

Electromagnetic Forces

Guiding Questions	Learning Objectives
1. What is a magnet? Are there magnetic charges? 2. What do field lines represent? Do field lines represent similar things for electric fields and magnetic fields?	(a) show an understanding that a magnetic field is an example of a field of force produced either by current-carrying conductors or by permanent magnets (b) sketch magnetic field lines due to currents in a long straight wire, a flat circular coil and a long solenoid (c) use $B = \frac{\mu_0 I}{2\pi d}$, $B = \frac{\mu_0 NI}{2r}$ and $B = \mu_0 nI$ for the magnetic flux densities of the fields due to currents in a long straight wire, a flat circular coil and a long solenoid respectively (d) show an understanding that the magnetic field due to a solenoid may be influenced by the presence of a ferrous core
3. Why is the word “flux” used when talking about a magnetic field?	(e) show an understanding that a current-carrying conductor placed in a magnetic field might experience a force (f) recall and solve problems using the equation $F = BIl \sin \theta$, with directions as interpreted by Fleming’s left-hand rule (g) define magnetic flux density as the force acting per unit current per unit length on a conductor placed perpendicular to the magnetic field (h) show an understanding of how the force on a current-carrying conductor can be used to measure the magnetic flux density of a magnetic field using a current balance (i) explain the forces between current-carrying conductors and predict the direction of the forces
4. Do magnetic fields have effects on electric charges?	(j) predict the direction of the force on a charge moving in a uniform magnetic field (k) recall and solve problems using the equation $F = BQv \sin \theta$ (l) describe and analyse deflections of beams of charged particles by uniform electric fields and uniform magnetic fields (m) explain how perpendicular electric and magnetic fields can be used in velocity selection for charged particles.

What is a magnet? Are there magnetic charges?

What do field lines represent?

1 Magnetic Field

The repulsion or attraction between two bar magnets is familiar to us. Magnets and the magnetic field play a very big role in our everyday lives, from simple motors to sophisticated electrical components in our phones.

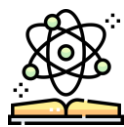
It is useful to describe forces in terms of fields and field lines because the description of fields helps us to intuitively understand forces. The magnetic force is no different.

A **magnetic field** is a region of space where a magnetic force is experienced by

- a magnetic material
- a current-carrying conductor (**not** placed parallel to the magnetic field lines)
- a moving charge (**not** moving parallel to the magnetic field lines)

Magnetic field lines have similar properties to those of electric field lines. These properties are:

- Magnetic field lines are drawn from the north seeking pole to the south-seeking pole. They represent the direction in which a north seeking pole at that point would move. In other words, the direction that a compass would point towards if it was placed at that point.
- The strength of the field is represented by the density of the field lines, i.e., the closer the field lines are to each other, the higher the field strength in that region.
- The field lines never cross each other.
- The lines always form a closed loop, and will continue inside a magnetic.



Nature of Science Moment

*There is a real possibility of confusion when talking about magnetic poles. This partly arises from the origin of our **observations** of magnetism. When we write “magnetic north pole” what we really mean is the magnetic pole that seeks the geographic north pole”.*

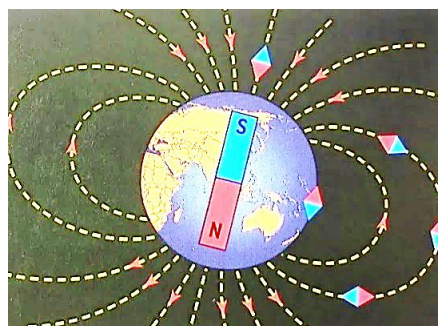


Fig. 1

We often talk loosely about a magnetic north pole pointing to the (geographical) north pole. Misunderstandings can occur because we also know that like poles repel and unlike poles attract. So we end up with the situation that a magnetic north pole is attracted to the “geographic north pole”, which seems wrong in the context of two poles repelling.

*More accurately, the north pole of the earth’s magnetic field points to the geographic south and vice versa (see **Fig. 1**).*

Adapted from Homer, D. & Bowen-Jones, M., 2014 Ed. Physics IB Course Companion

2 Magnetic Flux Pattern Due to Current Carrying Conductors

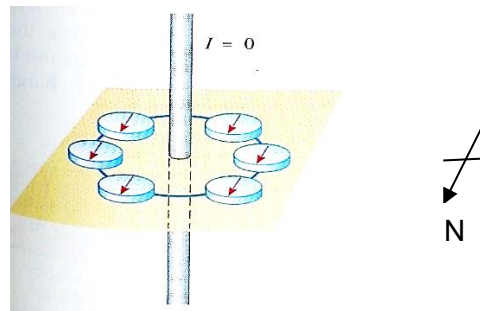
2.1 Long Straight Wire



Discovery of magnetic fields around current carrying wire.

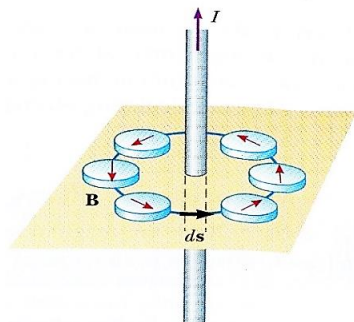
The magnetic field pattern due to a current in a long straight wire is a circular pattern centred on the wire. Measurements show that the field is strong close to the wire and becomes weaker with distance away from it.

Fig. 2.1(a)



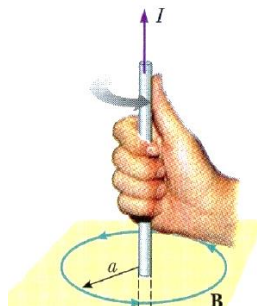
With no current in the wire, all compass needles point in the direction towards the geographic north pole (magnetic south pole) of the earth. This shows only the earth's magnetic field is present.

Fig. 2.1(b)



When current I flows, each compass needle deflects in a direction tangential to a circle. The net magnetic field in the region is circular around the wire. (Earth's magnetic field is negligible.)

Fig. 2.1(c)



The direction of the magnetic field could be deduced using the right-hand grip rule.

Imagine gripping the wire in your right hand, the thumb points in the current direction and the fingers curl in the direction of the field.

Magnetic field lines always form closed loops.

(Source: *Physics for Scientists and Engineers* by Serway)

A current flowing in an infinitely long, straight wire produces a magnetic field represented by concentric circular field lines as shown.

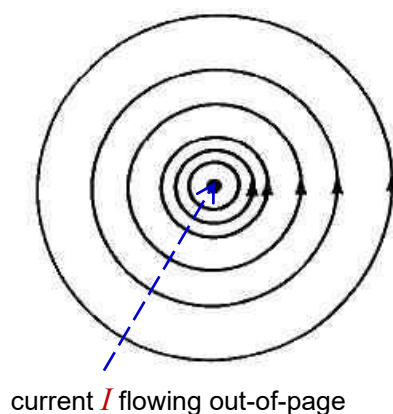


Fig. 2.2(a)

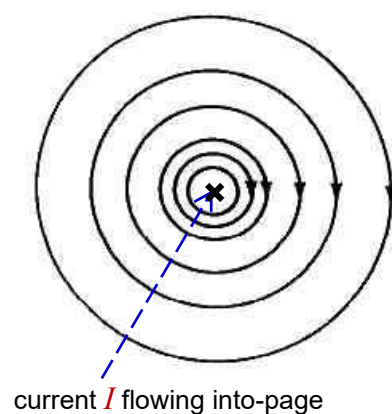


Fig. 2.2(b)

Points to note:

- The field lines are closer together nearer the wire, indicating that the field is stronger nearer the wire.
- Direction of $\mathbf{B}$ at any point is tangent to the flux line.
- Direction of $\mathbf{B}$ is always perpendicular to the radial line from the current.
- Using the right-hand grip rule, for current flowing out of the page, direction of $\mathbf{B}$ is anti-clockwise.

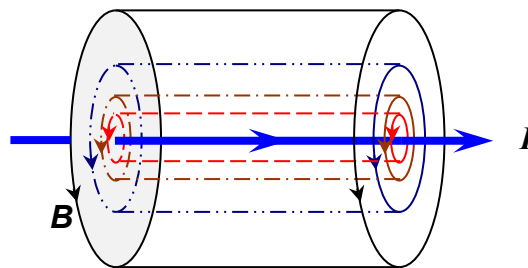


Fig. 2.3

In 3-dimension, the magnetic field is represented by a set of concentric cylindrical surfaces, where the perpendicular separation between each surface increases away from the wire, indicating that the field is stronger nearer the wire.

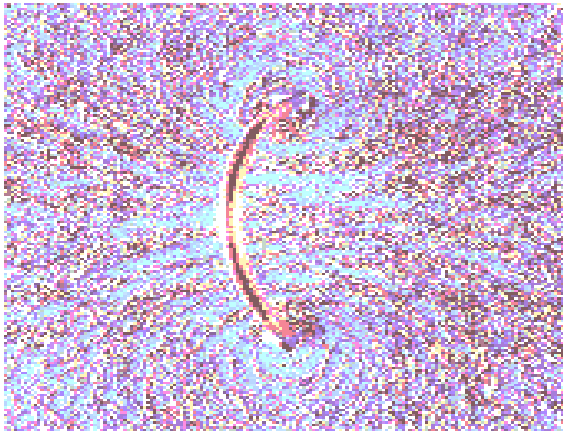
The magnitude for the magnetic flux density at a perpendicular distance d of infinite length carrying a current I is given by

$$B = \frac{\mu_0 I}{2\pi d}$$

where $\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$, is the permeability of free space.

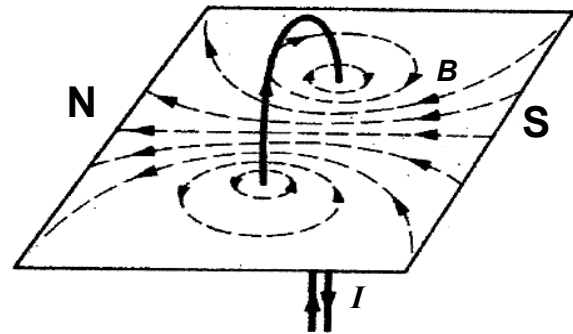
2.2 Flat Circular Coil

Fig. 2.4 shows the flux pattern around a flat circular coil carrying a current I . The pattern is in a horizontal plane, perpendicular to the coil and passing through its centre. The direction of the magnetic field $\mathbf{B}$ due to a current carrying conductor could be deduced using the right-hand grip rule.



(Source: *Physics for Scientists and Engineers* by Serway)

Fig. 2.4 (a)



(Source: *Advanced Physics* by Tom Duncan)

Fig. 2.4 (b)

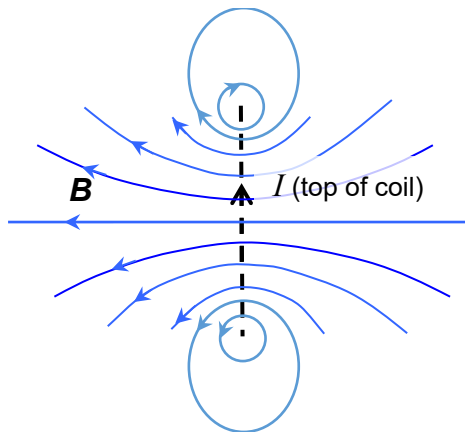


Fig. 2.4 (c) – Top view of the coil

The left side of the coil acts like a north pole and the right acts like a south pole.

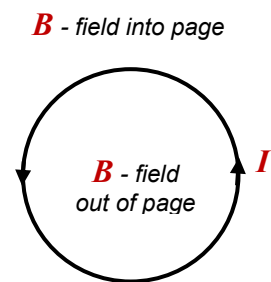


Fig. 2.4 (d) – Front view of the coil

The magnitude for the magnetic flux density at the centre of N tightly packed coils, each having radius r carrying a current I is given by

$$B = \frac{\mu_0 NI}{2r}$$

2.3 Long Solenoid

A long solenoid is a coil wound by a long wire where typically its length is 10 times that of the diameter of its cross-sectional area.

2.3.1 Loosely Wound Solenoid

Magnetic field lines look similar to that of a bar magnet. But, very close to the wires, the field lines are circular.

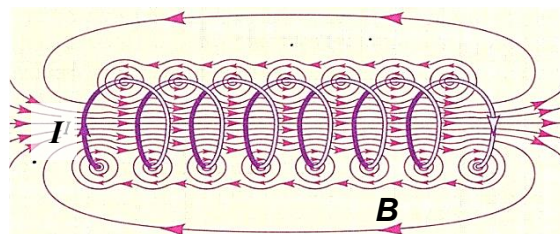


Fig. 2.5 (a)

2.3.2 Closely Wound and Long Solenoid

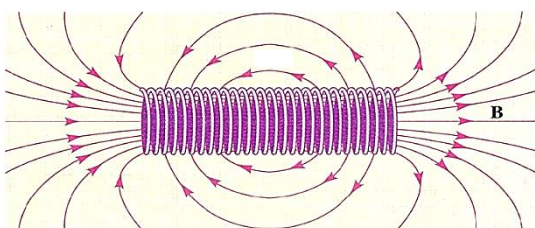


Fig. 2.5 (b) – Magnetic flux density pattern of a closely wound solenoid

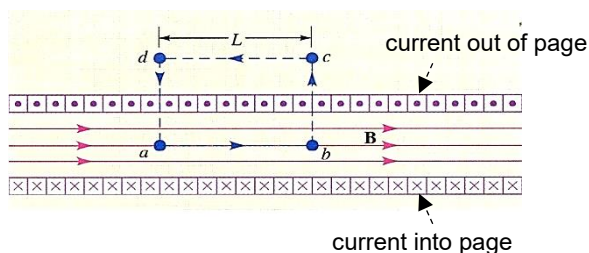


Fig. 2.5 (c) – Cross-section of an infinitely long solenoid

Some properties of a magnetic field set up by current flowing through the solenoid.

- The right hand grip is used to determine the direction of B-field created by each section of wire. The vector sum of the magnetic field of all sections results in the overall pattern of the magnetic field inside and outside of the solenoid.
 - Inside the solenoid, the magnetic field lines from the top and bottom current segments in Fig. 2.5(c) are in the same direction. They reinforce each other, so the field is strong.
 - Near the axis of the solenoid, the field is fairly uniform, so the field lines are parallel and equally spaced.
 - Outside the solenoid, the contributions from the top and bottom segments are in opposite directions, so they cancel out (not completely) and the field is much weaker.
 - The magnetic field lines are non-parallel and widely separated, indicating a non-uniform and weak field respectively. The lines space out more and more as they get further from the solenoid.

- For magnetic field to be uniform inside a solenoid
 - Coils must be closely wound
 - Solenoid must be sufficiently long

The magnitude for the magnetic flux density **inside** a closely wound solenoid of **infinite length** carrying a **current I** is given by

$$B = \mu_0 n I$$

where $n = \frac{N}{l}$ is the number of turns per unit length.

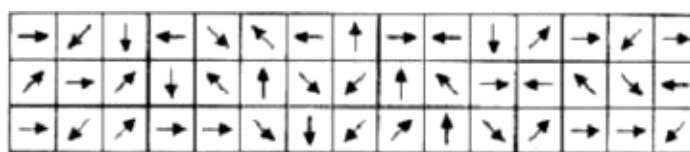
2.3.3 Effect of a Ferrous Core in a Solenoid

The motion of electrons in the ferromagnetic atoms produce a magnetic field. These atoms act as “atomic” magnets. A small group of “atomic” magnets is called a domain. Each domain acts like a tiny magnet.

In the absence of an external magnetic field, the domains are in random directions. Their magnetic fields cancel each other out and the core is unmagnetised. See Fig. 2.6(a).

When placed in an external magnetic field, the domains line up along this field pointing in the same direction. The resulting magnetic field is the vector sum of the magnetic field due to the current in the solenoid and that due to the ferrous core. See Fig. 2.6(b) and (c).

Fig 6 (a)



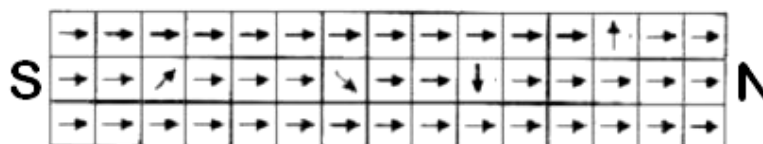
(a) Unmagnetised iron - random orientation, no resultant magnetic field

Fig 6 (b)



(b) Weak magnetic field - some domains align, material becomes a weak magnet

Fig 6 (c)



(c) Strong magnetic field - most domains align (saturation), material becomes a strong magnet.

Ferromagnetic materials can be classified into:

- hard magnetic material which requires a strong external magnetic field to align the domains but can retain its magnetism well.
- soft magnetic material which can be magnetised easily but will lose its magnetism easily when the external magnetic field is removed.

Example 2.1

Explain why the B-field inside a solenoid can be greatly increased by the insertion of a ferromagnetic material such as iron, cobalt or nickel into its core.

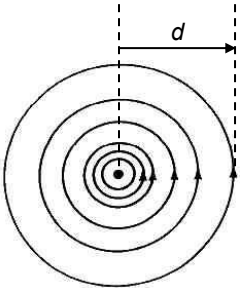
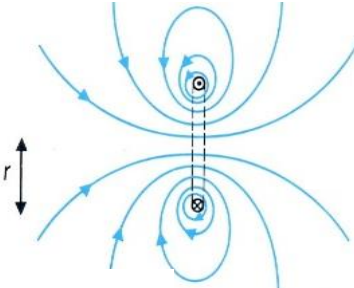
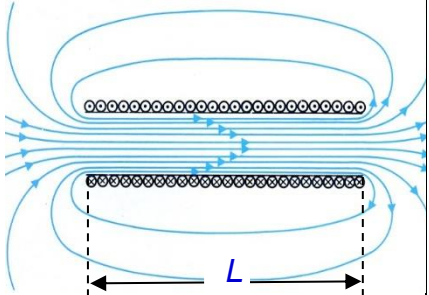
Solution

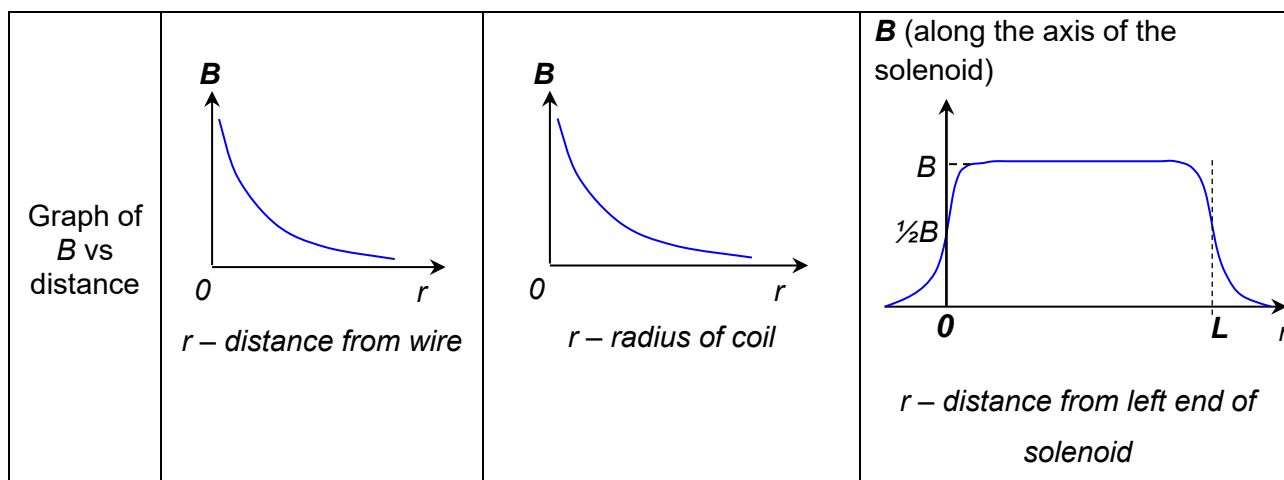
When a current is passed through the solenoid, it sets up an external magnetic field which aligns the domains of the ferromagnetic material in the same direction.

The magnetic field produced by the alignment of these domains strengthens the external magnetic field because the resulting magnetic field is the vector sum of the magnetic field due to the current in the solenoid and the magnetic field due to the ferromagnetic material.

Hence, the ferromagnetic material changes the medium's permeability from μ_0 to μ (where $\mu > \mu_0$). i.e., the ferromagnetic material provides a better magnetising medium than vacuum/air inside the solenoid.

2.4 Overview of Three Systems

	Long Straight Wire	Flat Circular Coil	Solenoid
Magnetic field lines	current I (out of page) 	N turns with current I and radius r 	current I with n turns per unit length, $n = \frac{N}{l}$ 
Equation	<i>B</i>-field at perpendicular distance r from long wire, $B = \frac{\mu_0 I}{2\pi d}$	<i>B</i>-field at centre of coil, $B = \frac{\mu_0 NI}{2r}$	<i>B</i>-field at central axis through coil, $B = \mu_0 nI$



Example 2.2

State and explain the value of the magnetic flux density at the ends of an infinitely long solenoid in terms of B_0 , the strength well within the solenoid.

Solution

If a long solenoid is cut at any point R in the middle into two equal “half” solenoids, each solenoid contributes equally to the field at R .

At R , each solenoid provides a flux density of $\frac{1}{2}B_0$. The field at the end of any long solenoid is

therefore half that at the centre and is given by $B = \frac{1}{2}\mu_0 nI$.

Why is the word “flux” used when talking about a magnetic field?

3 Magnetic Force on a Current-Carrying Conductor

3.1 Magnetic Flux Density and Tesla

Magnetic flux density is a vector quantity that quantifies the strength of a magnetic field and the direction which a north pole of the magnet seeks.

Its magnitude depends on

- magnitude of the current flowing
- the type of medium the magnetic field is permeating



The **magnetic flux density B** at a given region of space is defined as the force per unit length acting on a straight, current-carrying conductor, carrying unit current and placed at right angles to this external magnetic field.

In mathematical form,

$$B = \frac{F}{I l \sin(90^\circ)}$$

$$= \frac{F}{I l}$$

The S.I. unit of magnetic flux density is the tesla (T).

One tesla is the magnetic flux density of a magnetic field when a conductor carrying a current of 1 ampere placed perpendicular to this field experiences a force per unit length of 1 newton per meter in a direction perpendicular to both the field and the current.

Recalling the Field Strengths

- Gravitational field strength g at a point is defined as the force per unit mass: $g = \frac{F_g}{m}$.
- Electric field strength E is defined as the force per unit positive charge: $E = \frac{F_E}{q}$.
- In a similar way, magnetic flux density is defined in terms of the magnetic force experienced by a current-carrying conductor placed at right angle to the magnetic field; the flux density B is defined by the equation: $B = \frac{F_B}{I l}$

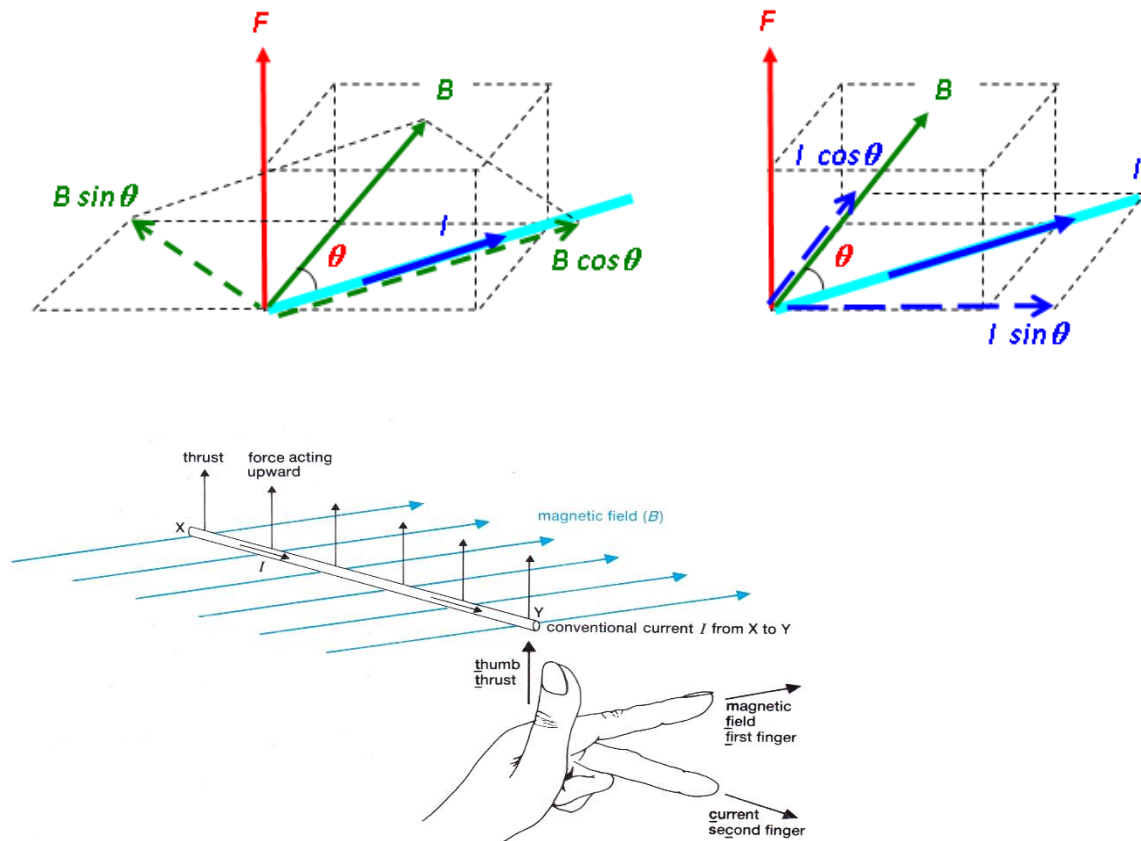
(Magnetic field strength is similar to magnetic flux density but their differences are beyond the scope of A levels.)

Hence the magnitude of the magnetic force F acting on a current-carrying conductor of current I and length l is given by

$$F = B I l \sin \theta$$

where θ is the angle between the magnetic flux density B and the direction in which the current I is flowing.

The direction of force F can be deduced using Fleming's left-hand rule as shown in Fig. 3.1.



(Source: Physics by Robert Hutchings)

Fig. 3.1 Fleming's Left-Hand Rule



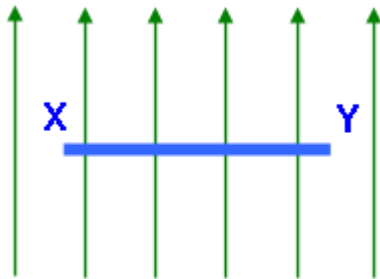
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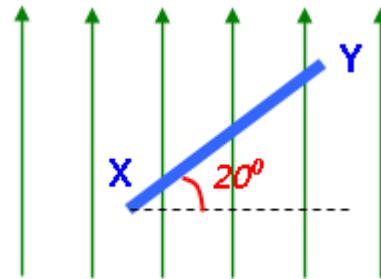
Jumping wire demo.

Example 3.1 (Source: Physics for Scientists & Engineers by Serway/Jewett)

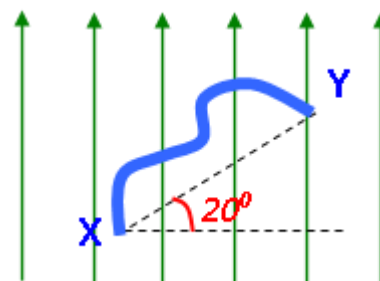
In each figure, the displacement between XY is 10 cm. The same current flows in each wire. Explain and place the magnitude of the magnetic force exerted on them, from the greatest to the least.



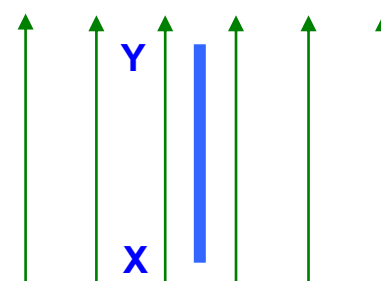
(a)



(b)



(c)



(d)

Solution

Need to calculate magnetic force on a current carrying conductor

$F = BIL \sin \theta$, where θ is the angle between B and I .

$$F_a = BIL \sin(90^\circ) = BIL \text{ as } I \text{ and } B \text{ are perpendicular.}$$

$$F_b = BIL \sin(70^\circ) \approx 0.940BIL$$

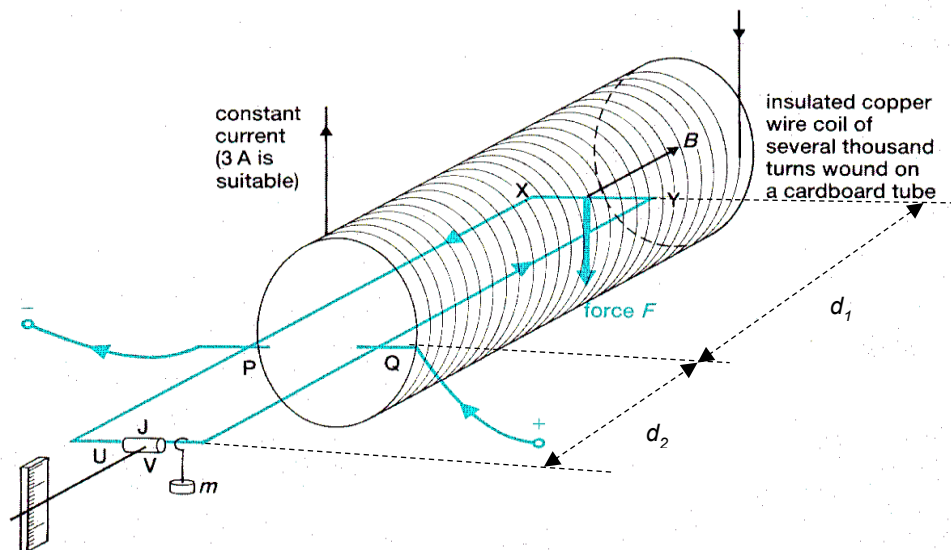
$$F_c = BIL \sin(70^\circ) \approx 0.940BIL$$

$$F_d = BIL \sin(0^\circ) = 0 \text{ N}$$

$$\text{Hence, } F_a > F_b = F_c > F_d$$

3.2 Current Balance

A simple set up of the current balance is shown in Fig. 3.3. In this example, the current balance will be used to determine the magnetic flux density within a solenoid.



(Source: Physics by Robert Hutchings)

Fig. 3.3

Fig. 3.3 shows a stiff copper wire rectangular frame UPXYQV placed in an external uniform magnetic field B . The frame is joined by an insulator J and is supported on knife edges at P and Q through which current can pass from a dc source.

The magnetic force F acts downwards (determined by Fleming's left hand rule) and its magnitude is given by

$$F = B I l$$

The frame can be balanced to the horizontal position by adjusting distance d_2 (or by changing mass m or changing I).

By principle of moments,

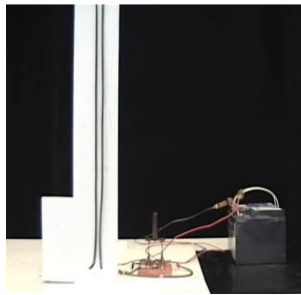
clockwise moment due to F = anti-clockwise moment due to mg

$$F d_1 = m g d_2$$

$$B I l d_1 = m g d_2$$

$$B = \frac{m g d_2}{I l d_1}$$

3.3 Forces between Long Parallel Current-Carrying Conductors



Watch how the forces on the current carrying conductors at work.

Fig. 3.4 shows two wires of length l carrying current I_1 and I_2 which are positioned a distance d apart in vacuum.

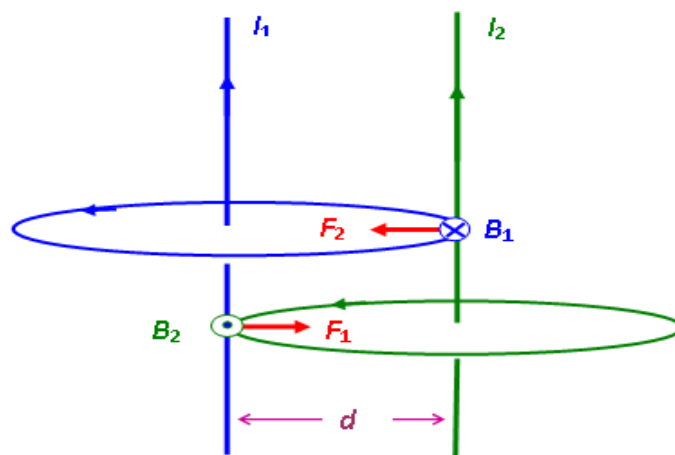


Fig. 3.4

Consider wire 1,

Magnitude of magnetic force on length l

$$F_{2 \text{ on } 1} = B_2 I_1 l$$

Using Fleming's left-hand rule,

$F_{2 \text{ on } 1}$ is directed to the right

Assuming that the wires are infinitely long

$$B_2 = \frac{\mu_0 I_2}{2\pi d}$$

Consider wire 2,

Magnitude of magnetic force on length l

$$F_{1 \text{ on } 2} = B_1 I_2 l$$

$F_{1 \text{ on } 2}$ is directed to the left.

Force per unit length on wire 1,

$$\frac{F_{2 \text{ on } 1}}{l} = \left(\frac{\mu_0 I_2}{2\pi d} \right) I_1$$

$$= \frac{\mu_0 I_2 I_1}{2\pi d}$$

$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

Force per unit length on wire 2,

$$\frac{F_{1 \text{ on } 2}}{l} = \left(\frac{\mu_0 I_1}{2\pi d} \right) I_2$$

$$= \frac{\mu_0 I_1 I_2}{2\pi d}$$

Mutual force acting per unit length on the infinitely long parallel wires is

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

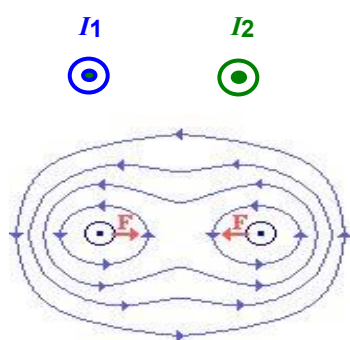
Note: **Newton's 3rd Law of motion**. Force on each wire is an action/reaction pair where they are equal in magnitude, opposite in direction, act on different wires and are of the same type.

Example 3.2

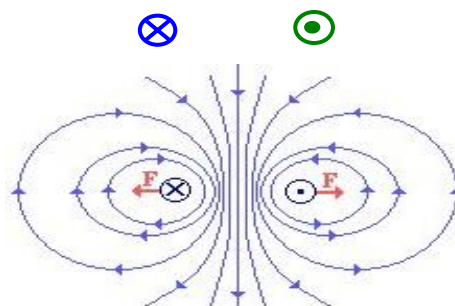
For each set of current carrying straight parallel wires below, sketch the magnetic fields and their directions around the wires and state if the forces are attractive or repulsive. Assume that $I_1 = I_2$.

Solution

(a)



(b)



(c)



(d)



Do magnetic fields have effects on electric charges?

4 Magnetic Force on Charged Particles

An electric current is regarded as a drift of electrons. Thus, the force experienced by a current-carrying conductor in a magnetic field is the resultant of the forces acting on all of the electrons in the conductor.

Consider a length L of a conductor containing N electrons, each of charge Q and average drift velocity v , as shown in Fig. 4.1.

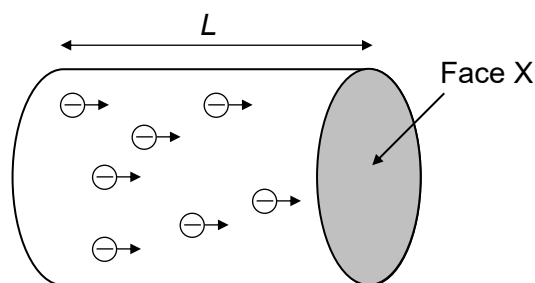


Fig. 4.1

Time required for all N electrons to reach face X, $t = \frac{L}{v}$

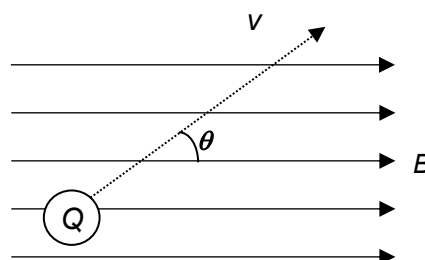
Rate of flow of charge through X, (i.e. current), $I = \frac{NQ}{t}$

If the conductor makes an angle θ with a uniform magnetic field B , then the force on the conductor is

$$\begin{aligned} F &= BIL \sin \theta \\ &= B \left(\frac{NQ}{t} \right) L \sin \theta \\ &= B \left(\frac{NQ}{t} \right) (vt) \sin \theta \\ &= NBQv \sin \theta \end{aligned}$$

Hence, the force on a single charged particle Q moving with velocity v at an angle θ with respect to the direction of a uniform magnetic field B is

$F = BQv \sin \theta$

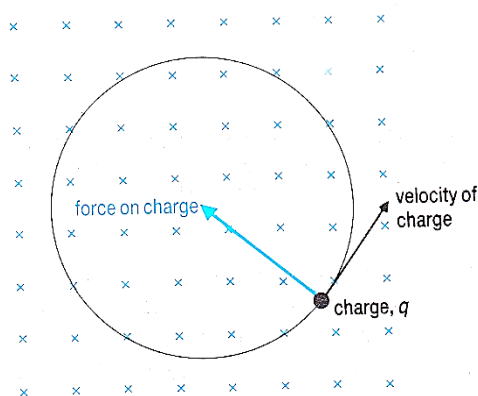


Motion of Charged Particle in a Uniform Magnetic Field



Watch how magnets affects the motion of electrons.

Consider a charge q of mass m travelling at a constant speed v in a circular path of radius r at right angles to a magnetic field of flux density B .



(Source: Physics by Robert Hutchings)

Fig. 4.2

The moving charge will experience a magnetic force of

$$F = Bqv$$

If this is a positive charge, the current will flow in the same direction as the velocity. Thus, using Fleming's left-hand rule, the force on the charge will be perpendicular to the velocity of the charge. This means that a moving charge in a magnetic field (that is perpendicular to the velocity) will move in a uniform circular motion.

Hence the magnetic force provides the centripetal force required to keep the charge in circular motion,

$$\frac{m v^2}{r} = B q v$$

$$r = \frac{m v}{B q}$$

Note: If the velocity is not entirely perpendicular to the B-field, there would be a component of velocity that is parallel to the B-field. This component of velocity will not result in a force (hence no acceleration) and will be constant. The resulting path would then be a helical path as shown in Fig. 4.3.

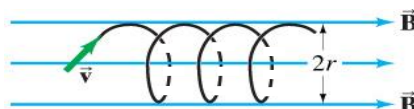


Fig. 4.3

Example 4.1

A proton moves with a speed of $1.0 \times 10^5 \text{ m s}^{-1}$ through the earth's magnetic field of flux density $50 \mu\text{T}$ at a particular location. When the proton moves eastward, the magnitude of the magnetic force acting on it is a maximum, and when it moves northwards, no magnetic force acts on it.

- (a)
 - (i) Determine the strength of the maximum magnetic force experienced by the proton.
 - (ii) State the direction of the magnetic field.
- (b) Calculate the electric field strength required to provide a force of equal magnitude to that provided by the magnetic field.

Solution

(a) *maximum magnetic force,* $F_B = B q v$

$$= (50 \times 10^{-6}) (1.60 \times 10^{-19}) (1 \times 10^5)$$

$$= 8.0 \times 10^{-19} \text{ N}$$

When proton moves eastwards, $F_B = \text{max}$ implies that B is perpendicular to v

When proton moves northwards, $F_B = 0$ implies that B is parallel to v .

- (b) *An electric field causes an electric force on the proton.*

Equating $F_E = qE$ to $F_B = B q v$

$$F_E = F_B$$

$$q E = B q v$$

$$(1.60 \times 10^{-19}) E = 8.0 \times 10^{-19}$$

$$E = 5.0 \text{ V m}^{-1}$$

5 Charged Particles in Uniform Electric and Magnetic Field

5.1 Velocity Selector

In a velocity selector, $\vec{B}$ and $\vec{E}$ are arranged such as to produce an electric force $\vec{F}_E$ and a magnetic force $\vec{F}_B$ that are in opposite directions to each other. To set this up, the fields have to be perpendicular to each other, referred to as crossed fields.

A particle with a charge q enters the fields at 90° to both fields.

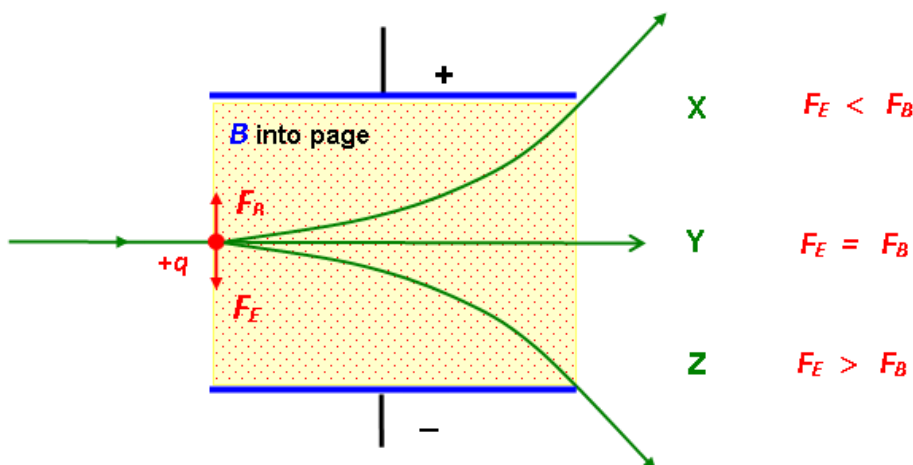


Fig. 5.1

When the **forces** exerted by the two fields on the charge q are of the *same magnitude* but in *opposite direction*,

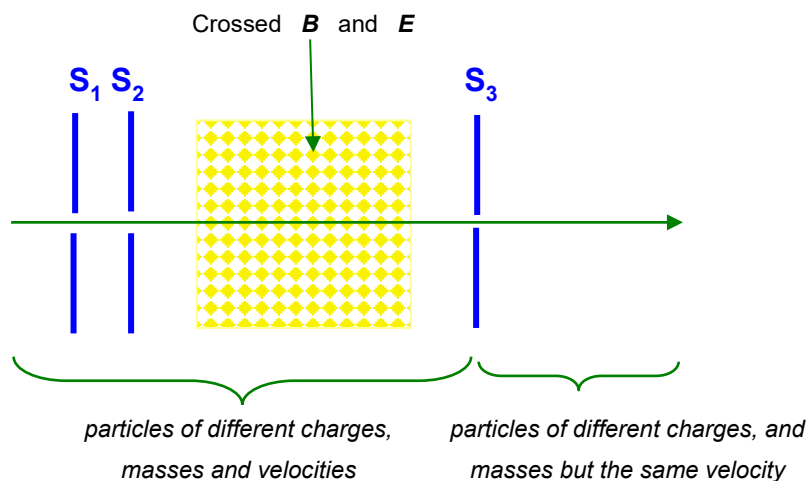
$$F_B = F_E$$

$$Bqv = qE$$

$$v = \frac{E}{B}$$

Hence, as long as the particles move at a speed $v = \frac{E}{B}$, the particles will pass through the crossed fields undeflected.

A velocity selector is used to select charge particles of a single velocity from a group of charged particles of different velocities.



Slits $\mathbf{S}_1$ and $\mathbf{S}_2$: to confine the charged particles to a narrow beam

Slit $\mathbf{S}_3$: to only allow charged particles moving with velocity

$$v = \frac{E}{B} \text{ to emerge}$$

Fig. 5.2

Example 5.1

Differently charged ions of several nuclides are all fired at the same speed into a region where there is a uniform magnetic field at right angles to their initial path.

Which ions travel in a circular path of the smallest radius?

- | | |
|--|--|
| A Singly charged ${}^6_3\text{Li}$ ions | C Doubly charged ${}^{12}_6\text{C}$ ions |
| B Doubly charged ${}^6_3\text{Li}$ ions | D Singly charged ${}^{16}_8\text{O}$ ions |

Example 5.2

An electron travelling with speed $4.3 \times 10^7 \text{ m s}^{-1}$ enters a magnetic field of uniform flux density 0.0086 T , in a direction at right angles to the field.

- Calculate the radius of the path of the electron in the field.
- Explain how it is possible to select electrons of a particular speed by the use of electric and magnetic fields.

Solution

- (a) *Each electron (charged particle) moving in a uniform magnetic field experiences a magnetic force.*

Apply $F_B = B q v \sin \theta$ where θ is the angle between B and v

Net force (magnetic force) provides the centripetal force, and electrons move in a circle.

$$\begin{aligned}\frac{m v^2}{r} &= B q v \\ r &= \frac{m v}{B q} \\ &= \frac{(9.11 \times 10^{-31})(4.3 \times 10^7)}{(0.0086)(1.60 \times 10^{-19})} \\ &= 2.85 \text{ cm}\end{aligned}$$

- (b) A set of parallel plates with a potential difference between them is set up in a plane perpendicular to the B -field.

On the electron, if F_E is equal in magnitude but opposite in direction to F_B , it will move in a straight line (undeflected) through a vertical slit placed in its path.

$$\begin{aligned}F_E &= F_B \\ q E &= B q v \\ v &= \frac{E}{B}\end{aligned}$$

Only electrons moving at speed $v = \frac{E}{B}$ can pass through the slit.

5.2 Application of the Velocity Selector – Mass Spectrometer

A mass spectrometer is used to measure the mass m of atoms.

Fig. 5.3 below shows the principle of a simple form of mass spectrometer. Ions each of charge Q are passed through narrow slits S_1 and S_2 and into a velocity selector (crossed E and B_1). The selected ions passing through the slit S_3 , are deflected by a uniform magnetic field B .

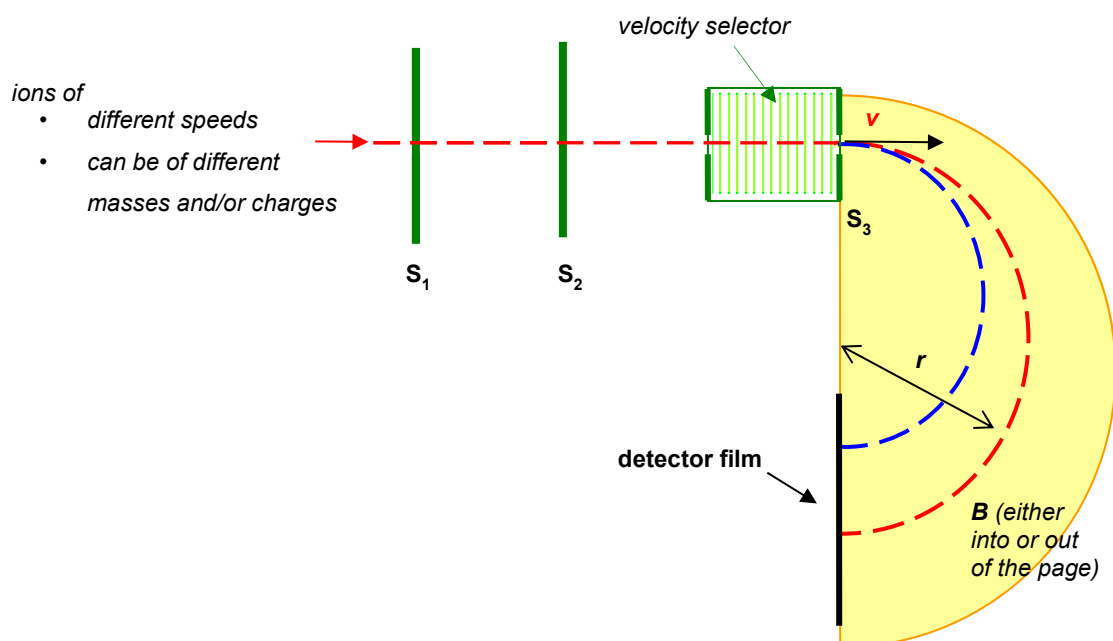


Fig. 5.3

In the velocity selector, ions that are un-deflected have velocity v ,

$$\begin{aligned} F_E &= F_B \\ qE &= B_1 q v \\ v &= \frac{E}{B_1} \end{aligned}$$

Inside the dome, the selected ions move in a circular path of radius r ,

$$\begin{aligned} F_C &= F_B \\ \frac{mv^2}{r} &= B_2 q v \\ r &= \frac{mv}{qB_2} \end{aligned} \quad \text{where } q \text{ is the specific charge of a particle.}$$

Example 5.3

For ions arriving at the detector in Fig. 5.2, which of the quantities “ q , $\frac{m}{q}$, m , mv ” is the same?

Solution

Since B , v are constants, we get $r \propto \frac{m}{q}$ which means only ratio $\frac{m}{q}$ is the same.

Example 5.4

A particle with a charge of magnitude Q enters a uniform magnetic field B with a velocity v .

Sketch the path of charge Q from the point of entry to the point of exit and indicate the direction/s of the force F on the charge due to the magnetic field at various locations of the charge's path in the following situations. **Fig. 1** has a stronger magnetic field.

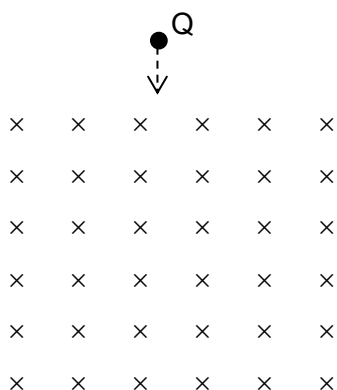


Fig. 1

Q is negative and B -field is normal
and directed into the page

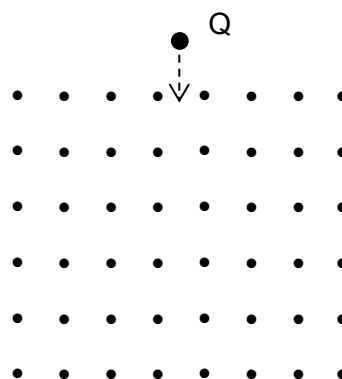
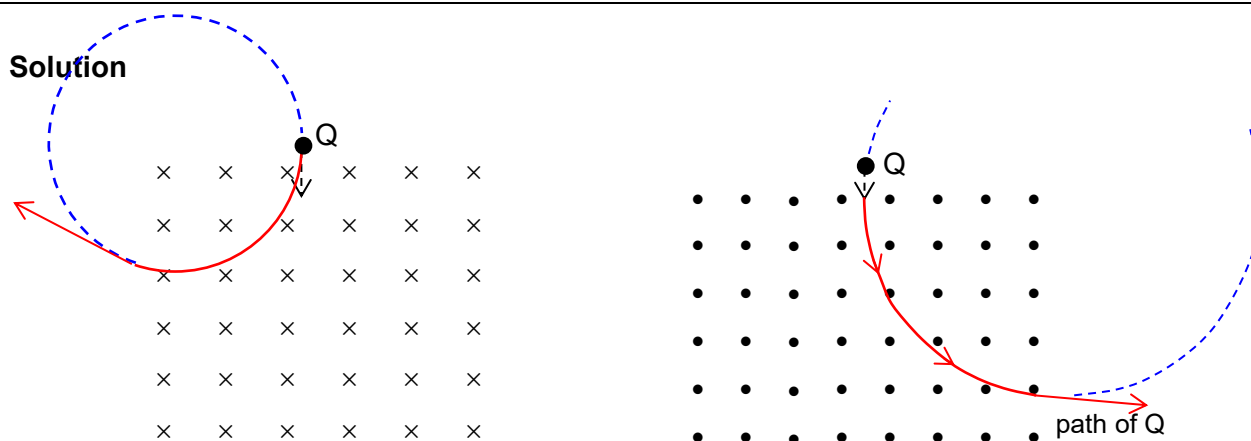


Fig. 2

Q is negative and B -field is normal
and directed out of the page



5.3 Application of the Velocity Selector – Hall Probe

When a current carrying conductor (or semiconductor) is placed in a magnetic field, a p.d. is generated in a direction perpendicular to both the current and magnetic field. The direction of the p.d. generated can be used to determine the nature of the charge carriers in the conductor (or semiconductor). The phenomenon responsible for this is known as the *Hall Effect*.

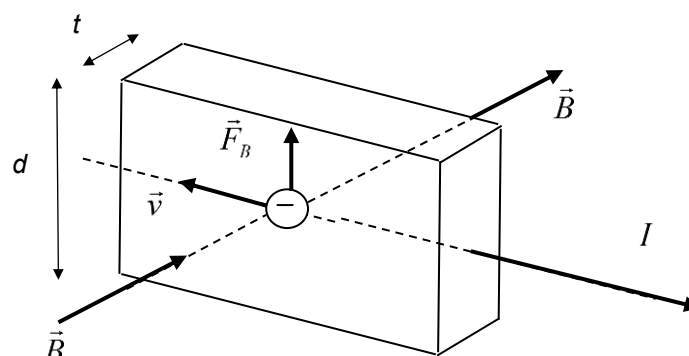


Fig. 5.4

Fig. 5.4 shows a current-carrying conductor, with a magnetic field passing through the conductor perpendicular to the current. If the majority charge carriers are electrons moving with drift velocity v , using Fleming's left-hand rule, we can deduce that they will experience an upward force $F = Bqv$.

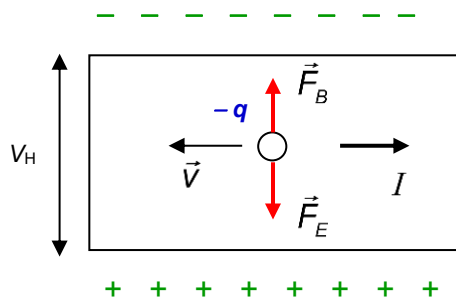


Fig. 5.5 (a) Negative majority charge carriers

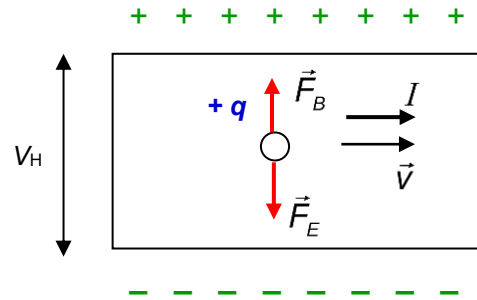


Fig. 5.5 (b) Positive majority charge carriers

In Fig. 5.5 (a), the accumulation of negative charge (due to accumulation of electrons) on the upper edge, leaves an excess of positive charge (due to loss of electrons) at the lower edge. This accumulation of charge at the edges sets up an electric field, which increases until the electric force F_E on subsequent charge carriers balance their magnetic force F_B .

When this equilibrium condition is reached, subsequent electrons are no longer deflected upwards.

$$\begin{aligned}
 F_E &= F_B \\
 qE &= Bqv \\
 q \frac{V_H}{d} &= Bqv \\
 V_H &= Bdv \\
 &= \frac{B d I}{n e A} && \text{where } I = nevA \\
 &= \frac{B I}{n e t} && \text{where } A = dt
 \end{aligned}$$

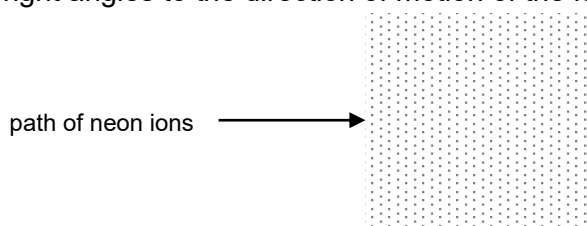
In Fig. 5.5 (b), with positive charge carriers (e.g. when using p-type semiconductors), the polarity of V_H is reversed. This indicates that the polarity of V_H depends on the type of charge carriers

A sensitive voltmeter can be used to measure Hall voltage V_H .

Review Questions

Question 1 (J99/3/6 – modified)

- (a) A stationary negatively-charged particle experiences a force in the direction of the field in which it is placed.
State, with a reason in each case, whether or not the field is
- (i) magnetic,
 - (ii) electric,
 - (iii) gravitational.
- (b) (i) Calculate the magnitude of the electric field strength required to maintain an electron in a fixed position in the gravitational field of the Earth, near its surface.
[$5.6 \times 10^{-11} \text{ N C}^{-1}$]
- (ii) Hence explain why gravitational effects are ignored when considering motion of electrons in electric fields.
- (c) Neon atoms are ionised into positive ions by the removal of one electron from each atom. Some neon ions are accelerated from rest in a vacuum through a potential difference of 1400 V. They are then injected into a region of space where there are uniform electric and magnetic field acting at right angles to the direction of motion of the ions.



- (i) Copy the figure above and indicate clearly on your diagram the directions of the electric and magnetic fields so that the ions pass un-deflected through the region.
 - (ii) Calculate the speed of the accelerated ions on entry into the region of the electric and magnetic fields. (mass of a neon atom = $3.3 \times 10^{-26} \text{ kg}$) [$1.2 \times 10^5 \text{ m s}^{-1}$]
 - (iii) If the electric field strength E is $6.2 \times 10^3 \text{ V m}^{-1}$, calculate the magnitude of the magnetic flux density B so that the ions are not deflected in the region of the fields. [0.0517 T]
- (d) The mechanism by which the neon atoms are ionised is changed such that each atom loses two electrons. State the change in
- (i) the speed of the ions entering the region of the electric and magnetic fields in (c),
 - (ii) the path of the ions in the two fields.

Solution

- (a) (i) No, as charge is stationary it will not experience a magnetic force.
(ii) No, negative charged particle will experience electric force opposite to direction of field.
(iii) Yes, gravitational force is in the direction of gravitational field.

- (b) (i) Near Earth's surface, $g = 9.81 \text{ m s}^{-2}$

$$\begin{aligned} qE &= mg \\ E &= \frac{mg}{q} \\ &= \frac{(9.11 \times 10^{-31})(9.81)}{(1.6 \times 10^{-19})} \\ &= 5.6 \times 10^{-11} \text{ N C}^{-1} \end{aligned}$$

- (ii) Mass of electrons are very small, hence gravitational effects on electrons are insignificant compared to the electric force and can be ignored.

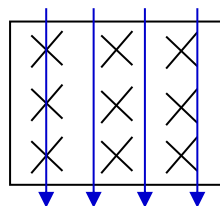
- (c) (i) Electric field downwards

Magnetic field into the paper

Or

Electric field upwards

Magnetic field out of the paper



- (ii) Assume gain in KE = loss in EPE

$$\begin{aligned} \frac{1}{2}mv^2 &= q(\Delta V) \\ v &= \sqrt{\frac{2q(\Delta V)}{m}} \\ &= \sqrt{\frac{2(1.6 \times 10^{-19})(1400)}{(3.3 \times 10^{-26})}} \\ &= 1.2 \times 10^5 \text{ m s}^{-1} \end{aligned}$$

- (iii) $Bqv = qE$

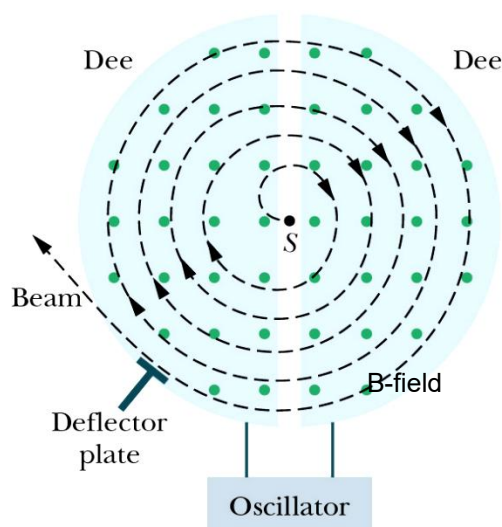
$$\begin{aligned} B &= \frac{E}{v} \\ &= \frac{(6.2 \times 10^3)}{1.2 \times 10^5} \\ &= 0.0517 \text{ T} \end{aligned}$$

- (d) (i) KE of the ions would double, speed will increase by $\sqrt{2}$.

- (ii) $F_B > F_E$,

Net F is not 0. Ions are deflected in the direction of the magnetic force.

Question 2 (Fundamentals of Physics (Halliday & Resnick))



A cyclotron is an accelerator in which a beam of energetic particles is produced.

The charged particles describe a spiral path of many turns at right angles to a constant magnetic field, and are given acceleration always in the same sense, from an alternating electric field, each time they cross the gap between the two conductors (dees). These charged particles then leave the cyclotron at the outer edge of the cyclotron.

Note that within the Dees, the charged particles are shielded from the electric fields by the copper walls of the Dees.

- If the charged particles were protons, explain clearly how the alternating electric field provides the acceleration.
- Show that the frequency of the applied voltage is independent of the radius r of the orbit.
- Show that the kinetic energy gained per cycle is $2qV_0$ where V_0 is the magnitude of the voltage applied but V_0 does not affect the final energy of the charged particles. Comment on your answer.
- Discuss what would happen to the particles if V_0 is increased.

Solution

- (a) For the proton to accelerate across the gap, whenever the proton reaches the gap (exiting one dee), the opposite dee must be at a lower potential.

In each cycle, since the proton crosses the gap twice, the direction of electric field must change twice (changing every half cycle).

- (b) In the dees, magnetic force provides the centripetal force,

$$\begin{aligned}\frac{mv^2}{r} &= Bqv \\ Bq &= \frac{mv}{r} \\ &= \frac{m(r\omega)}{r} \\ &= m(2\pi f_c)\end{aligned}$$

Hence the frequency of circular motion,

$$f_c = \frac{Bq}{2\pi m}$$

The polarity of the p.d. changes every half cycle, and the frequency of alternating voltage is f_v

For each proton to be accelerated as it crosses the gaps, the p.d. must alternate where

$$\begin{aligned}f_v &= f_c \\ &= \frac{Bq}{2\pi m}\end{aligned}$$

which is independent of r .

(c) Each time, proton crosses the gap,

$$\begin{aligned}\text{Assume, gain in KE} &= \text{loss in electric potential energy} \\ &= qV_0\end{aligned}$$

In each cycle, each particle crosses the gap twice,

$$\text{gain in KE per cycle} = 2qV_0$$

The KE of the particle as it exits the cyclotron is equals to the KE when the proton moves through its last half-cycle.

$$\begin{aligned}KE_{\text{exit}} &= KE_{\text{max}} \\ &= \frac{1}{2}mv_{\text{exit}}^2 \quad \text{where } \frac{mv_{\text{exit}}}{R} = Bqv_{\text{exit}} \\ &= \frac{B^2 q^2 R^2}{2m} \quad \text{where } KE_{\text{exit}} \text{ is independent of } V_0.\end{aligned}$$

(d) When V_0 increases, gain in KE every cycle will increase ($2qV_0$). This means the radius of circular path will increase more when it transits from one dee to the other. Hence, particles will take fewer revolutions to reach the exit at the outer edge of the cyclotron.

Question 3 (2012/HCI/Prelim/P2/4)

An alpha particle is injected at a speed of v at an angle of 60° into a uniform magnetic field of flux density B of 0.519 T . It describes a helical path of radius $r = 2.00 \text{ cm}$ as shown in Fig. 3.1 (a).

Fig. 3.1 (b) shows the projection of its trajectory onto the x - y plane.

The charge and mass of an alpha particle are $3.2 \times 10^{-19} \text{ C}$ and $6.64 \times 10^{-27} \text{ kg}$ respectively.

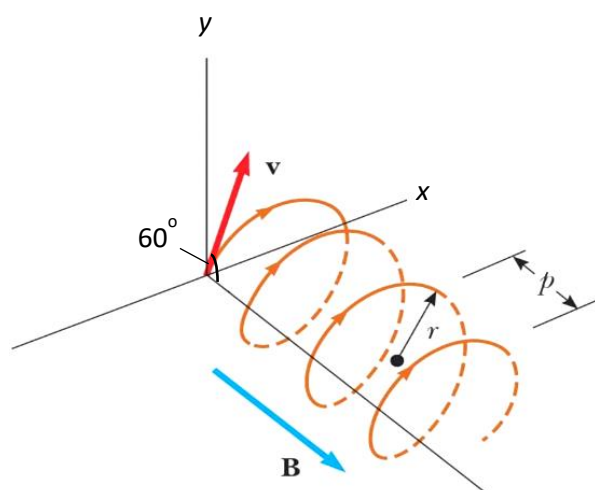


Fig. 3.1 (a)

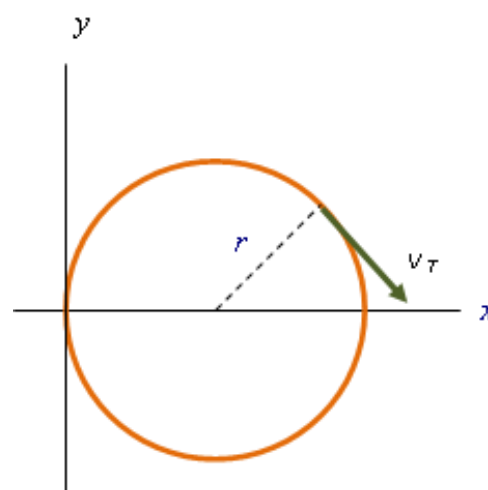


Fig. 3.1 (b)

- (a) The velocity of the particle can be further resolved into the transverse velocity v_T (Fig. 3.1 (b)) and a component parallel to the magnetic field v_z (Fig. 3.1 (a)).

Determine

- (i) the magnitude of the transverse velocity v_T of the alpha particle, [$5.00 \times 10^5 \text{ m s}^{-1}$]
- (ii) the time for the alpha particle to complete one revolution in the helix, [$2.51 \times 10^{-7} \text{ s}$]
- (iii) the velocity component of the alpha particle parallel to the field, v_z and [$2.89 \times 10^5 \text{ m s}^{-1}$]
- (iv) the pitch, p , of the helix. [7.25 cm]

- (b) A uniform electric field in the opposite direction to the magnetic field is turned on after the particle has completed a number of revolutions.

Describe and explain the subsequent motion of the alpha particle.

Solution

- (a) (i) For α particle, centripetal force (in x-y plane) is provided by the magnetic force

$$\begin{aligned}
 B q v_T &= \frac{m v_T^2}{r} \\
 v_T &= \frac{B q r}{m} \\
 &= \frac{0.519 (3.2 \times 10^{-19}) 0.02}{6.64 \times 10^{-27}} = 5.00 \times 10^5 \text{ m s}^{-1}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad T &= \frac{2\pi r}{v_T} \\
 &= \frac{2\pi (2.00 \times 10^{-2})}{5.00 \times 10^5} = 2.51 \times 10^{-7} \text{ s}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad \frac{v_r}{v_z} &= \tan(60^\circ) \\
 v_z &= \frac{v_T}{\tan 60^\circ} \\
 &= \frac{5.00 \times 10^5}{\tan 60^\circ} = 2.89 \times 10^5 \text{ m s}^{-1}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad \text{pitch, } p &= v_z T \\
 &= (2.89 \times 10^5)(2.51 \times 10^{-7}) = 0.0725 \text{ m}
 \end{aligned}$$

- (b) The α particle experiences an electrical force in the opposite direction to the magnetic field, (reducing v_z but not affecting the transverse speed v_T).

Hence, the pitch of the helical trajectory will decrease, but the period of revolution will stay the same.

Eventually the α particle subscribes a helical path in a **reversed** direction to the B-field with an increasing pitch, but with the same period of revolution.