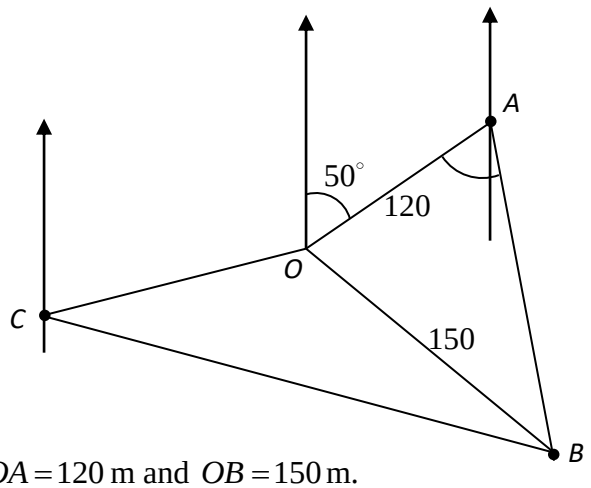
Topical Revision Paper 12

Term 3 Unit 12: Bearings, Sine Rule, Cosine Rule and Area of Triangle

Question 1:

The diagram below shows the position of three locations *A*, *B* and *C* with respect to a reference point *O*. The bearings of *A*, *B* and *C* from *O* are 050°, 135° and 240° respectively.



It is also given that $OA = 120$ m and $OB = 150$ m.

(i) Find the bearing of *B* from *A*. [2]

(ii) If the bearing of *B* from *C* is 100°, calculate the length of *OC*. [2]

(iii) A mast stands vertically at *B* such that the angle of elevation of the top of the mast from *O* is 62°. Find the height of the mast. [2]

Solution:

(i)

$\angle BOA = 135^\circ - 50^\circ = 85^\circ$

$AB = \sqrt{150^2 + 120^2 - 2(150)(120)\cos 85^\circ} = 183.7$

$\frac{\sin OAB}{150} = \frac{\sin 85^\circ}{AB} \Rightarrow OAB = 54.4^\circ$

Hence bearing of *B* from $A = 360^\circ - (180^\circ - 50^\circ) - 54.4^\circ = 175.6^\circ$

(ii)

$\angle COB = 240^\circ - 135^\circ = 105^\circ$ and $\angle OCB = 100^\circ - (180 - 120)^\circ = 40^\circ$

Hence by sine rule :

$$\frac{OC}{\sin 35^\circ} = \frac{150}{\sin 40^\circ}$$

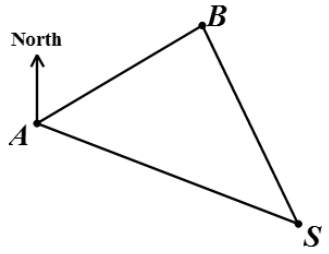
$OC = 133.8$ m

(iii) $\tan 62^\circ = \dfrac{\text{height}}{150}$

height $= 282.1$ m

Question 2:

In the diagram *A, B* and *S* represent three points in the sea. *B* is 3 km from *A* on a bearing of 060°. The bearings of *S* from *A* and *B* are 110° and 155° respectively.



(i) Calculate the distance *AS*. [2]

(ii) A ship at *S* sails directly to *A* at a steady speed of 2 kmh^{-1}. After 40 minutes, the ship is at *T*, a point on *AS*.

(a) Calculate the distance *BT*. [3]

(b) Find the bearing of *B* from *T*. [2]

(iii) A hot air balloon, *H*, is lowering at a point 600 m vertically above ***B***. Calculate the greatest angle of elevation of ***H*** from an observer on the ship as the ship sails from ***S*** to ***A***. [2]

Solution:

(i)

$\angle BAS = 110^0 - 60^0 = 50^0$

$\angle ABS = 60^0 + (180^0 - 155^0) = 85^0$

$\angle ASB = 180^0 - 85^0 - 50^0 = 45^0$

$$\frac{AS}{\sin 85^0} = \frac{3}{\sin 45^0}$$

$$AS = \frac{3siin85^0}{\sin 45^0} = 4.226 \approx 4.23km$$

(iv)
(a)
Distance = Speed x Time

$$= 2 \times \frac{40}{60} = \frac{4}{3} \text{km}$$

AT = 4.226 – 1.333 km = 2.893 km

Using cosine rule,

$$BT^2 = 3^2 + 2.893^2 - 2(3)(2.893)\cos 50^0$$

$$BT = 2.492$$

$$BT = 2.49km$$

(b)

$$\frac{\sin \angle ABT}{2.892} = \frac{\sin 50^0}{2.492}$$

$$\angle ABT = \frac{2.892 \sin 50^0}{2.492} = 62.74^0$$

$$\angle y = 62.74^0 - 60^0 = 2.74^0$$

The bearing of B from T is $360^0 - 2.74^0 = 357.26^0 \approx 357.3^0$

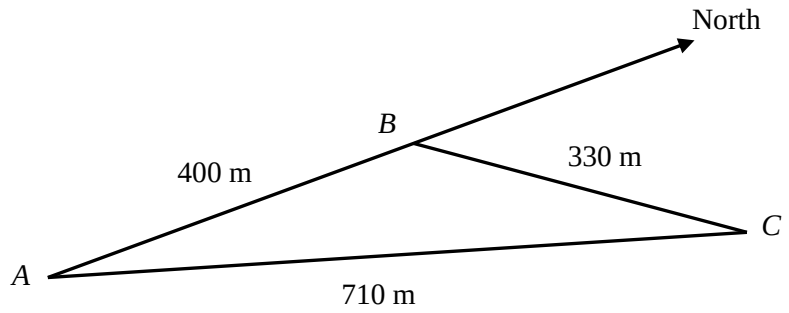
(iii)
Shortest distance from observer to $B = 3\sin 50^0$

$$\theta = \tan^{-1}\left(\frac{0.6}{3\sin 50^0}\right)$$

$$\approx 14.6^0$$

Question 3:

The diagram shows three points, *A, B* and *C* on a piece of horizontal land. It is given that *AB* = 400 m, *BC* = 330 m, *AC* = 710 m and *B* is due North of *A*.



(i) Calculate

(a) $\angle ABC$, [2]

(b) the area of triangle ABC, [2]

(c) the bearing of C from A. [2]

(ii) BZ is a tower crane standing vertically at B and the angle of elevation of Z from A is $18°$. Calculate the height of the tower crane. [2]

(iii) A car moves along the path AC until it reaches a point P which is nearest to the tower crane BZ. Calculate the largest angle of depression of the point P from Z. [2]

Solution:

(i)

(a)

$$\cos \angle ABC = \frac{400^2 + 330^2 - 710^2}{2 \times 400 \times 330}$$

$$\angle ABC = 153.0° \text{ (1 d.p)}$$

(b)

$$\text{Area of } \Delta ABC = \frac{1}{2} \times 400 \times 330 \times \sin(152.99)$$

$$= 30000 \text{ m}^2 \text{(3 s.f)}$$

(c)

$$\frac{\sin \angle BAC}{330} = \frac{\sin(152.988)}{710}$$

$$\angle BAC = 12.2°$$

Bearing is $012.2°$

(ii)
Let BZ be the height of the tower crane

$$\tan 18° = \frac{BZ}{400}$$
$$BZ = 130 \text{ m (3s.f)}$$

(iii)
BP is the shortest distance from B to path AC

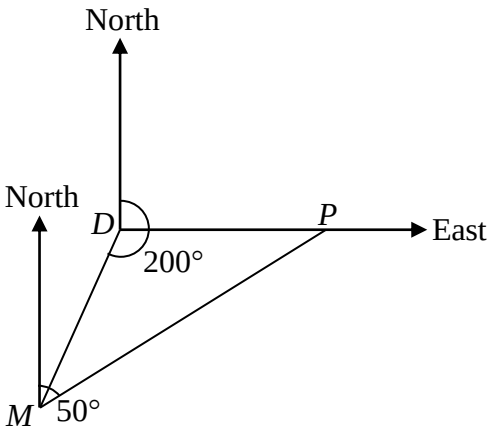
$$\frac{1}{2} \times 710 \times BP = 29975.69$$
$$BP = 84.44 \text{ m}$$

Let largest angle of depression be θ,

$$\tan \theta = \frac{129.97}{84.44}$$
$$\theta = 57.0° \text{ (1 d.p)}$$

Question 4:

A man, at point *M*, is 80 metres at a bearing of $200°$ from an observation deck, *D*, which stands 25 metres tall. He begins to walk to a point *P* which is due East of *D* at a bearing of $050°$.



Ignoring the man's height, calculate

(i) the angle of elevation of the top of the deck from the man at *M*, [2]

(ii) the distance he walks when the angle of elevation of the top of the deck from his position is the greatest, and [2]

(iii) the distance DP. [2]

Solution:

(i)

Angle of elevation

$= \tan^{-1}\frac{25}{80}$

$= 17.4°$ (1 d.p.)

(ii)

By Sine Rule, $\frac{DP}{\sin 30°} = \frac{80 \text{ m}}{\sin(180° - 30° - 110°)}$

$DP = \frac{80}{2\sin 40°}$ m

$DP = 62.2$ (3 s.f.) m

(iii)

$\measuredangle DMP$

$= 50° - (200° - 180°)$

$= 30°$

Distance he walks $= 80\cos 30°$ m

$= 40\sqrt{3}$ or 69.3 (3 s.f.) m