JURONG PIONEER JUNIOR COLLEGE
9478 H2 PHYSICS

COLLISIONS

Content

(1) Impulse
(2) Conservation of momentum and energy

Learning Outcomes

Students should be able to:

(a) recall that impulse is given by the area under the force-time graph for a body and use this to solve problems

(b) state the principle of conservation of momentum

(c) apply the principle of conservation of momentum to solve simple problems including inelastic and (perfectly) elastic interactions between two bodies in one dimension (knowledge of the concept of coefficient of restitution is not required)

(d) show an understanding that, for a (perfectly) elastic collision between two bodies, the relative speed of approach is equal to the relative speed of separation

(e) show an understanding that, whilst the momentum of a closed system is always conserved in interactions between bodies, some change in kinetic energy usually takes place.

Impulse

Objective:
a) recall the impulse is given by the area under the force-time graph for a body and use this to solve problem.

This learning outcome was covered in the previous topic "Motion and Forces".

Recall: Newton's second law states that:

The rate of change of momentum of a body is (directly) proportional to the resultant force acting on the body and is in the same direction as the resultant force.

where the **momentum *p* of a body is the product of its mass *m* and instantaneous velocity *v***. Mathematically:

$$p = m\,v$$

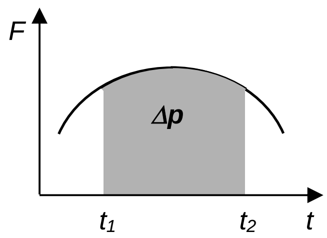
Momentum is a **vector** quantity and its unit is kg m s^{-1} or N s.

Recall Newton's second law:

$$F = \frac{dp}{dt}$$

Integrating both sides give:

$$dp = F\,dt$$
$$\int dp = \int F\,dt$$
$$\Delta p = \int F\,dt$$



If a force-time (F–t) graph is available, the **change in momentum of a body, Δp,** could be obtained from the **net area under the graph**. Suppose the variation of an applied force $F(t)$ with time t is as shown in the given graph.

The change in momentum, Δp, due to $F(t)$ from time t_1 to t_2 is then given by:
$\Delta p = p_2 - p_1$ = area under F–t graph
where p_1, p_2 = momentum at t_1 , t_2 respectively

Alternatively, if $\langle F \rangle$ is the average force acting, the change in momentum can also be given by the area of the rectangle $\langle F \rangle \Delta t$, which has the same value as the area below the curve.

This area is also known as the impulse.

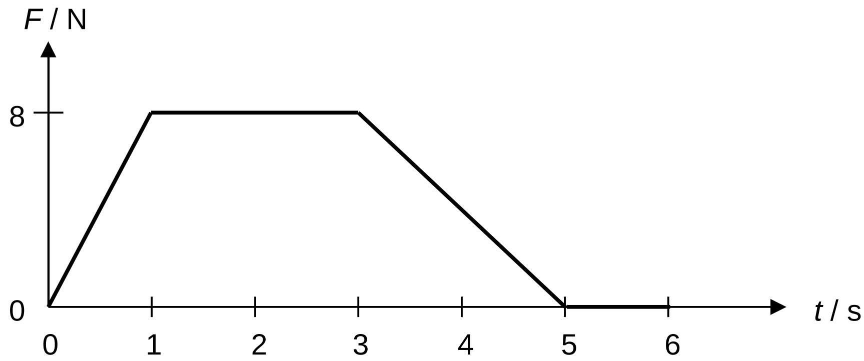
Impulse is the product of the average net force and the time it acts on a body.

Note:
Impulse of the force acting on a body can also be seen as the **change in momentum of the body caused by the force**. Its unit is **N s or kg m s^{-1}**.

When you hit a tennis ball with your racket the ball changes its momentum. We say that the racket has delivered an **impulse, I** to the ball.

Example 1

A body of mass 2.0 kg initially at rest is acted on by a force F which varies with time t as shown below.



Determine the momentum of the body at time $t = 6$ s.

Solution

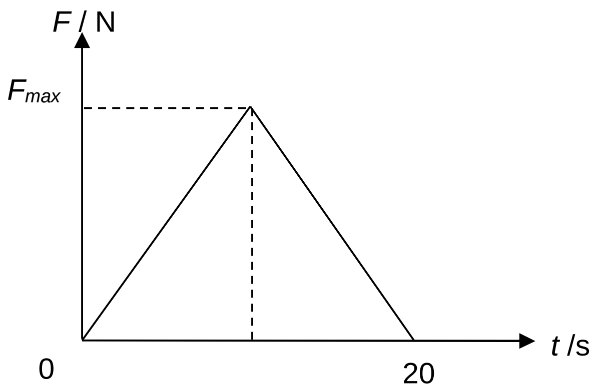
The area under the F–t graph is the change in momentum Δp experienced by the body.

Example 2

To stop a car of mass 1500 kg travelling at 30 m s^{-1}, the driver applies his brakes so that F, the total stopping force increases steadily to a maximum and then decreases to zero as shown on the graph.

Determine the following quantities:

a) The momentum of the car when it is travelling at 30 m s^{-1},
b) the change in momentum between $t = 0$ and $t = 10$ s,
c) the value of F_{max}, and
d) the value of $F_{average}$



Solution

Conservation of momentum and energy

Objectives:

(b) State the principle of conservation of momentum.

(c) Apply the principle of conservation of momentum to solve simple problems including inelastic and (perfectly) elastic interactions between two bodies in one dimension (knowledge of the concept of coefficient of restitution is not required).

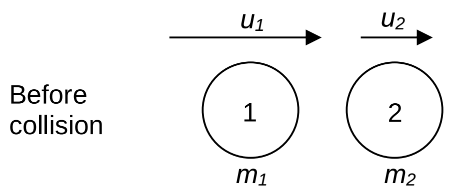
(d) Show an understanding that, for a (perfectly) elastic collision between two bodies, the relative speed of approach is equal to the relative speed of separation.

(e) Show an understanding that, whilst the momentum of a closed system is always conserved in interactions between bodies, some change in kinetic energy usually takes place.

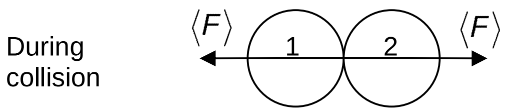
- Applying **Newton's third law** to closed collision systems will lead us to the **principle of conservation of momentum**.

- By a **system**, we mean a set of chosen objects which may interact with each other. A **closed (or isolated) system** is one in which the only (significant) forces are those between the objects in the system. Typically, we consider a closed system of two interacting bodies.

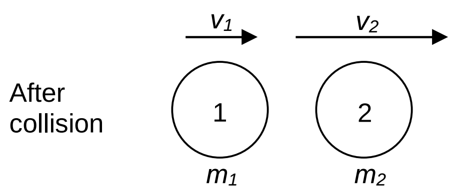
- Consider an object 1, of mass m_1 and velocity u_1, colliding with object 2, of mass m_2 and velocity u_2, moving in the same direction. (u_1 will have to be larger than u_2 in order for them to collide.)



- During the collision, an average force $\langle F \rangle$ is exerted by object 1 on object 2 (right). By Newton's third law, an equal and opposite $\langle F \rangle$ is exerted by object 2 on object 1 (left).



- We consider average force $\langle F \rangle$ in this case because the force may not be constant during the duration of contact.
- In the very short time interval Δt that both objects are in contact with each other, they will experience the same magnitude of impulse $\langle F \rangle \Delta t$ but opposite in direction.
- If object 1 moves with a reduced velocity v_1 after collision, object 2 will move with an increased velocity v_2 :



Taking right to be the positive direction:

Change in momentum of object 1: $-\langle F \rangle \Delta t = \Delta p_1 = m_1v_1 - m_1u_1$ -------------- (1)
Change in momentum of object 2: $\langle F \rangle \Delta t = \Delta p_2 = m_2v_2 - m_2u_2$ -------------- (2)

NOTE: Δp_1 is negative; Δp_2 is positive.
Substituting (2) into (1) gives:

$$m_1v_1 - m_1u_1 = -(m_2v_2 - m_2u_2)$$
$$m_1v_1 - m_1u_1 = -m_2v_2 + m_2u_2$$

Therefore $\boxed{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$

Total momentum of the system before collision = Total momentum of the system after collision

- The **principle of conservation of momentum** states that:

The total momentum of a system is constant, provided no external resultant force acts on the system.

- The principle of conservation of momentum can also be understood using the following equation:

$$\boxed{\Delta p_1 + \Delta p_2 = 0}$$

- The principle of conservation of momentum is applicable to <u>any system</u> as long as there is no net external force acting on the system.

- The principle of conservation of momentum was proven by assuming that the <u>directions</u> of travel of objects 1 and 2 are the <u>same</u> before and after the collision, but the results are <u>valid</u> even if they were <u>travelling in opposite directions</u>.

- There are two common closed systems within the scope of this syllabus:
  - collisions and
  - disintegration.

Collisions

There are two types of collisions: elastic collisions and inelastic collisions.

(Perfectly) Elastic Collisions

- (Perfectly) elastic collisions are those in which the **<u>total kinetic energy is conserved</u>**.

- Truly elastic collisions can only occur in practice on an atomic scale i.e. collisions between atoms and molecules.

- By principle of conservation of momentum:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad \text{---- (1)}$$

- By conservation of K.E.:
K.E. of system before collision = K.E. of system after collision

$$\frac{1}{2}m_1u_1^{\ 2} + \frac{1}{2}m_2u_2^{\ 2} = \frac{1}{2}m_1v_1^{\ 2} + \frac{1}{2}m_2v_2^{\ 2} \qquad \text{---- (2)}$$

These two equations can be used to solve problems involving elastic collisions.
From equations (1) and (2), the following equation can be derived (refer to Appendix):

$$\boxed{u_1 - u_2 = v_2 - v_1} \qquad \text{---- (3)}$$

(**NOTE**: (1) and (3) are **<u>vector</u>** equations)

which shows that the **relative speed of approach ($u_1 - u_2$) before collision is equal to the relative speed of separation ($v_2 - v_1$) after collision.**

Hence for **<u>(perfectly) elastic collisions</u>,** it is sufficient to use equations (1) and (3), instead of (1) and (2).

Inelastic Collisions

- Inelastic collisions are those in which **<u>total kinetic energy is not conserved</u>**: it may be converted to heat that is dissipated to the surroundings, and to a lesser extent, sound energy.

- In the real world inelastic collisions are the most common type of collision.

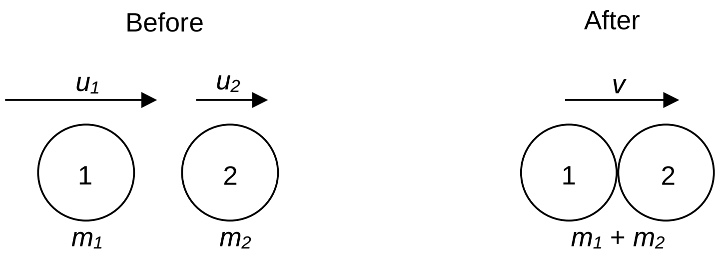
- By principle of conservation of momentum:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad \text{---- (1)}$$

Only equation (1) can be used for inelastic collisions to solve problems.

Equation (2) <u>cannot</u> be used since K.E. is not conserved.

- A special form of inelastic collision called **<u>completely / perfectly inelastic collision</u>** is one in which the **<u>two bodies stick together (or coalesce) after impact</u>** (e.g. a bullet being embedded in a target).



Taking right to be positive:

By principle of conservation of momentum:

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

where
v is the **<u>common velocity</u>** after collision.

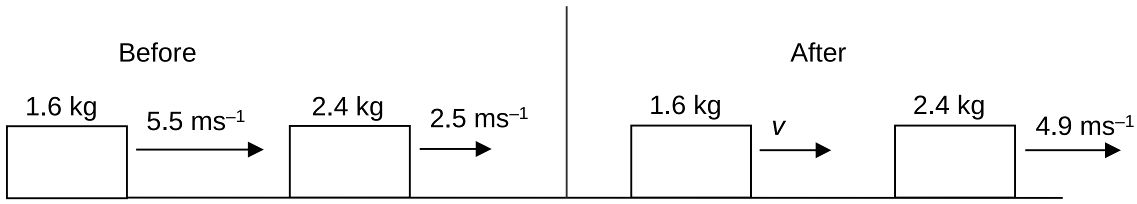
*Note that total momentum of the system is **always** conserved for both (perfectly) elastic and inelastic collisions.

Summarising the information in a table,

Interaction	Total Momentum	Total Kinetic Energy	Remarks
Elastic / Perfectly elastic	conserved	conserved	**the relative speed of approach = relative speed of separation.** The objects <u>always separate</u> after interaction
Inelastic		**not conserved** [some KE changed to other forms of energy]	The objects <u>always separate</u> after interaction
Perfectly inelastic			The objects <u>stick together (or coalesce)</u> after interaction.

<u>Example 3</u>

The blocks in the figure below slide without friction.
(a) What is the velocity v of the 1.6 kg block after the collision?
(b) Is the collision elastic?



<u>Solution</u>

By principle of conservation of momentum and taking right as positive:
(a) $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

(b) K.E. before collision $= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 =$

K.E. after collision $= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 =$

Since K.E. is conserved, the collision is elastic.

OR

$u_1 - u_2 =$

$v_2 - v_1 =$

Since relative speed of approach = relative speed of separation, the collision is elastic.

Example 4

Two trolleys X and Y are about to collide. The momentum of each trolley before impact is given in the figure below.

After the collision, the trolleys travel in opposite directions and the momentum of trolley X is 2 kg m s^{-1}. What is the momentum of trolley Y?

Solution

Before: After:

By conservation of linear momentum, taking right as positive:

The momentum of trolley Y is

Example 5

A particle of mass m moving with speed u makes a head-on collision with an identical particle which is initially at rest. The particles coalesce (stick together) and move off with a common velocity.

(a) Determine the common speed of the particles after the collision.
(b) Determine the ratio of the kinetic energy of the system after the collision to that before it.
(c) Explain the difference in kinetic energy of the system before and after the collision.

Note: A **head-on** collision is one where the objects remain moving along the same straight line joining their centres of mass before and after collision. **(i.e. motions are collinear).**

Solution

(a) By principle of conservation of momentum and taking right positive,

=

where v is the common speed after collision.

$\therefore\ v =$

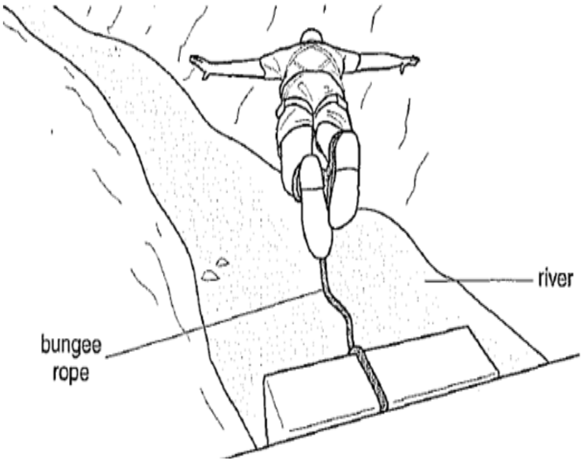
(b) Initial kinetic energy, $E_1 = \frac{1}{2}mu^2$

Final kinetic energy, $E_2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = \frac{1}{2}(2m)v^2 =$

$\Rightarrow$ Hence, the ratio of $\frac{E_2}{E_1} =$

(c) Half of the initial kinetic energy is converted to heat that is dissipated to the surroundings, and to a lesser extent, sound energy.

Example 6



The momentum of the person increases during free fall. Explain whether or not this is a violation of the principle of conservation of momentum.

Solution

This is **not** a violation because total momentum is conserved for a system of interacting bodies, and not just for one body. For the man alone, his momentum will change as there is an external resultant force (i.e. gravitational force) acting on him.

If we consider the man, Earth and bungee rope as the system, then the total momentum remains constant although the man's momentum changes.

As the man falls, the Earth moves upwards with a very tiny velocity (due to its large mass). Hence total momentum remains unchanged.

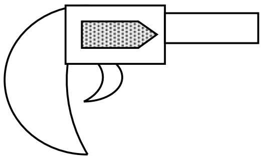
Disintegration

- Disintegrations are systems that are initially intact (with no external force acting on them) that subsequently disintegrate (break apart) into smaller pieces. The pieces are scattered in different directions. Since no net external force acts on these systems, we can apply the principle of conservation of momentum to such systems.

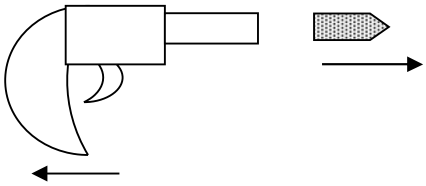
- However, total K.E. is obviously not conserved in this case – the system may start off with no K.E. at all but its fragments certainly possess K.E. upon disintegration. The K.E. may originate from the chemical reactions taking place within the systems or even from the change in mass of the reactants and products as in the case of spontaneous decay in nuclear reactions.

Examples of disintegrations

a. Before a gun is fired, the total momentum of the system, consisting of the gun and the shell is zero. The total momentum must remain at zero and so the gun recoils with a momentum equal and opposite to that of the shell immediately after firing.

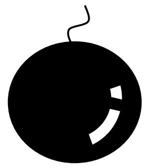


Before firing

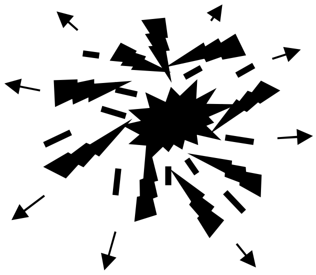


After firing

b. When a bomb explodes in mid-air, the total momentum of all the fragments just after the explosion must equal the total momentum of the complete bomb just before explosion.

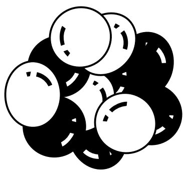


Before explosion

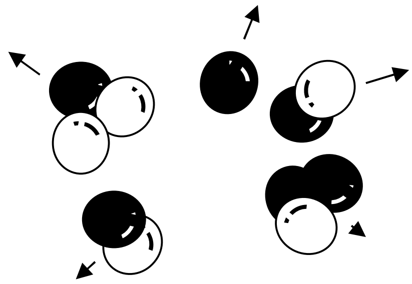


After explosion

c. When a stationary nucleus disintegrates, the total momentum of the resulting nucleus and the emitted particles must add up to zero. Apparent discrepancies led to the discovery of new fundamental particles.



Before disintegration

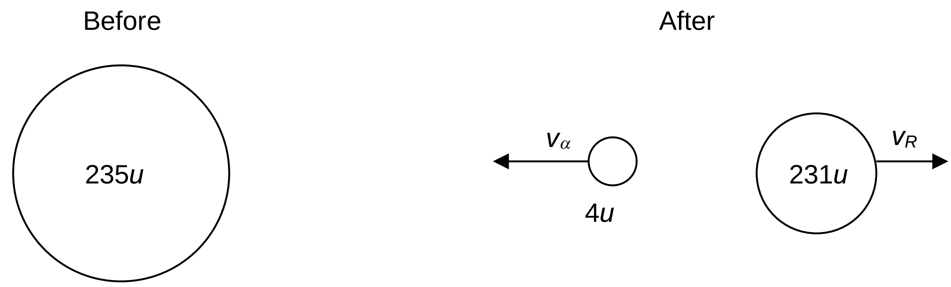


After disintegration

Example 7

Uranium-235 disintegrates by emitting an alpha particle of mass 4*u* and leaving a residual nucleus of mass 231*u*, where *u* is the atomic mass unit.

Calculate the ratio of the kinetic energy of the alpha particle to that of the residual nucleus in this disintegration.



Solution

By conservation of momentum and taking right to be positive:

---------------- (1)

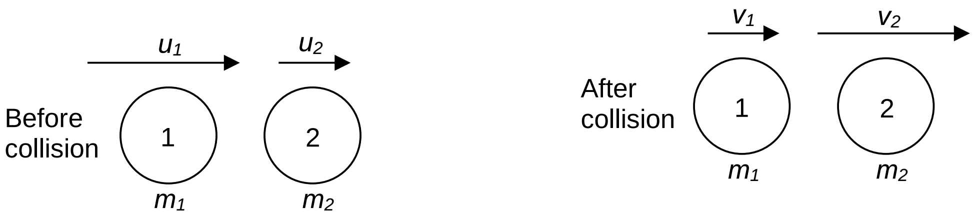
Rearranging gives :

---------------- (2)

$$\frac{\text{K.E. of alpha particle}}{\text{K.E. of nucleus}} =$$

Appendix

For an **elastic collision**, show that relative speed of approach = relative speed of separation.



- By the principle of conservation of momentum,

  Momentum of system before collision = Momentum of system after collision

  $$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad \text{------- (1)}$$

- For a perfectly elastic collision, the kinetic energy of the system is conserved.

  K.E. of system before collision = K.E. of system after collision

  $$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \qquad \text{------- (2)}$$

  From (1), $\quad m_1(u_1 - v_1) = m_2(v_2 - u_2) \qquad$ ---- (3)

  From (2), $\quad m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \qquad$ ---- (4)

  Assuming $u_1 \neq v_1$ and $u_2 \neq v_2$, (4) ÷ (3) :

  $u_1 + v_1 = v_2 + u_2$

  $\Rightarrow u_1 - u_2 = v_2 - v_1$

  which shows that the relative speed of approach $u_1 - u_2$ before collision is equal to the relative speed of separation $v_2 - v_1$ after collision