external force on the box by person, W
$= F_{ext}\, h$
$= mgh$

Hence, Increase in $E_p = mgh$

Note:

- This derivation applies only to objects that stay near to the surface of the Earth during the motion, and h is small relative to the diameter of the Earth, so that the gravitational acceleration g on the body is practically constant during the motion. (
- The potential energy is associated with the gravitational force between the Earth and the mass m. Hence the gravitational potential energy E_p belongs to the mass-Earth system and not simply to the mass m alone. It is a property of the system as a whole. In general, a system is one or more objects that we choose to study.
- E_p is a relative and not an absolute quantity. It is always taken in relation to a reference level which can be chosen arbitrarily. What is more important is the change in the E_p with reference to this level (as shown in the figure below). Consider the change in potential energy ΔE_p of a 1.0 kg mass along path 1 and 2.



Choosing L_2 as the reference level, $\Delta E_p = (1.0)(9.81)(10) - (1.0)(9.81)(0) = 98.1$ J
Choosing L_1 as the reference level, $\Delta E_p = (1.0)(9.81)(20) - (1.0)(9.81)(10) = 98.1$ J

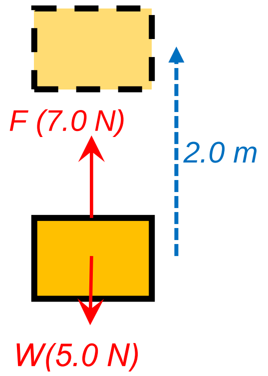
- The *work done on the mass* in moving it between two positions *is dependent only on the initial and final positions* and *independent of the path taken by the mass*.

  ΔE_p remains the same for path 1 and 2. When the starting and ending points are the same, the total work is zero, and there is no change in the gravitational potential energy.

Example 2

A mass of weight 5.0 N is lifted by a vertical force of 7.0 N through a vertical distance of 2.0 m.

(a) Calculate the work done by the lifting force.
(b) Calculate the work done by the gravitational force.
(c) Calculate the change in gravitational potential energy.
(d) Calculate the total work done on the mass.
(e) Assuming it was initially at rest. calculate the final speed of the mass.

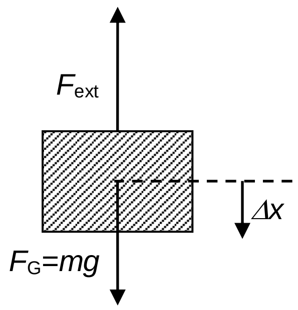


[+14 J, -10J, +10J, 4J, 4.0 m s^{-1}]

5.3.2 Relationship between Gravitational Force and Gravitational Potential Energy

Consider the same box on p.10, we define the change in GPE as the work done by the external force on the box. The change in GPE can also be expressed as *the negative of the work done on the mass by gravitational force*.

$$\Delta E_p = -W_G$$



Consider a small downward displacement, Δx, at constant velocity so that $F_{ext} = F_G$.
the work done by the gravitational force, $W_G = F_G\,\Delta x$ is positive.
GPE decreases in the process.

Thus $\Delta E_p = -W_G = -F_G\,\Delta x \Rightarrow F_G = -\dfrac{\Delta E_p}{\Delta x}$

When we take the limit as $\Delta x \to 0$, $\boxed{F_G = -\dfrac{dE_p}{dx}}$

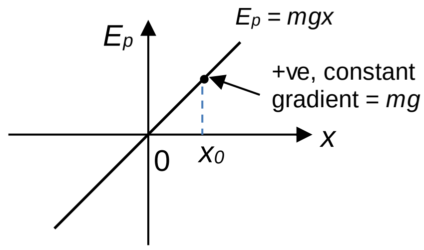
***the gravitational force is the negative of the gradient of the potential energy - position graph*.**

The magnitude of gradient gives the magnitude of the gravitational force, while the negative sign suggests that the gravitational force always acts in the direction towards lower potential energy.

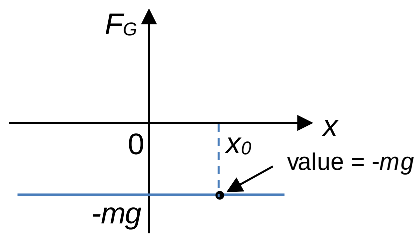
Let us now apply the above relationship in *the uniform gravitational field* near the surface of Earth, where gravitational acceleration is g.

Consider a mass m at a height x_0 from the ground, taking GPE = 0 J at the ground level.

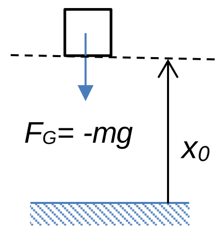
Since GPE, $E_p = mgx$, the E_p - x graph is a straight line graph passing through the origin, with gradient mg.



GPE decreases as x decreases.
Gravitational force F_G at the point x_0 is -ve of the gradient of E_P – x graph, implying F_G is a constant value acting in the direction of decreasing GPE.



For all x, $F_G = -mg$ (constant)



At x_0, $F_G = -mg$ (constant) is directed towards decreasing potential energy.

The <u>gravitational force</u> at x_0 is the <u>negative of the gradient of E_p -x graph</u>, or $F_G = -\dfrac{dE_p}{dx} = -mg$.

In general, the force-potential energy relationship is applicable in any conservative[1] system.

Conservative force, $F = -\dfrac{dE_p}{dx}$ is the negative of the gradient of the E_p -x graph.

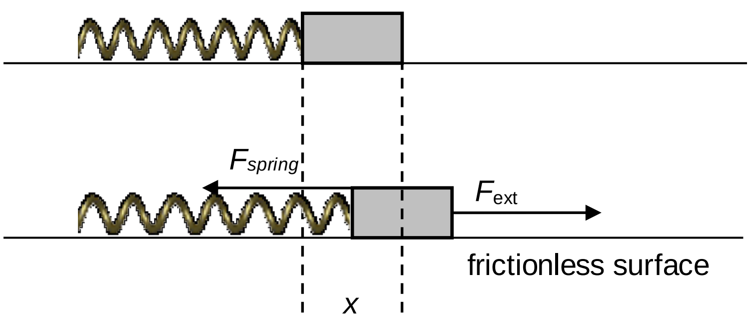
We will re-visit this relationship when we study the chapters on Gravitational Field and Electric Field later. In those chapters, potential energy is often denoted by the symbol U. You will encounter situations in which you are given a potential energy - distance graph and you can use this general relationship to find the corresponding force.

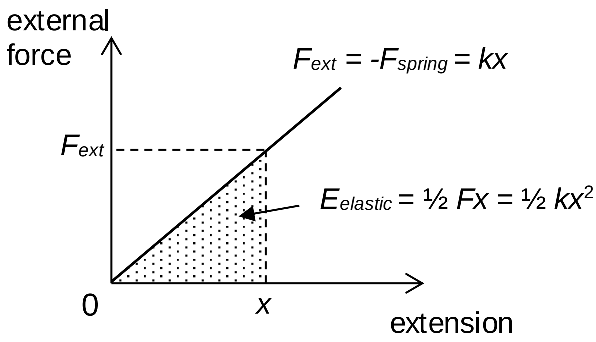
[1] See Appendix 1 for more details regarding conservative system and conservative force

5.3.3 Elastic Potential Energy

In order to stretch a spring, we must exert an increasingly larger force to pull both ends of the spring apart as we stretch the spring. This is because the force we exert must at least be equal to if not larger than the resistive spring force that the rest of the spring exerts on the ends of the spring. Indeed, the resistive spring force gets stronger the more the spring is extended or compressed.

In fact, in most situations the resistive spring force needed to stretch the spring **increases in proportion to the extension**. Whenever this is true, we say that **the spring obeys Hooke's law, i.e. the spring force F_{spring} is proportional to the spring's extension x,** i.e. $F_{spring} = -kx$ where k is the spring constant. The negative sign appears because F_{spring} is in the direction opposite to the displacement x. A stiffer spring that requires a larger force to stretch it has a larger spring constant. A stiffer spring also means that more elastic potential energy is stored per unit of extension.





Let a light spring be stretched through an extension x, very slowly (at practically zero velocity) by applying an external force F_{ext}.

To just overcome the spring force, the external force $F_{ext} = -F_{spring}$.

$$W = \int_0^x F_{ext}\ dx \quad = \text{Area under the } \textit{force–extension graph}$$

$$= ½\ F\ x$$

$$= ½\ (k\ x)\ (x)$$

$$E_{elastic} = ½\ kx^2$$

Hence, elastic potential energy $E_{elastic} = \frac{1}{2}Fx = \frac{1}{2}kx^2$

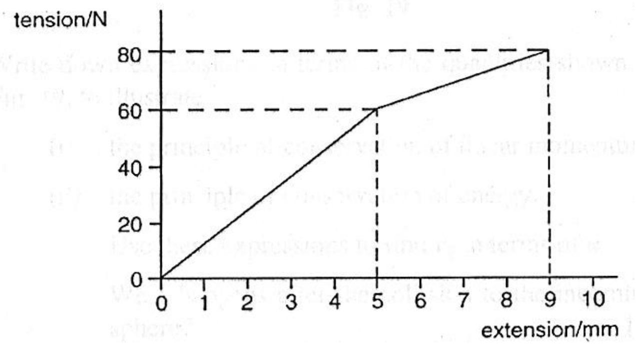
Note, the spring force is a conservative force.

$F_{spring} = -\dfrac{dE_{elastic}}{dx}$ is applicable.

Verifying: $F_{spring} = -\dfrac{d}{dx}\left(\dfrac{1}{2}kx^2\right) = -kx$

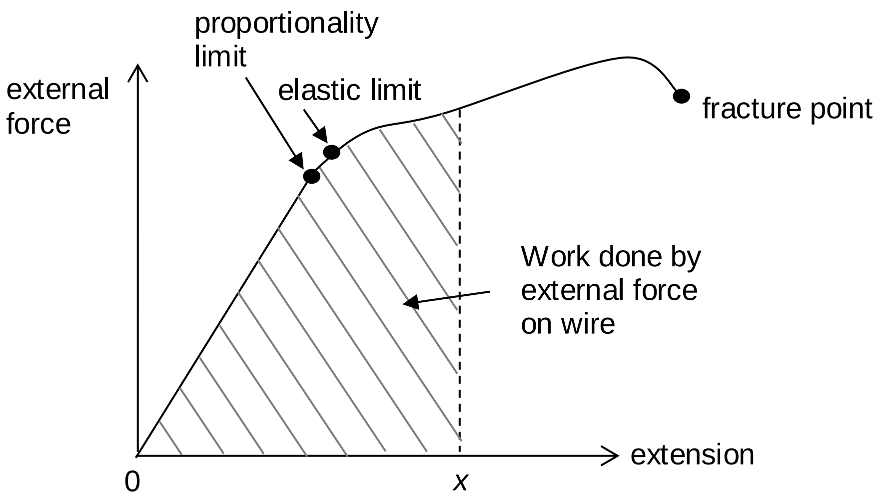
<u>Example 3 (mod J99/I/23)</u>

A sample is placed in a tensile testing machine. It is extended by known amounts and the tension is measured. (i) What is the work done in stretching the sample to the first point where it no longer obeys Hooke's Law. (ii) What is the work done on the sample when it is given a total extension of 9.0 mm?
[0.15 J, 0.43 J]



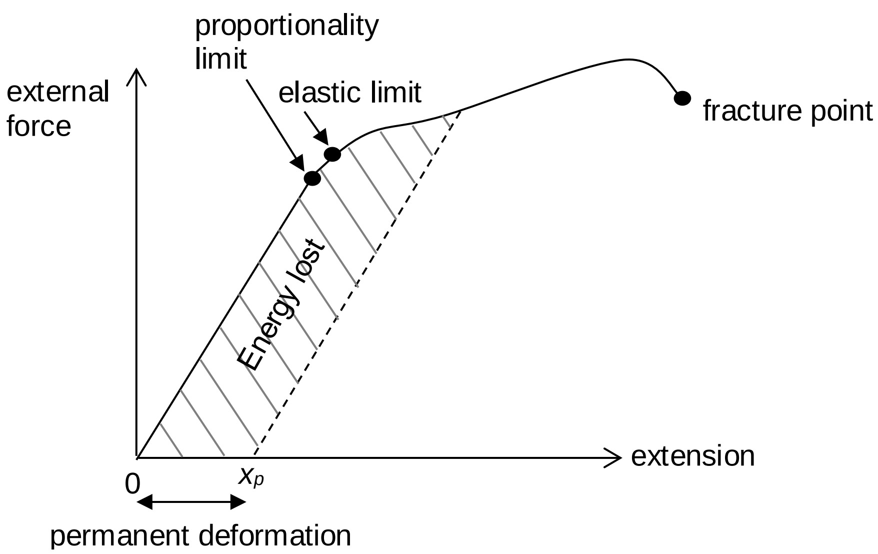
5.3.4 Work done by External Force on a Deformed Material

An elastic material (e.g. a wire) usually obey Hooke's Law closely unless the deformation is excessively large. If the material is stretched beyond its proportionality limit, it will no longer obey Hooke's Law and the force-extension graph will not be a straight line from that point onwards. If an elastic material (e.g. a wire) is stretched further **beyond its elastic limit** (also known as yield point), it will **suffer a permanent deformation** and not snap back to its original length when the external force is removed. If the wire is stretched even further, it will eventually hit fracture point and the wire will simply snap. The force-extension graph will look like the one shown in the figure below.



In such cases, to determine work done by external force on wire, one cannot simply use the equation $E = ½\, kx^2$ since Hooke's Law is no longer obeyed. Instead, the work done can simply be worked out by estimating the area under the force-extension graph (shaded area).

When the wire is **stretched beyond its elastic limit** and the external force is subsequently removed, the extension is only partially recovered. Work done by external force on wire will be more than the elastic potential energy. Energy is therefore lost as heat.



Try this out ! The effect of hysteresis is usually very small for metals, but is noticeable for polythene, glass and rubber. You can easily investigate this using a rubber band. By simply stretching it and then holding it against your lips you can detect a rise in temperature!

5.4 Principle of Conservation[2] of Energy

When solving problems via the energy approach, remember that this approach is based on the principle of conservation of energy:

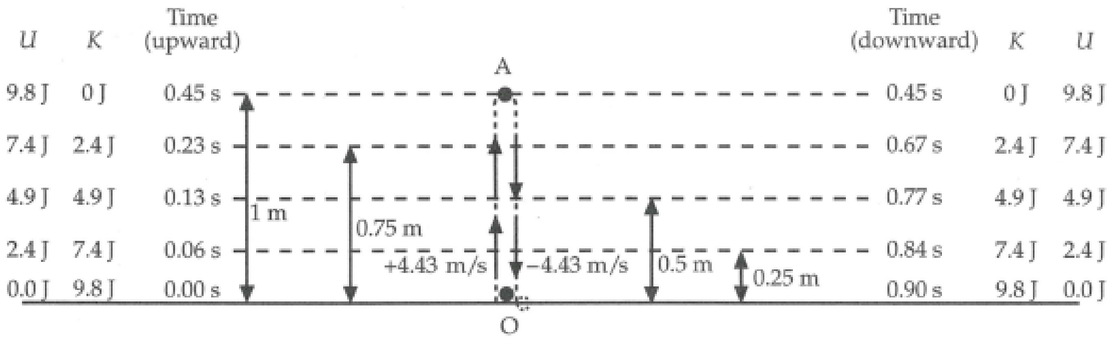
Energy can be **converted from one form to another** but **cannot be created or destroyed**.

In other words, the total energy is neither increased nor decreased in any process.

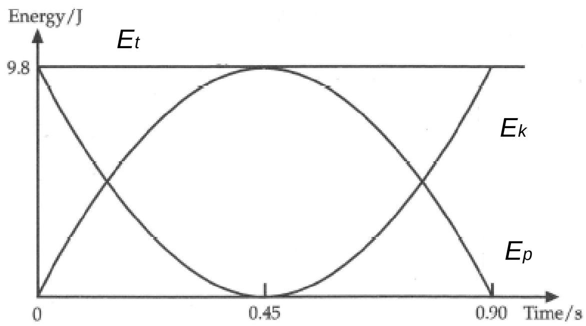
- **When the only force that does work is gravitational force in the system, the total mechanical energy *E* (i.e. sum of kinetic energy and potential energy) is constant.**

  Mathematically, $(E_k + E_p)_{\text{initial}} = (E_k + E_p)_{\text{final}}$ **or** $\Delta E_k + \Delta E_p = 0$

  Example: Consider the motion of a 1.0 kg mass projected vertically upwards from the ground with an initial speed of 4.43 m s^{-1}. Given that air resistance is negligible, the kinetic energy of the mass is converted to gravitational potential energy as it rises from 0 to A. As it falls back to 0, potential energy is converted into kinetic energy.



  A graph of energy against time is as shown. At any point of time, the total mechanical energy E_t, that is the sum of E_k and E_p, is the same.



- **When there is presence of external forces doing work in the system, change in mechanical energy = net work done by external forces,** W_{ext}.

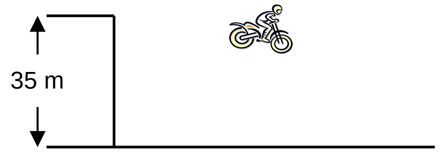
  Mathematically, $(E_k + E_p)_{\text{initial}} + W_{\text{ext}} = (E_k + E_p)_{\text{final}}$ **or** $\Delta E_k + \Delta E_p = W_{\text{ext}}$

Note that an external force can be doing positive work, adding to the total mechanical energy, or negative work, reducing the total mechanical energy of the system.

[2] A quantity that always has the same value is called a conserved quantity.

<u>Example 4 (no external force, mechanical energy is conserved)</u>

(a) A motorcyclist drives horizontally off a cliff at a speed of 38.0 m s^{-1}. Ignoring air resistance, find the speed of the motorcycle just before it reaches the ground.
[46.2 m s^{-1}]



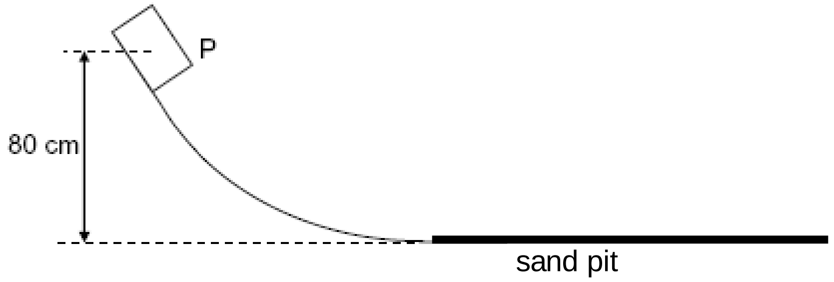
(b) Would your answer in (a) change if the motorcyclist leaps across at the same 35 m height with an initial upward angle while maintaining the same initial speed of 38.0 m s^{-1}?

<u>Example 5 (external force present, mechanical energy is lost)</u>

A car of mass 800 kg moving at 30 km h^{-1} along a horizontal road is brought to rest by a constant retarding force of 5000 N. Calculate the distance the car moves before coming to rest. [5.6 m]

<u>Example 6 (external force present, mechanical energy is lost)</u>

A block of mass 4.0 kg is released at point P on a curved frictionless track as shown. The block slides down a vertical distance of 80 cm and enters a sand pit. Determine the distance the block can move along the sand pit before coming to rest if the friction acting on the block is 85 N. [0.37 m]



5.5 Power

The same amount of work is done in raising a given body through a given height, regardless whether it takes one second or one year to do so. However, *the rate at which work is done* is greater if less time is taken.

Power is defined as the rate at which work is done.

$$\text{Instantaneous power, } P = \frac{dW}{dt}$$

$$\text{Average power, } \langle P \rangle = \frac{\Delta W}{\Delta t}$$

Notes:

- The SI unit of power is the **watt** (W). Power is a **scalar** quantity.
- Work can also be expressed in units of (power x time). This is the origin of the term *kilowatt-hour*. One *kilo-watt hour* is the work done in 1 hour by an agent working at a constant rate of 1 kW.

Example 7 (Average Power)

A 1.10 x 10^3 kg car, starting from rest, accelerates for 5.00 s. The magnitude of the acceleration is 4.60 m s^{-2}. Determine the average power generated by the net force that accelerates the vehicle. [5.82 x 10^4 W]

5.5.1 Relationship between Power, Force and Velocity

Consider a force F that acts on a body for a small time interval Δt. The body moves a small displacement Δx in the direction of the force.

Work done by the force F during Δt, $\quad \Delta W = F\,\Delta x$

Power delivered by that force F during the time interval Δt

$$\begin{aligned} P &= \Delta W / \Delta t \\ &= (F\,\Delta x) / \Delta t \\ &= F(\Delta x/\Delta t) \\ &= Fv \end{aligned}$$

where v is the instantaneous velocity of the body.

When a force F acts on a body that is moving with velocity v, in the direction of the force, work is done on the body at the rate given by $P = Fv$

Notes:

- A force F that acts in the direction of the displacement Δx, does positive work on the body. It transfers energy to the body at a rate proportional to the speed of the body.
- If F or v varies with time, then P may not be constant. However, the product Fv at any moment will gives the instantaneous power of F at that instant.
- Where F acts against the motion of the body, it does negative work on the body. It extracts energy out of the body at a rate equal to the magnitude of Fv.

<u>Example 8</u>

A 1000 kg car experiences a total resistive force of 2000 N moving with a constant velocity of 20.0 m s^{-1} along a straight horizontal road.
What is the power developed by the engine of the car when it travels at 20.0 m s^{-1}
(a) horizontally, and
(b) up a slope with gradient of 5°?
[40 kW; 57.1 kW]

5.6 Efficiency (of energy conversion), η

No practical engine can transform energy from one form to another without some energy going to internal energy and other non-useful forms of energy.

The efficiency of any energy conversion process is the ratio of

- the useful energy output to the total energy input, or
- the useful power output to the power input

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}} \quad , \quad \eta = \frac{E_{output}}{E_{input}} = \frac{P_{output}}{P_{input}}$$

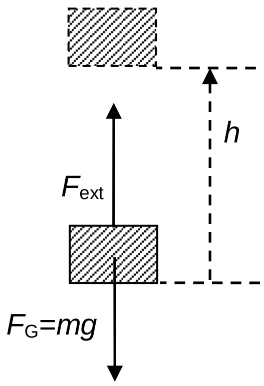
Note:
η (Greek small letter, Eta) is a ratio between two quantities of the same type. It has no unit.

Example 9
A small motor is used to raise a weight of 2.0 N at constant speed through a vertical height of 80 cm in 4.0 s. The efficiency of the motor is 20 %. What is the electrical power supplied to the motor? [2.0 W]

Appendix 1 Potential Energy (U) and Conservative Force (Good to know)

From the derivation on p. 10, the ***gain in gravitational potential energy*, Δ*U*** can be taken as:

the positive work done on the mass by an external force when the mass moves (a distance h) to higher position.



$\Delta U = F_{ext}\, h$

or:

the negative of the negative work done on the mass by gravitational force when the mass moves to a higher position.

$\Delta U = -(-F_G\, h)$

The above is applicable only to a conservative system, or a system that involves a conservative force, such as gravitational force, electric force and spring force. An essential feature of conservative forces is that their work is always reversible. Another important aspect of conservative forces is that a body may move from point 1 to point 2 by various paths, but the work done by a conservative force is the same for all of these paths. Thus, if a body stays close to the surface of the earth, the gravitational force mg is independent of height, and the work done by this force depends only on the change in height. If the body moves around a closed path, ending at the same point where it started, the total work done by the gravitational force is always zero.

The work done by a conservative force always has four properties:
1. It can be expressed as the difference between the initial and final values of a potential-energy function.
2. It is reversible.
3. It is independent of the path of the body and depends only on the starting and ending points.
4. When the starting and ending points are the same, the total work is zero.

When the only forces that do work are conservative forces, the total mechanical energy $E = E_k + E_p$ is constant.

Not all forces are conservative. Consider the friction force acting on the crate sliding on a ramp. When the body slides up and then back down to the starting point, the total work done on it by the friction force is not zero. When the direction of motion reverses, so does the friction force, and friction does negative work in both directions. The lost kinetic energy cannot be recovered by reversing the motion or in any other way, and mechanical energy is not conserved. There is no potential-energy function for the friction force.

Friction is a non-conservative force that acts to reduce the mechanical energy in a system.

However, non-conservative forces do not always reduce the mechanical energy; a non-conservative force changes the mechanical energy, so a force that increases the total mechanical energy, like the force provided by a motor or engine, is also a non-conservative force.

In general,

When there are only conservative forces,

Change in total mechanical energy of the system = 0

When there are presence of conservative forces and non-conservative forces,

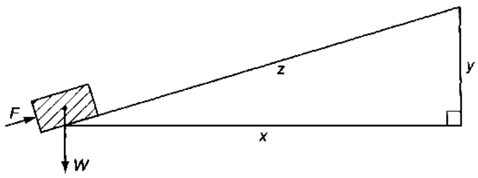
Change in total mechanical energy of the system

= work done on the system by all non-conservative forces

Topic 5 Work Energy Power
Self Review Questions

Work and Energy

S1 **[H12007P1Q14]** A mass of weight W rests on a rough slope of dimensions x, y and z as shown.



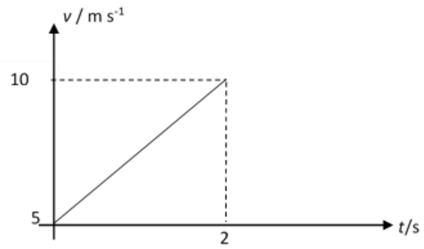
A force F pushes the mass all the way up the slope. The mass starts and finishes at rest.

How much work is done by the force F?

A Wy **C** Wz

B $Wy+Fz$ **D** Fz

S2 **[NYJC 2011 Prelim P1 Q10]** The graph shows the variation with time of the velocity of a body when it is acted on by a force on a smooth horizontal surface.



If mass of the body is 2.0 kg, the work done by the force on the body is

A 25 J **C** 75 J

B 50 J **D** 100 J

S3 **[H2 2011 P1Q10]** A wire is stretched elastically by a force of 200 N, causing an extension of 4.00 mm. The force is then steadily increased to 250 N. The wire still behaves elastically.

How much extra work is done in producing the additional extension?

A 0.225 J **C** 225 J

B 0.250 J **D** 250 J

Conservation of Energy

S4 **[H1 2007P1Q15]** A toy car is released from rest at the top of a length of flexible track. The track is shaped differently for three trials X, Y and Z.

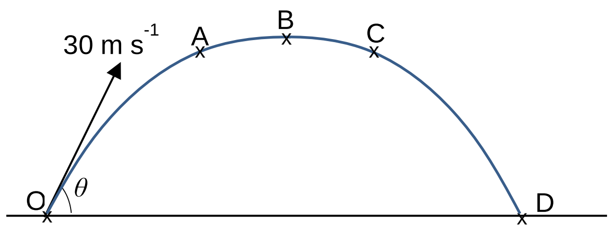
In each trial, the top of the track is the same vertical height above the ground. The speeds v_X, v_Y and v_Z at the end of the track are measured. Friction is negligible.

Which of the following is correct?

A v_X, $= v_Y = v_Z$ **C** $v_Y > v_Z > v_X$

B v_X, $> v_Y > v_Z$ **D** $v_Z > v_Y > v_X$,

S5 On a horizontal plane, a 2.0 kg particle is fired from point O at an angle of 60° above the horizontal with an initial speed of 30 m s^{-1}. When the particle is at point B, it is at its maximum height from the ground. Assuming that the effect of air resistance is negligible, at which point on the particle's trajectory is its kinetic energy minimum?



S6 **[H1/2012/1/15]** A car of mass 1000 kg travels up a hill at constant speed. It gains 87 m in height travelling a distance of 500 m. The total frictional force is 580 N. What is the work done by the driving force?

A 0.85 MJ **C** 4.90 MJ

B 1.14 MJ **D** 5.19 MJ

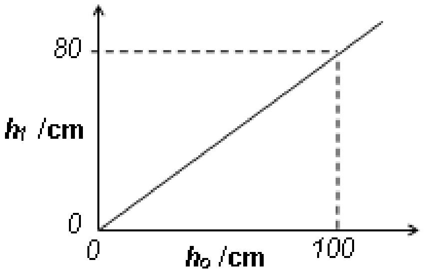
S7 **[J77/2/30]** 500 kJ of heat was produced when a vehicle of total mass 1600 kg was brought to a rest on a level road. The speed of the vehicle just before the brakes were applied was

A 0.625 m s^{-1} **D** 62.5 m s^{-1}

B 0.79 m s^{-1} **E** 625 m s^{-1}

C 25 m s^{-1}

S8 **[N81/P1/Q2]** A ball released from a height h_o above a horizontal surface rebounds to a height h_1 after one bounce. The graph that relates h_o to h_1 is shown.



If the ball (of mass *m*) was dropped from an initial height *h* and made three bounces, the kinetic energy of the ball immediately after the third impact with the surface was

A $(0.8)^3\, mgh$

B $(0.8)^2\, mgh$

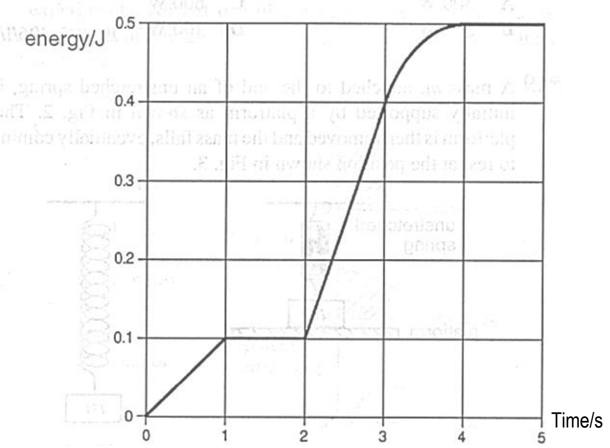
C $0.8\, mg\,(h/3)$

D $[1 – (3 \times 0.2)\, mgh]$

E $[1 – (0.8)^3\, mgh]$

Power

S9 **[J94/1/6]** A bicycle dynamo is started at time t = 0 s. The total energy transformed by the dynamo during the first 5 seconds increases as shown in the graph.



What is the maximum power generated at any instant during these first 5 seconds?

A 0.10 W

B 0.13 W

C 0.30 W

D 0.50 W

S10 **[NYJC/2007/Prelim/P1/Q10]** To travel at constant speed, a car engine provides 25 kW of useful power. The driving force on the car is 600 N. At what speed does it travel?

A 2.5 m s^{-1}

B 4.2 m s^{-1}

C 25 m s^{-1}

D 42 m s^{-1}

S11 **[H1/N2009/1/12]** The drag force acting on a car moving at velocity *v* through still air is proportional to v^2. When the car is travelling at 20 m s^{-1} on a level road, the power required to overcome the drag force is 4800 W.

What power is required when the car travels at 25 m s^{-1}?

A 6000 W

B 7500 W

C 8000 W

D 9400 W

S12 **[J93/1/6]** A small metal sphere of mass *m* is moving through a viscous liquid. When it reaches a constant downward velocity *v*, which of the following describes how the kinetic energy and gravitational potential energy of the sphere changes with time?

A K.E.: constant and equal to $\frac{1}{2} mv^2$
G.P.E.: decreases at a rate of mgv

B K.E.: constant and equal to $\frac{1}{2} mv^2$
G.P.E.: decreases at a rate of $(mgv - \frac{1}{2} mv^2)$

C K.E.: constant and equal to $\frac{1}{2} mv^2$
G.P.E.: decreases at a rate of $(\frac{1}{2} mv^2 – mgv)$

D K.E.: increases at a rate of mgv
G.P.E.: decreases at a rate of mgv

E K.E.: increases at a rate of mgv
G.P.E.: decreases at a rate of $(\frac{1}{2} mv^2 – mgv)$

Discussion Questions

Work and Energy

D1 **[2008/P3/Q1(part)]** Fig 6.1 shows the variation with displacement d of the force *F* applied to an object. *F* and *d* are in the same direction. On Fig 6.2, draw a graph showing the variation with *d* of the work done.

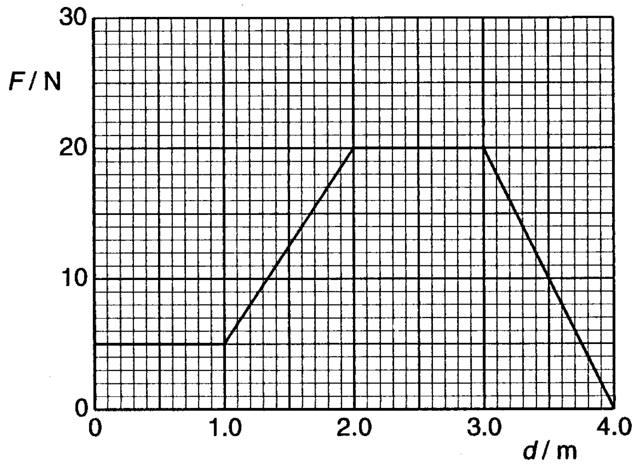


Fig. 6.1

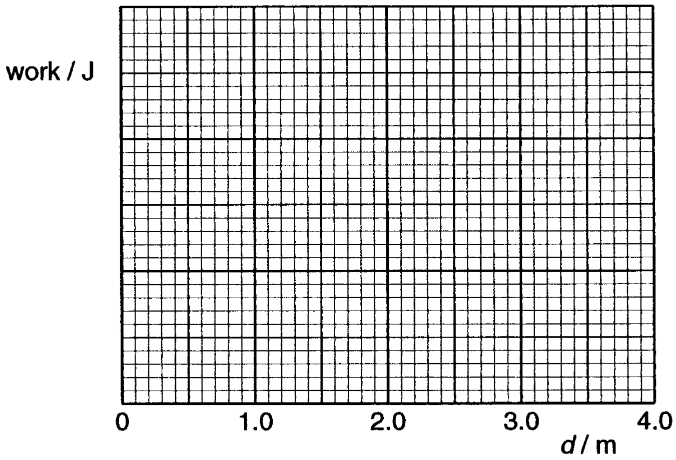
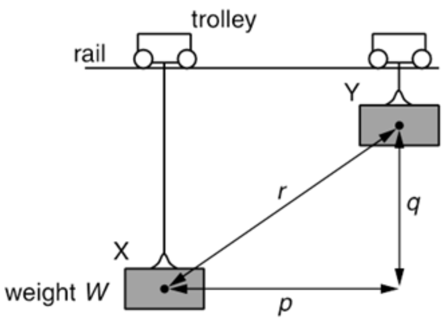


Fig. 6.2

D2 **[CIE June 2003]** A weight *W* hangs from a trolley that runs along a rail. The trolley moves horizontally through a distance *p* and simultaneously raises the weight through height *q*.

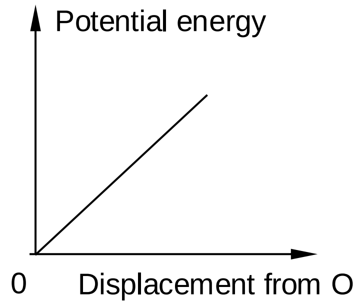
As a result, the weight moves through a distance *r* from X to Y. It starts and finishes at rest. How much work is done on the weight by the trolley during this process?



A *Wp* **B** *W(p+q)* **C** *Wq* **D** *Wr*

D3 The graph shows the way the potential energy of a body varies with its displacement from a point O.

Which feature of the graph means that the force on the body is directed towards O? Explain your answer.



A The graph is linear.

B The graph passes the origin.

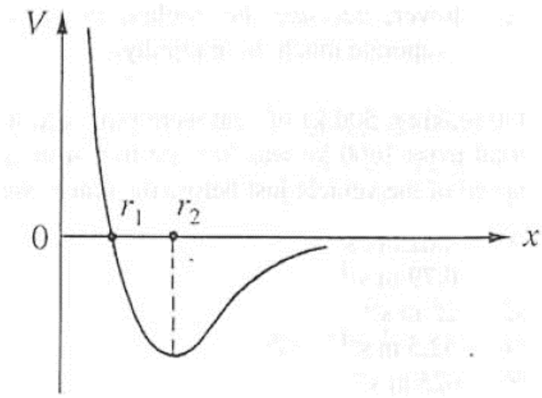
C The potential energy increases as the body moves further from O.

D The value of the potential energy is always positive.

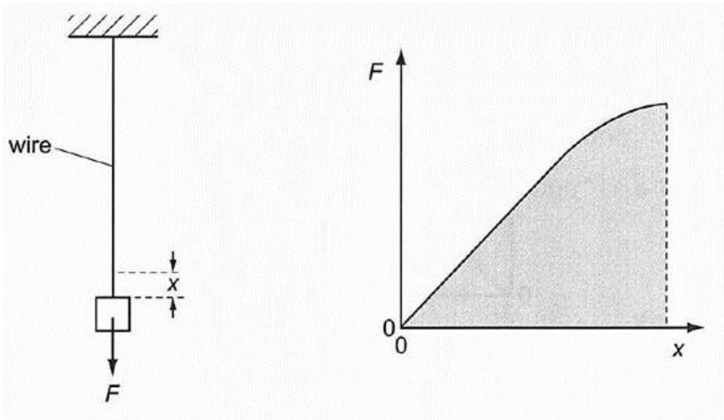
D4 [J8911/6] The potential energy V of two molecules separated by distance x is shown in the diagram.

Which of the following correctly describes the force between the molecules?

	attractive for	repulsive for
A	$x < r_1$	$x > r_1$
B	$x > r_1$	$x < r_1$
C	$x < r_1$	$x > r_2$
D	$x < r_2$	$x > r_2$
E	$x > r_2$	$x < r_2$



D5 **[H1 2009 P1 Q9 & 2015 P1 Q11]**) A wire, fixed at its upper end, is subjected to an increasing load *F* by increasing the mass attached to its lower end. A graph of *F* against the extension *x* of the wire is shown.



The wire is stretched so that it is permanently deformed. What does the shaded area on the graph represent?

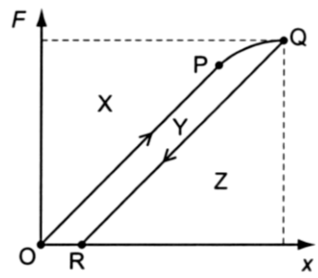
A the amount of elastic potential energy stored in the wire

B the amount of heat produced in the wire

C the loss of gravitational potential energy of the mass

D the work done by *F* on the wire

D6 **[H1 2010 P1 Q11]** A metal wire is stretched by a varying force F, causing its extension x to increase as shown by the line OPQ on the graph. The force is then gradually reduced to zero and the relation between the force and extension is indicated by the line QR.

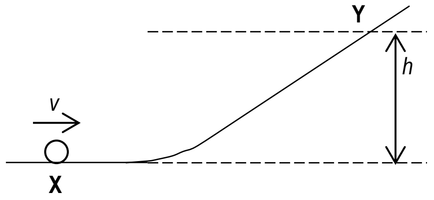


What area represents the elastic potential energy in the wire at Q?

A X **B** Y **C** Z **D** Y + Z

Conservation of Energy

D7 **[J1987/P1/Q5]** An object of mass m passes point X with velocity v and slides up a frictionless incline to stop at point Y which is at a point h above X.

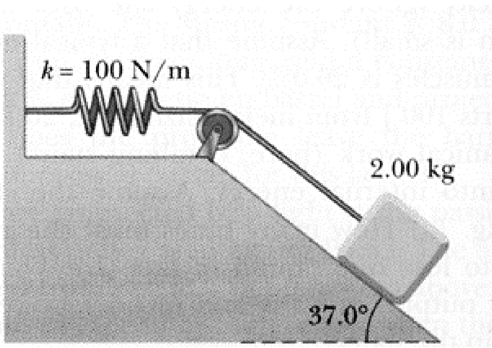


A second object of mass ½ m passes X with a velocity ½ v. To what height will it rise?

A ¼ h **B** ½ h **C** $\frac{1}{\sqrt{2}}h$ **D** h **E** $h\sqrt{2}$

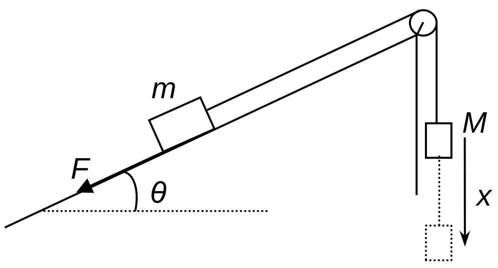
D8 **[College Physics by Serway & Faughn]**

A 2.00 kg block situated on a rough incline is connected to a spring of negligible mass having a spring constant of 100 N m^{-1}. The block is released from rest when the spring is un-stretched and the pulley is frictionless. The block moves 20.0 cm down the incline before coming to rest. Ignore air resistance.



Find the average resistive force due to friction between the block and the incline. [3]

D9 **[J88/1/6 modified]** A mass m moves on a rough plane inclined at an angle θ to the horizontal. It experiences a constant frictional force F. Mass M is attached to it by a light inelastic cord running over a smooth pulley. During motion, the mass M is allowed to fall a vertical distance x, causing m to move up the plane as shown in the diagram.



(a) How much heat is generated by the friction in this process?

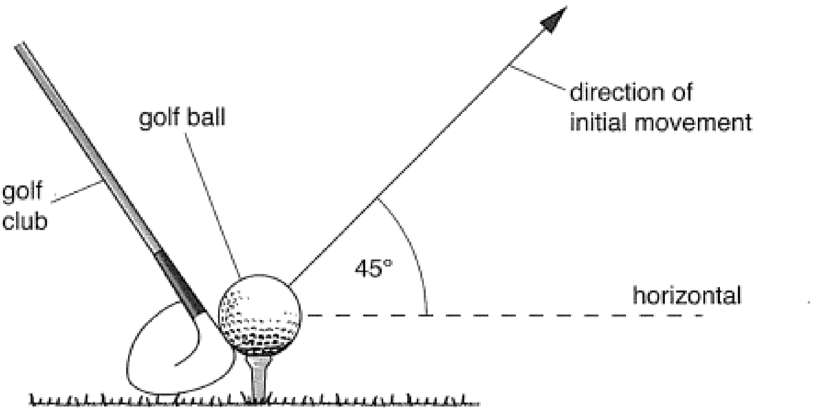
A Fx **C** $Mgx \sin\theta$ **E** $Mgx \sin\theta + Fx$

B mgx **D** $Mgx \sin\theta - Fx$

(b) Determine the expression for the total kinetic energy gained by the two masses. Express your answers in terms of *m, M, F, g, x and θ.*

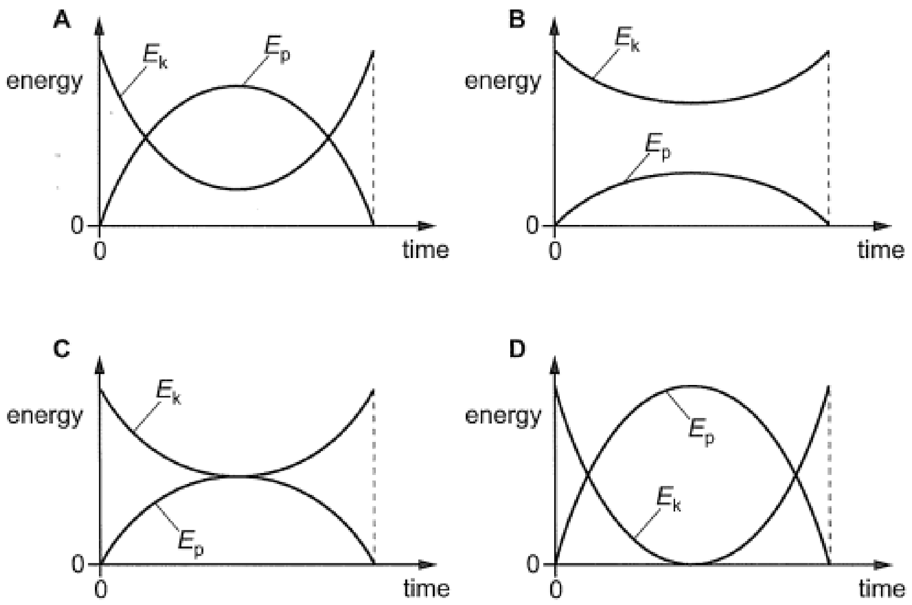
D10 [2016 Specimen P1Q6]

A golf ball, on level ground, is hit and starts to move at an angle of 45° to the horizontal.



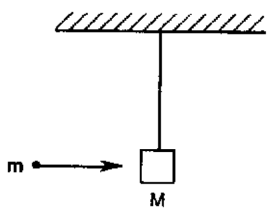
Which graph shows how the kinetic energy E_k and the gravitational potential energy E_p of the ball vary from the time of impact until just before it reaches the ground?

Ignore any effects of air resistance.



D11 [HCI Promo 1999/1/6]

A bullet of mass m = 10 g is fired into a ballistic pendulum of mass M = 2.0 kg as shown. The bullet remains in the block after the collision and the whole system rises to a maximum height of 20 cm.



Find the initial speed of the bullet in m s^{-1}.

A 2.0 **B** 28.0 **C** 398 **D** 719

D12 **[N2012/1/10, modified]** The top end of a spring is attached to a fixed point and a mass of 4.2 kg is attached to its lower end. The mass is released and after bouncing up and down several times it comes to rest at a distance 0.29 m below its starting point. Which row gives the gain in the gravitational potential energy of the mass E_p and the gain in the elastic potential energy of the spring E_s?

	E_p / J	E_s / J
A	–12	+12
B	–12	+6
C	+12	+12
D	+12	+6

D13 **[H1 2009 P2 Q7]** Fig 7.1 shows a man doing a bungee jump. The man has a mass of 75 kg and falls a distance of 41 m before the elastic rope attached to him starts to exert any force on him.

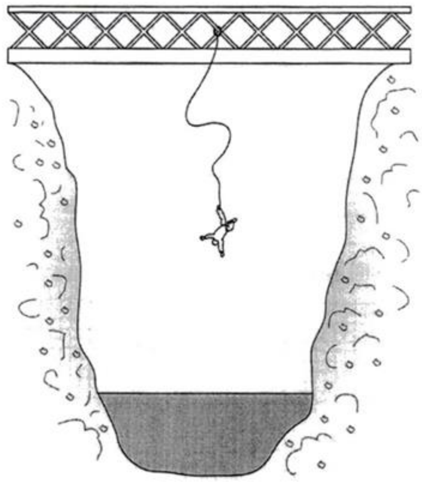


Fig. 7.1

(ai) Calculate the theoretical time taken for the fall of this distance. [2]

(ii) The actual time for this fall is 2.9 s. State the deduction you can make by comparing the actual time with your answer to (i). [1]

(b) A force-extension graph for the elastic rope used for the bungee jump is shown in Fig 7.2.

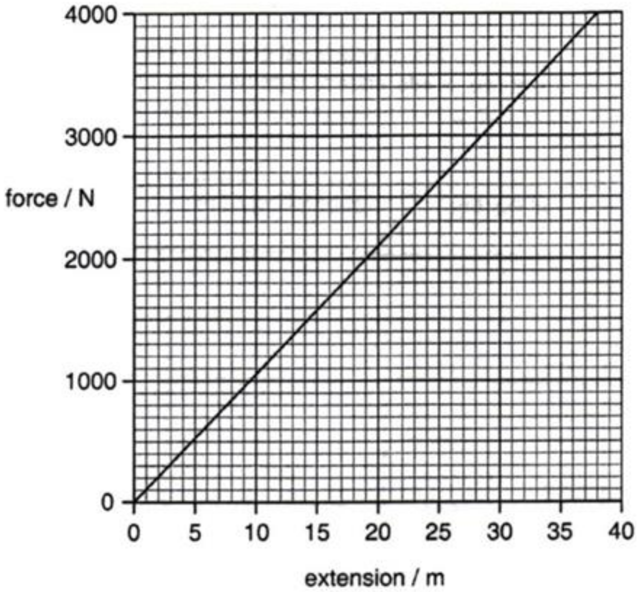


Fig. 7.2

The total distance of fall for the man before he stops for the first time is 73 m. Deduce

(i) the extension of the rope when the man stops for the first time. [1]

(ii) the elastic potential energy stored in the rope at this time. [2]

(ci) Complete the table below to show how the gravitational potential energy and the kinetic energy of the man and the elastic potential energy stored in the rope change with the distance fallen.

	at the top	after falling 41 m	after falling 73 m
GPE/J			0
EPE/J			
KE/J			

(ii) Calculate

1. how far the man has fallen from the top when he has maximum kinetic energy [2]

2. the maximum kinetic energy of the man during the fall [3]

(iii) Use your values to sketch three graphs on Figure 7.4 showing how the three different types of energy vary with distance fallen. Label each graph. [4]

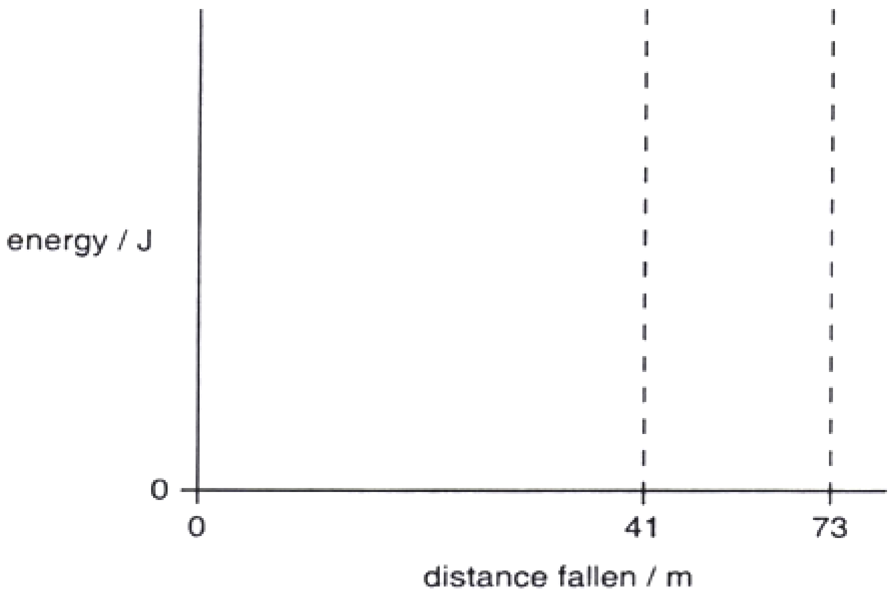
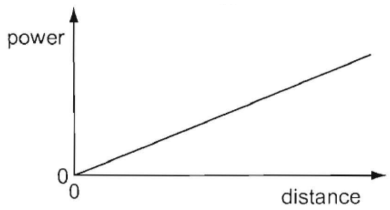


Fig. 7.4

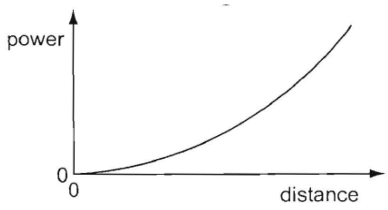
Power

D14 **[H2 2010 P1Q11]** A vehicle starts from rest and accelerates uniformly. Which graph shows how the power output of the vehicle varies with distance travelled?

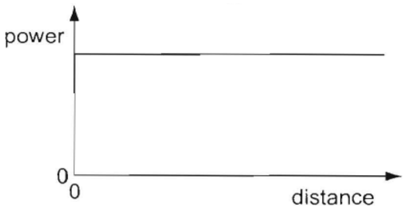
A



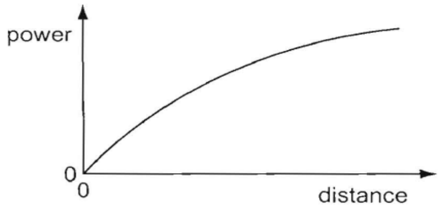
C



B



D



D15 **[2009/P1/Q12]** A driving force of 200 N is needed for a car of mass 800 kg to travel along a level road at a speed of 20 m s^{-1}. What power is required to maintain the car at this speed up a gradient in which the car rises 1 m for each 8 m of travel along the road?

A 6.0 kW **C** 20 kW

B 7.2 kW **D** 24 kW

D16 **[2009/P1/Q13]** A speed boat with two engines, each of power output 36 kW, can travel at a maximum speed of 12 m s^{-1}. The total drag D on the boat is proportional to the square of the speed of the boat.

What is the maximum speed of the boat when only one engine is working?

A 3.0 m s^{-1} **C** 8.5 m s^{-1}

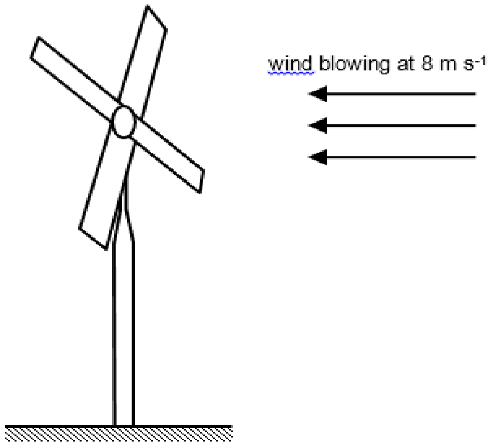
B 6.0 m s^{-1} **D** 9.5 m s^{-1}

D17 **[CIE/N2011/1/16, modified]** A man of mass m ties himself to one end of a rope which passes over a single fixed pulley. He pulls on the other end of the rope to lift himself up at an average speed of v. Which of the following expressions gives the average useful power at which he is working?

A $\frac{1}{2}\ mgv$ **C** $mgv + \frac{1}{2} mv^2$

B mgv **D** $mgv - \frac{1}{2} mv^2$

D18 **[HCI/2007/BT2/P1/Q7]** Giant windmills have been proposed as power sources. One such windmill sweeps out a circle of 60.0 m radius. If 60% of the available energy from the wind is converted into useful energy, calculate the power generated when the wind blows at 8.00 m s^{-1} (Density of air = 1 kg m^{-3}).



A 2.90 MW

B 3.48 MW

C 1.48 MW

D 1.74 MW

Numerical Solutions

Self Review Questions

S1	D	**S5**	B	**S9**	C
S2	C	**S6**	B	**S10**	D
S3	A	**S7**	C	**S11**	D
S4	A	**S8**	A	**S12**	A

Discussion Questions

D8 1.81 N

D13 (a) 2.89 s
(b)(i) 32.0 m (ii) 54.4 kJ
(c) (ii) 1. 48 m 2. 32.7 kJ