



Topic 18: Alternating Current

Content:

- A Characteristics of alternating currents
- B Rectification with a diode
- C The transformer

Learning Outcomes:

Candidates should be able to:

- (a) show an understanding of and use the terms period, frequency, peak value and root-mean-square (r.m.s.) value as applied to an alternating current or voltage
- (b) deduce that the mean power in a resistive load is half the maximum (peak) power for a sinusoidal alternating current
- (c) represent an alternating current or an alternating voltage by an equation of the form $x = x_0 \sin \omega t$
- (d) distinguish between r.m.s. and peak values and recall and solve problems using the relationship $I_{rms} = I_0 / \sqrt{2}$ for the sinusoidal case
- (e) show an understanding of the principle of operation of a simple iron-cored transformer and recall and solve problems using $N_s / N_p = V_s / V_p = I_p / I_s$ for an ideal transformer
- (f) explain the use of a single diode for the half-wave rectification of an alternating current.

A Characteristics of Alternating Currents

Understanding the Nature of Scientific Knowledge

During the 'War of the Currents' in the late 1880s, Thomas Edison (1847-1931) favoured direct current while George Westinghouse (1846-1914) argued for alternating current. Direct current at 110V used to be the standard for electricity distribution in United States in terms of safety and practicality. However, the use of alternating current eventually prevailed as it was more effective in minimising power loss with the use of transformers and high voltage in long-distance transmission. Historical competition between companies for electrical power distribution and other factors resulted in different standards for the mains voltage in different countries. Having different voltages and frequencies between countries, and for some, within the country, can create confusion and result in extra costs for electrical appliances and adaptors. In Singapore, 230V at 50 Hz is used. In line with technological changes in the way the society uses electricity, there is ongoing research to continue to find ways to minimize power losses in electrical power transmission. It appears that direct current now has an opportunity to dominate many modern electrical digital devices. The method that will eventually triumph in the future will be the one that results in the least power losses.



A.1 Introduction to Alternating Currents

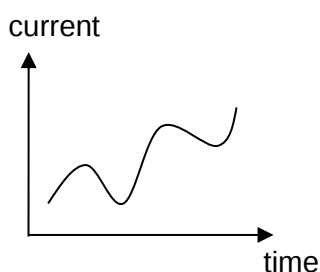
An **Alternating Current** is a current that varies periodically in direction.

Direct Current	Alternating Current
Polarity of d.c. supply remains constant with time.	Polarity of a.c. supply changes with time.
Current is unidirectional, from positive to negative terminal.	Current changes direction.
Current may increase and decrease in magnitude.	Current may increase and decrease in magnitude.

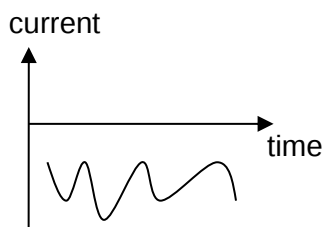
- The effects of a.c. are essentially the same as those of d.c. Both can be used for heating and lighting purposes.

Check Your Understanding 1

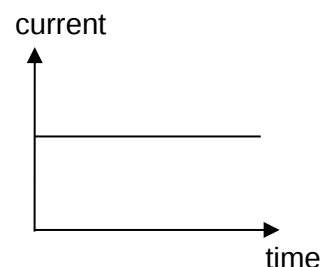
Which of the following are examples of direct current and which are alternating current?



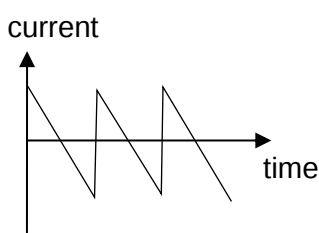
A



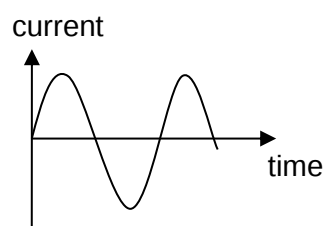
B



C



D



E

A.2 Root-Mean-Square Value of an Alternating Current / EMF

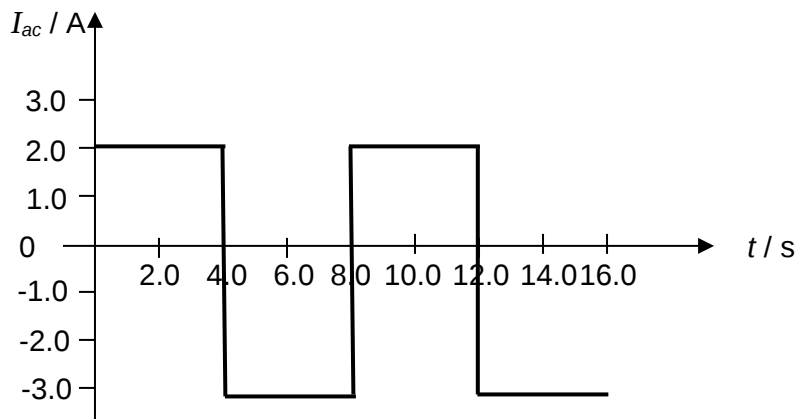
The **root-mean-square value of an alternating current** is the value of the steady direct current which produces heat at the same rate as the alternating current in a given resistor.

The **root-mean-square voltage** is defined as the steady direct voltage which produces heat at the same rate as the alternating voltage across a given resistor.



Worked Example

Consider a periodic alternating current waveform I_{ac} with period T of 8.0 s, passing through a resistor as shown below.



The root-mean-square current I_{rms} is mathematically determined by calculating the square root of the mean value of the square of the instantaneous current over one cycle.

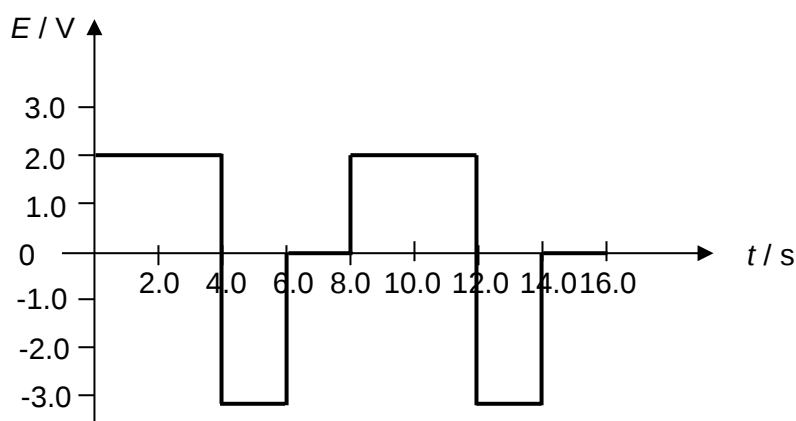
$$I_{rms} = \sqrt{\langle I_{ac}^2 \rangle} = \sqrt{\frac{\int_0^T I^2 dt}{T}} = \sqrt{\frac{\text{area under } I^2 \text{ curve for one cycle}}{\text{one period}}}$$
$$I_{rms} = \sqrt{\frac{2^2 \times 4 + (-3)^2 \times 4}{8}} = 2.55 \text{ A}$$

The root-mean-square value is therefore the effective value of the alternating current that produces the same average power dissipation in a resistor as a steady direct current of the same value. This means that for the waveform above, the same average power is dissipated in the resistor if the alternating current is replaced by a steady direct current of 2.55 A.

Note that this formula also applies to the r.m.s. value of an alternating e.m.f. source, as shown in Example 1.

Example 1

The variation with time t of an alternating e.m.f. source E is as shown below.



Recall the concept of root-mean-square from Thermal Physics (root-mean-square speed of molecules).

Steps to calculate r.m.s. value:
1. square
2. average
3. root



Determine, for the alternating e.m.f. source, the

- | | |
|-----------------------------|------------|
| (a) Frequency | [0.125 Hz] |
| (b) Peak e.m.f. | [3.0 V] |
| (c) Root-mean-square e.m.f. | [2.06 V] |

Thinking Process

Parts (a) and (b) involve direct application of concepts and reading off the graph.

Key question on objective:

How can the root-mean-square e.m.f. be calculated from a graph?

- The sequence should be:
 1. Square the values of e.m.f. for one complete cycle of the a.c.
 2. Average the squared values over one period
 3. Square root the average

Key questions to solve the problem:

What is meant by 'squaring the values' when applied to a graph?

- For this graph, 'square' refers to squaring the value of E at each point in time
- Graphically, this can be seen by sketching a E^2 against time graph.

What is meant by 'averaging over one period' when applied here?

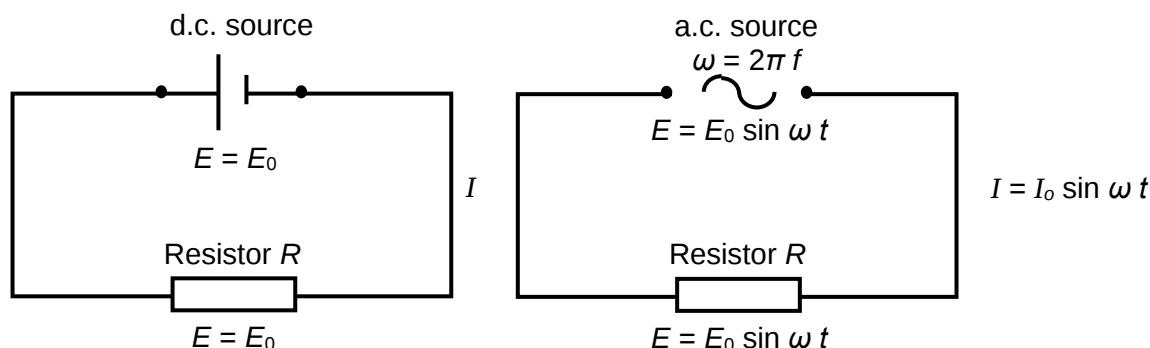
- Sum up all the values of E^2 multiplied by the time they are sustained over, then divide by the period
- Graphically, this can be seen as dividing the area under the E^2 - t graph for one period by one period.

Solutions



A.3 Power Dissipation by a Sinusoidal Alternating Current / EMF

- The simplest alternating e.m.f. can be represented by the sinusoidal waveform.
- Consider two circuits: the first with a d.c. source of E_0 , the second a sinusoidal a.c. source with peak value E_0 and frequency f connected to a resistor in the diagrams below.



ω is the angular frequency of the a.c. It can be calculated by $\omega = 2\pi f$

Note that we have encountered equations of this form for simple harmonic oscillations.

- The table below illustrates the differences in current, e.m.f. and power dissipation for both circuits.

Quantity	DC Circuit	AC Circuit
Instantaneous EMF	$E_{dc} = E_0$ Instantaneous, peak and mean e.m.f. are the same in value.	$E_{ac} = E_0 \sin \omega t$
Peak EMF		E_0
r.m.s. EMF		$E_{rms} = \sqrt{\langle E_{ac}^2 \rangle} = \frac{E_0}{\sqrt{2}}$ See mathematical proof on next page
Instantaneous Current	$I_{dc} = \frac{E_0}{R}$ Instantaneous, peak and mean current are the same in value.	$I_{ac} = \frac{E_{ac}}{R} = \frac{E_0 \sin \omega t}{R} = \left(\frac{E_0}{R} \right) \sin \omega t$
Peak Current		$I_0 = \frac{E_0}{R}$
r.m.s. Current		$I_{rms} = \sqrt{\langle I^2 \rangle} = \frac{I_0}{\sqrt{2}}$
Instantaneous Power Dissipated in Resistor	$P_{dc} = I_{dc}^2 R = \left(\frac{E_0}{R} \right)^2 R$ Instantaneous, peak and mean power are the same in value.	$P_{ac} = I^2 R = I_0^2 R \sin^2 \omega t$
Peak Power Dissipated in Resistor		$P_0 = I_0^2 R$
Mean Power Dissipated in Resistor		$\langle P \rangle = I_{rms} E_{rms} = \left(\frac{I_0}{\sqrt{2}} \right) \left(\frac{E_0}{\sqrt{2}} \right)$ $\langle P \rangle = \frac{I_0 E_0}{2} = \frac{P_0}{2}$ Mean power = $\frac{1}{2}$ Peak Power (only for sinusoidal ac)

It would not be meaningful to talk about mean value instead of r.m.s. value for sinusoidal alternating voltage or current because the mean value would be zero.



Proof of Formula $E_{rms} = \frac{E_0}{\sqrt{2}}$

- For a sinusoidal alternating e.m.f. $E_{ac} = E_0 \sin \omega t$, the root-mean-square e.m.f. is

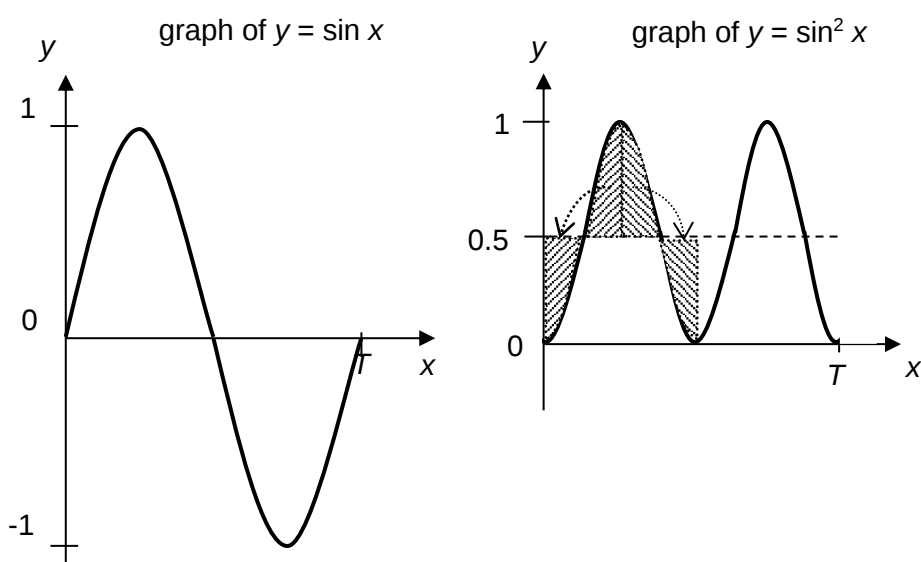
$$E_{rms} = \sqrt{\langle E_{ac}^2 \rangle} = \sqrt{\langle E_0^2 \sin^2 \omega t \rangle}$$

- Since E_0 is a constant,

$$E_{rms} = E_0 \sqrt{\langle \sin^2 \omega t \rangle} = \frac{E_0}{\sqrt{2}} \quad \text{as the mean value of } \sin^2 \omega t \text{ is } \frac{1}{2}$$

Note: The mean power is half the peak power is due to the symmetry (rotational) about the midpoint of the sinusoidal $V-t$ or $I-t$ graph.

Proof that $\sqrt{\langle \sin^2 x \rangle} = \frac{1}{\sqrt{2}}$:



- The area under the $\sin^2 x$ graph over one period T can be rearranged into a rectangle of constant height of 0.5.
- Therefore, the mean value of $\sin^2 x$ is $\langle \sin^2 x \rangle = \frac{1}{2}$
- The root of this mean value is $\sqrt{\langle \sin^2 x \rangle} = \frac{1}{\sqrt{2}}$

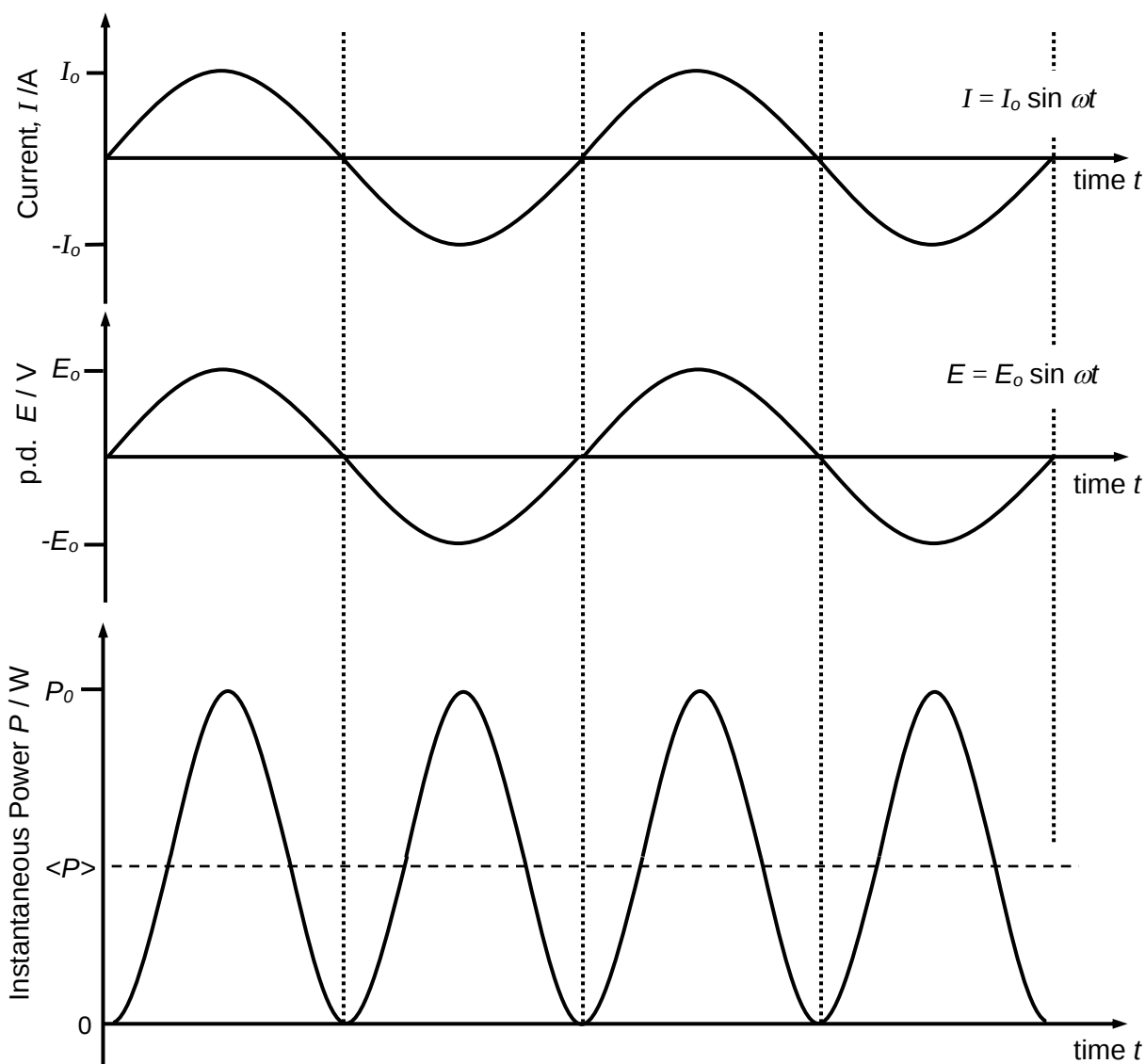


Check Your Understanding 2

A constant potential difference V and a sinusoidal potential with peak value V_0 dissipates the same average power in a given resistor. What is the relationship between V and V_0 ?

- A $V_0 = \frac{V}{2}$ B $V_0 = \frac{V}{\sqrt{2}}$ C $V_0 = V$ D $V_0 = \sqrt{2}V$ E $V_0 = 2V$

A.4 Graphical Representations of an Alternating Current or EMF and Power Dissipation in Resistor



Note: The current-time and the p.d.-time graphs are in phase.

Note: The period of the power-time graph is half that of the alternating current and voltage.

Since the power varies over a cycle, the mean power dissipation is a more meaningful representation.

We can therefore say that the a.c. through the resistor has a **constant mean power dissipation of $\langle P \rangle$** .

**Worked Example**

A sinusoidal current is represented by the equation

$$I = I_0 \sin \omega t$$

- (a) Write down the expression of the current if both the frequency and amplitude are doubled.
- (b) Determine the new root-mean-square current in terms of I_0 .

Thinking Process

- (a) Key question on objective:

How is a sinusoidal a.c. represented by an equation?

- The amplitude is represented by I_0
- The frequency is related to ω by $\omega = 2\pi f$

- (b) Key question on objective:

How is root-mean-square current calculated for a sinusoidal a.c.?

- Direct application of equation

Solutions

- (a) When the frequency is doubled, the angular frequency ω would be doubled as well. When the amplitude is doubled, the peak current I_0 would be doubled. Hence, the new expression would be: $I = 2I_0 \sin 2\omega t$

- (b)

$$\text{root-mean-square current for sinusoidal a.c.} = \frac{\text{peak current}}{\sqrt{2}}$$

$$\text{Here, } I_{rms} = \frac{2I_0}{\sqrt{2}} = \sqrt{2}I_0 = 1.41I_0$$

Example 2

A steady current of 2.0 A dissipates a certain power in a variable resistor. The variable resistor is now connected to a sinusoidal alternating current.

- (a) If the power dissipation remains the same, state, with a reason, the r.m.s. value of the alternating current. [2.0 A]
- (b) If the resistance has to be halved in order to obtain the same power dissipation, calculate the r.m.s. value of the alternating current. [2.83 A]



Thinking Process

(a) Key question on objective:

What is the significance of the root-mean-square value in terms of power dissipation?

- By definition, an a.c. with root-mean-square current of 2.0 A will dissipate the same power as a constant d.c. of 2.0 A.

(b) Key questions to solve problem:

How is power dissipation related to resistance?

- When a current I passes through a resistance R , the power dissipated is I^2R .

Since power is the same for the a.c. and d.c. in this question, how is current related to resistance?

- Resistance should be inversely related to the square of the current.

Solutions

Check Your Understanding 3

A resistor is connected in series with a sinusoidal alternating current supply of negligible internal resistance. The peak value of the supply voltage is V_0 and the peak value of the current in the resistor is I_0 . The average power dissipation in the resistor is

A $\frac{I_0 V_0}{2}$

B $\frac{I_0 V_0}{\sqrt{2}}$

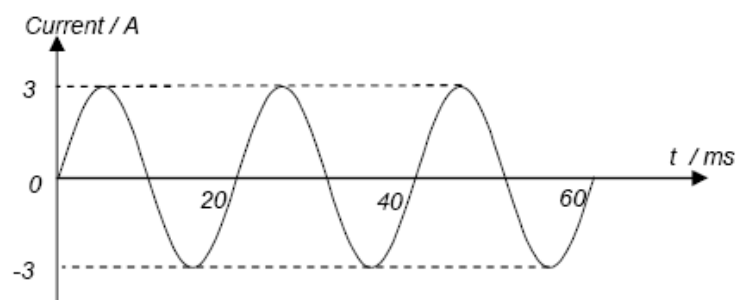
C $I_0 V_0$

D $2I_0 V_0$



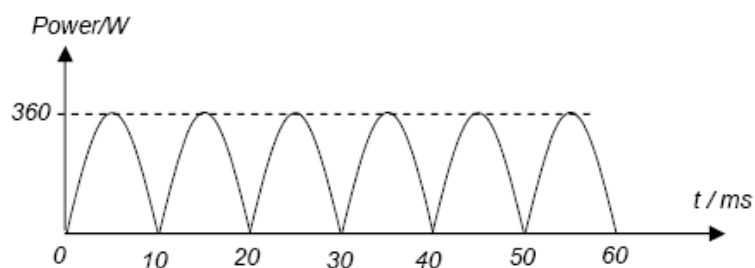
Check Your Understanding 4

A sinusoidal alternating current as shown below flows through a resistor of $80\ \Omega$.

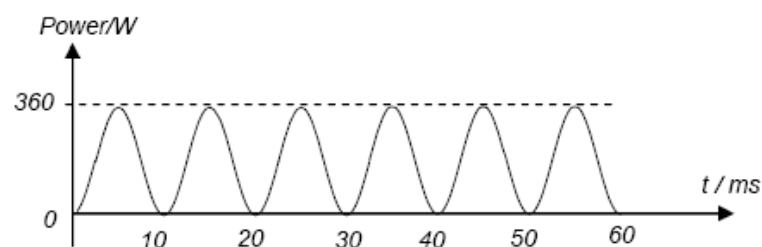


The power dissipated in the resistor is given by

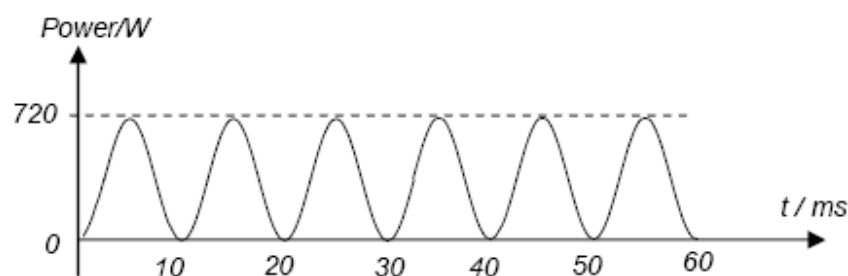
A



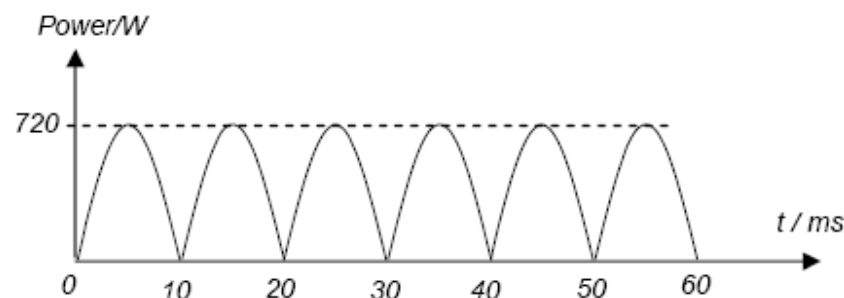
B



C



D



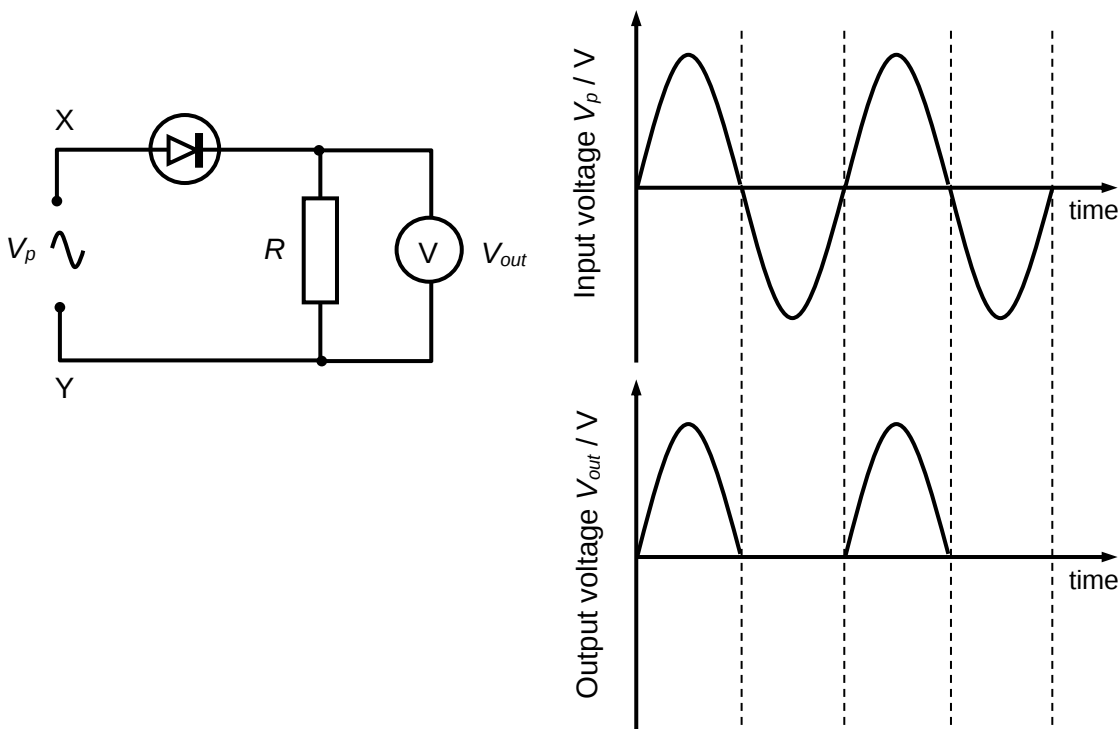


B Rectification with a Diode

- The e.m.f. obtained from the mains is alternating in nature and a means of obtaining d.c. from a.c. is necessary. This is because processes such as electroplating and battery charging, as well as electronic devices such as computers and television sets, require direct current to work.
- The process of converting a.c. to d.c. is known as rectification. This is achieved through the use of rectifiers which conduct appreciable amounts of current in one direction only.
- An example of a rectifier is the diode.
 - When a diode is forward-biased, it conducts electricity with minimal resistance.
 - When a diode is reverse-biased, it allows little or no current flow.
- Hence, devices connected to an a.c. source via a diode experience current in one direction only.

Half-wave Rectification using a Diode

- In the figure below, the diode conducts only during the half of the cycle when X is at a higher potential than Y (i.e. X is positive with respect to Y).
- The output p.d. across the resistor R is a pulsating d.c.

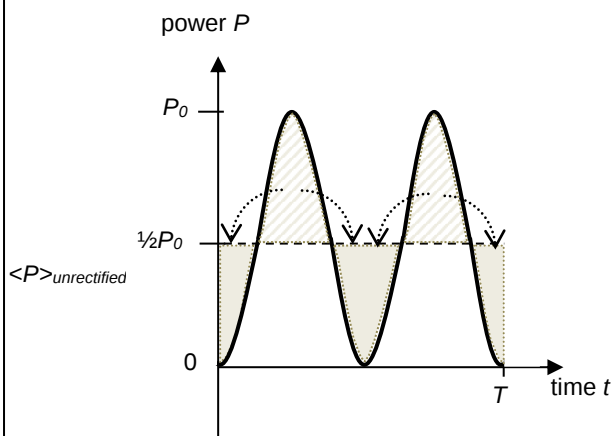


There is also full-wave rectification, which can be achieved through a Wheatstone Bridge. (Refer to Appendix)

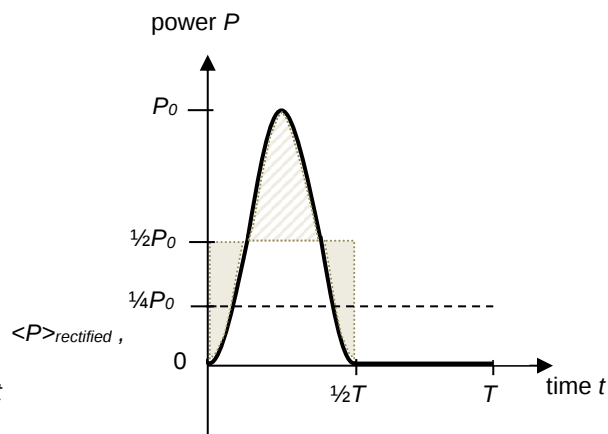
- For half-wave rectification, the resistor only receives current half of the time and there is power dissipation only half of the time.



power dissipation **before** rectification



power dissipation **after** rectification



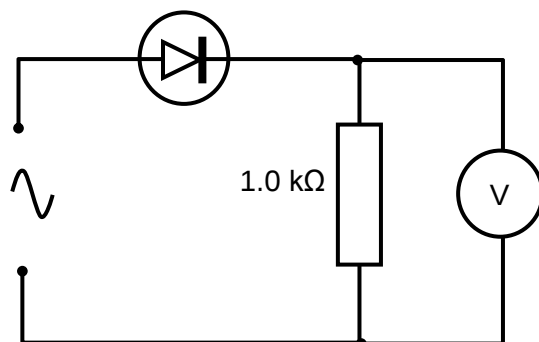
- For a sinusoidal a.c., the maximum power for rectified circuit remains the same as that before rectification.
- Since there is power dissipated only during half of the cycle, the mean output power for the rectified circuit would be half the mean output power before rectification.
- The following table summarizes the relationship between important quantities for a sinusoidal a.c. before and after half-wave rectification.

	Sinusoidal a.c.	Half-wave Rectified Sinusoidal a.c.
Peak Voltage		V_0
Peak Current		I_0
Maximum Power		P_0
		} remains the same after rectification
Root-mean-square current I_{rms}	$\frac{I_0}{\sqrt{2}}$	$\frac{I_0}{2}$
Root-mean-square voltage V_{rms}	$\frac{V_0}{\sqrt{2}}$	$\frac{V_0}{2}$
Mean power $\langle P \rangle$	$\frac{P_0}{2}$	$\frac{P_0}{4}$



Example 3

The figure below shows a voltmeter that reads 65 V when measuring the potential difference across a $1.0 \text{ k}\Omega$ resistor connected to a sinusoidal power source with angular frequency ω .



- (a) Calculate the average power dissipated by the resistor in the above circuit. [4.23 W]
(b) Write down the equation for the variation of the instantaneous power (in watts) with respect to time provided by the source. [$16 \sin^2 \omega t$]

Thinking Process

(a) Key question on objective:

How can average power dissipated be calculated from the voltmeter reading?

- Recall that the voltmeter works based on heating effect of the a.c.
- Hence, the reading of 65 V is the 'd.c. equivalent' of the a.c., i.e. the root-mean-square value
- Average power can be found by using the r.m.s. value in the normal equation $P = V^2/R$

(b) Key question on objective:

How does the presence of a diode affect the circuit?

- The a.c. is rectified such that there is p.d. across the resistor only during half of each cycle.
- The average power calculated earlier is only half the value of the average power for the a.c. source before rectification, or one quarter of the maximum power.

Key questions to solve the problem:

What form should the equation take?

- Again using the equation $P = V^2/R$, the form of the equation should be $P = (V_0^2 / R) \sin^2 \omega t$
- V_0^2 / R is the maximum power dissipated for the a.c. before rectification

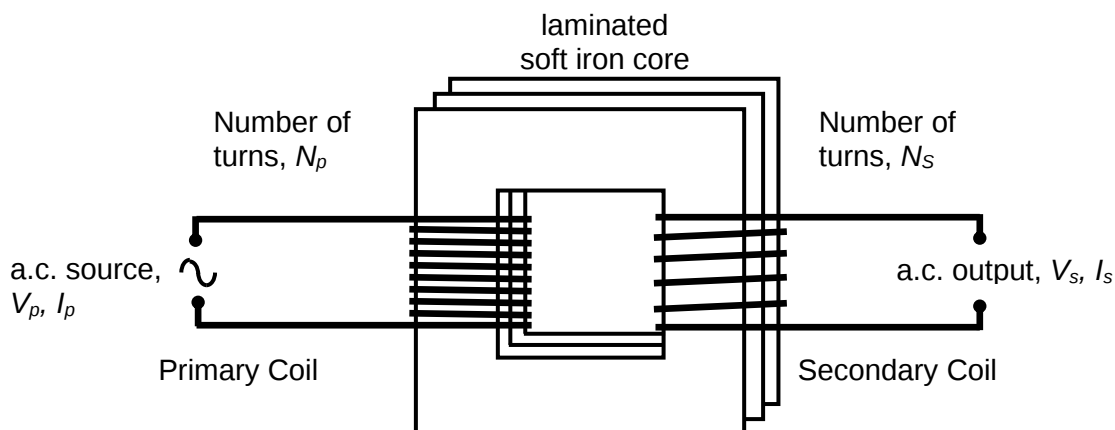
Solutions



C The Transformer

C.1 How a Transformer Works

- A transformer is a device for increasing or decreasing an a.c. voltage using the principles of electromagnetic induction.



- Two insulated copper coils, called the primary and secondary coils, are wound on separate limbs of a laminated soft iron core.
- The coils are not electrically connected to one another. Hence, there is no current flow from the primary coil to the secondary coil.
- The soft iron core ensures that most of the magnetic flux associated with one coil also passes through the other i.e. no flux leakage. It is also usually laminated to reduce energy losses caused by induced currents in the core.
- For a transformer with an alternating e.m.f. V_p connected to the primary coil, an alternating current I_p flows in the primary coil.
- This alternating current sets up a changing magnetic field.
- The iron core links the magnetic flux ϕ associated with the primary coil to the secondary coil.
- The secondary coil with N_s turns experiences a changing magnetic flux linkage.
- By Faraday's Law, an e.m.f. V_s is induced in the secondary coil where

$$V_s = - N_s \frac{d\phi}{dt}$$

Take note that the topic on transformer is often cross-linked with the topic of electromagnetic induction.

Note that the secondary coil is not connected to a power supply and the secondary voltage is induced.



- For an ideal transformer (i.e. input power in primary circuit is equal to output power in secondary circuit), since V is proportional to N ,

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

Where V_p = input e.m.f. at primary circuit,
 V_s = output e.m.f. at secondary circuit,
 N_p = number of turns in primary coil and
 N_s = number of turns in secondary coil.

- If an external load is connected to the secondary coil, an induced current I_s will flow in the secondary circuit.
- If I_p is the primary current for an ideal transformer,

Power input = Power output

$$I_p V_p = I_s V_s \quad \Rightarrow \quad \frac{V_s}{V_p} = \frac{I_p}{I_s}$$

- In summary, for an ideal transformer,

$$\frac{V_s}{V_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p}$$

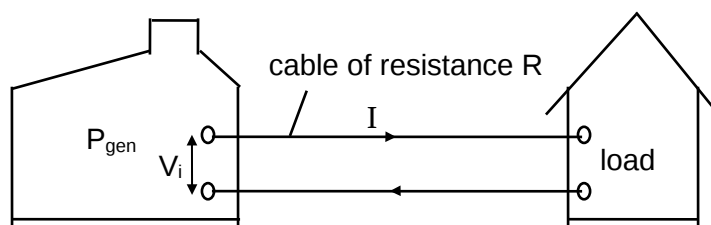
- Note that the above formula can be used either when comparing the peak values of the current and e.m.f., or when comparing the rms values of the current and e.m.f.
- If $N_s > N_p$, the transformer is called a step-up transformer since $V_s > V_p$. If $N_s < N_p$, the transformer is called a step-down transformer since $V_s < V_p$.
- The equation tells us that an ideal transformer that steps up the voltage simultaneously steps down the current and a transformer stepping down the voltage steps up the current.
- However, the power is neither stepped up nor stepped down in an ideal transformer.



- The following table shows the causes of power loss for practical transformers, and ways to reduce the power loss:

Causes of Power Loss	Ways to Reduce Power Loss
Power loss due to <u>resistance in the windings</u>	Minimise resistance in windings by using: - Wires with larger cross-sectional area - Wires with lower resistivity
<u>Magnetic flux leakage</u> , where not all the magnetic flux associated with the primary coil passes through the secondary coil	The soft iron core concentrates the magnetic field lines, channeling more of the magnetic flux from the primary coil to the secondary coil.
Power loss due to <u>induced currents</u> in the iron core Soft iron core is a conductor, hence current can be induced in it when the magnetic flux changes.	The soft iron core is laminated to reduce the eddy currents.
<u>Hysteresis losses</u> due to repeated magnetization of the core The core may retain some magnetic properties even when the current is attempting to induce a magnetic field in the opposite direction.	

C.2 Use of Transformer in Electrical Power Transmission



One of the problems in the transmission and distribution of electrical energy from the power station to the consuming loads (households and factories) is the **power loss** as Joule heating (I^2R) in the grid cables. This loss should be minimised for efficiency and economy.

- One solution is to have very **thick cables** so that the **resistance R is low**. Then the power lost as heat in the cables will be a minimum. However, there is a limit to the extent to which this can be done: the thicker the cable used, the heavier is the weight to be supported and hence the higher the construction cost.



2. Another solution is to **reduce the current I in transfer**. This can be done by **using transformers to step up the voltage** at which the electrical power is to be transmitted.

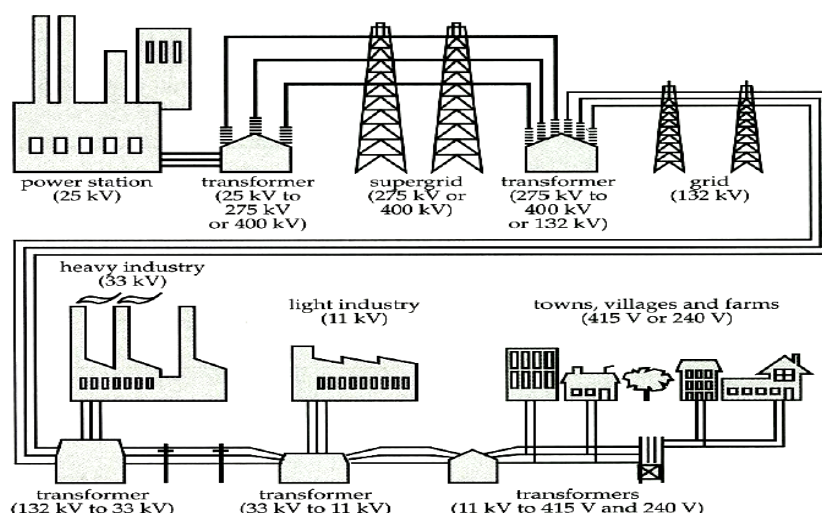
Suppose the electrical power generated, P_{gen} is to be delivered at a p.d. of V_i by the supply

lines of total resistance R . Then the current I in the supply line will be: $I = \frac{P_{gen}}{V_i}$

Hence, the power lost as heat will be given by: $P_{loss} = I^2 R = \left(\frac{P_{gen}}{V_i} \right)^2 R$

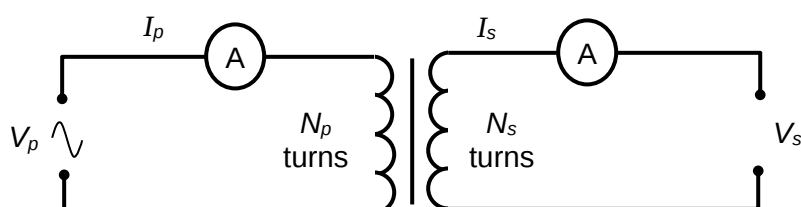
Thus the greater the value of V_i , the lower the power loss. Hence, electrical power can be transmitted more economically at high voltage and low current.

For transmission of electrical power, step-down transformers can be used subsequently to step down the voltage in stages to provide the required voltages for industries and homes as shown in the diagram below, as most home appliances do not operate on high voltages.



Example 4

In a laboratory experiment to test a transformer, a student used the circuit shown in the diagram to take measurements.



Two of the original entries in the student's results table are missing as shown.

V_p / V	I_p / mA	N_p turns	V_s / V	I_s / mA	N_s turns
240	2.0			50	50



Assuming the transformer was 100% efficient, complete the table above.

Thinking Process

Direct application of the equation for ideal transformers

Solutions**Example 5**

A hydroelectric plant generates electrical power of 15.0 MW and transmits it at a potential of 765 kV r.m.s. to home consumers. The transmission cables have an effective resistance of 320 Ω . Calculate

- (a) the peak current passing through the cables.
- (b) the power loss through the cables.
- (c) the potential difference across the cables.

Thinking Process

Key question on objective:

Does the high potential of 765 kV refer to the potential difference across the cables?

- No, the high potential refers to the potential when the cables leave the power plant.
- Ideally the potential difference across cables should be low in order to minimize power loss.

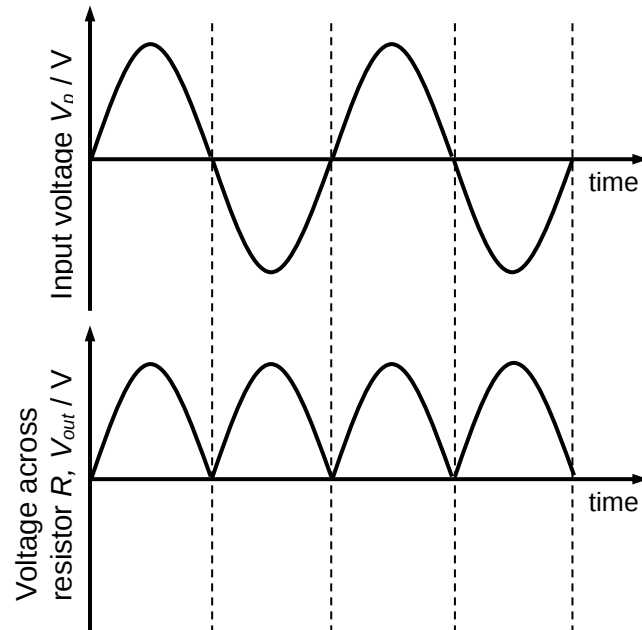
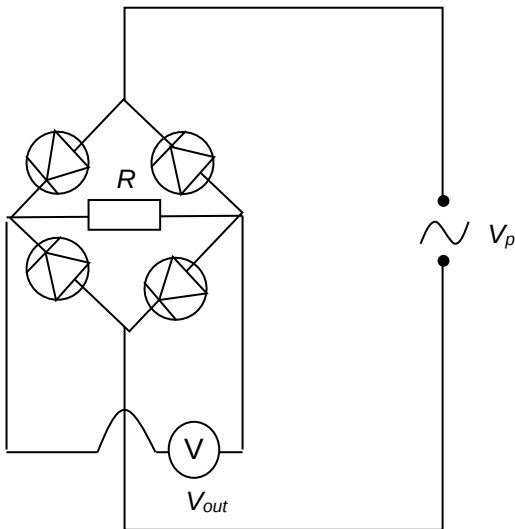
Solving of question involves simple application of equations from Current of Electricity, together with the new equations from this chapter.

Solutions



Appendix

The following setup is a Wheatstone Bridge made up of 4 diodes, which is capable of doing a full wave-rectification. Although the power supply is an a.c. source, the current flowing through resistor R is a direct current.





Summary of Equations

Terms / Concepts	Equations	Description
equation representing instantaneous voltage or current of sinusoidal a.c.	$x = x_0 \sin \omega t$	x = instantaneous voltage or current of a sinusoidal a.c. x_0 = maximum voltage or current of a sinusoidal a.c.
generic equation for calculating r.m.s values	$I_{rms} = \sqrt{\langle I_{ac}^2 \rangle} = \sqrt{\frac{\int_0^T I^2 dt}{T}} = \sqrt{\frac{\text{area under } I^2 \text{ curve for one cycle}}{\text{period}}}$	I_{rms} = rms current $\langle I_{ac}^2 \rangle$ = average of the squared value of the instantaneous a.c. over one period I = instantaneous a.c. current
r.m.s. value of a sinusoidal a.c.	$I_{rms} = \frac{I_0}{\sqrt{2}}$ $V_{rms} = \frac{V_0}{\sqrt{2}}$	I_{rms} = rms current I_0 = maximum instantaneous current V_{rms} = rms voltage V_0 = maximum instantaneous voltage
mean power dissipation in an a.c. circuit	$\langle P \rangle = I_{rms} V_{rms}$ $\langle P \rangle = \left(\frac{I_0}{\sqrt{2}} \right) \left(\frac{V_0}{\sqrt{2}} \right) = \frac{1}{2} I_0 V_0$ $\langle P \rangle = \frac{1}{2} P_0$	$\langle P \rangle$ = mean power dissipation P_0 = maximum instantaneous power I_{rms} = rms current I_0 = maximum instantaneous current V_{rms} = rms voltage V_0 = maximum instantaneous voltage
mean power dissipation in an half-wave rectified a.c. circuit	$\langle P \rangle_{rectified} = \frac{1}{2} \langle P \rangle_{unrectified}$ $\langle P \rangle_{rectified} = \frac{1}{4} P_0 = \frac{1}{4} I_0 V_0$	$\langle P \rangle_{rectified}$ = mean power dissipation for a.c. after half-wave rectification $\langle P \rangle_{unrectified}$ = mean power dissipation for a.c. before rectification P_0 = maximum instantaneous power I_0 = maximum instantaneous current V_0 = maximum instantaneous voltage



Relationship between primary and secondary values in an ideal transformer	$\frac{V_s}{V_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p}$	V_p = potential difference across primary coil V_s = potential difference across secondary coil
For step-up transformers	$V_s > V_p$ $N_s > N_p$ $I_s < I_p$	I_p = current flowing in primary coil I_s = current flowing in secondary coil N_p = number of turns in primary coil N_s = number of turns in secondary coil
For step-down transformers	$V_s < V_p$ $N_s < N_p$ $I_s > I_p$	

Glossary

Period	Time taken for the alternating current to complete one cycle.
Frequency	The number of complete cycles of the alternating current per unit time.
Amplitude or peak value	Maximum value of the a.c. in either direction.
Peak-to-peak value	Sum of the maximum values of the a.c. in both directions (ignore the negative sign).
Root-mean-square value of an alternating current	The value of the <u>steady direct current</u> which produces <u>heat at the same rate</u> as the alternating current in a given resistor.
Rectification	The process of obtaining a d.c. from an a.c.