**RAFFLES INSTITUTION**
**H2 Mathematics (9758)**
**2025 Year 5**

## Additional Practice Questions for Chapter 1: Basics

**1** Find the set of values of $p$ for which the equation $x^2+2x+p=3px-1$ has no real roots.

[ $\left(0,\dfrac{16}{9}\right)$ ]

**2** Find the range of values of $t$ for which the equation $x+\dfrac{2}{x}=t$ has real roots, leaving your answer in surd form.

$\left[t\le -2\sqrt{2} \text{ or } t\ge 2\sqrt{2}\right]$

**3** Given that $q$ is a constant, show that $(2+qx)^2+x(2x-q)$ is positive for all real values of $x$.

**4** Express $\dfrac{1-2x+5x^2}{(1-2x)(1+x^2)}$ as partial fractions.

[ $\dfrac{1}{1-2x}-\dfrac{2x}{1+x^2}$ ]

**5** A curve has equation $y=\left(2-\sqrt{3}\right)x^2+x-1$. The $x$-coordinate of a point $A$ on the curve is $\dfrac{\sqrt{3}+1}{2-\sqrt{3}}$.

**(a)** Show that the coordinates of $A$ can be written in the form $\left(p+q\sqrt{3}, r+s\sqrt{3}\right)$, where $p$, $q$, $r$ and $s$ are integers.

**(b)** Find the $x$-coordinate of the stationary point on the curve, giving your answers in the form $a+b\sqrt{3}$, where $a$ and $b$ are rational numbers.

$\left[\textbf{(a)}\left(5+3\sqrt{3}, 18+11\sqrt{3}\right) \quad \textbf{(b)}\ -1-\dfrac{1}{2}\sqrt{3}\right]$

**6** In this question, $a$, $b$, $c$ and $d$ are positive constants.

**(a) (i)** It is given that $y=\log_a(x+3)+\log_a(2x-1)$. Explain why $x$ must be greater than $\dfrac{1}{2}$.

**(ii)** Find the exact solution of the equation $\dfrac{\log_a 6}{\log_a(y+3)}=2$.

**(b)** Write the expression $\log_a 9+\left(\log_a b\right)\left(\log_{\sqrt{b}} 9a\right)$ in the form $c+d\log_a 9$, where $c$ and $d$ are integers.

$\left[\textbf{(a)(ii)}\ -3+\sqrt{6} \quad \textbf{(b)}\ 2+3\log_a 9\right]$

**7** Let $\mathrm{f}(t)=1-\frac{1}{2}\mathrm{e}^{-2t}$.

**(a)** By solving $\mathrm{f}(t)=0$, show that $t=-0.347$, correct to 3 significant figures.

**(b)** Sketch the graph of $y=\mathrm{f}(t)$ for $t\geq -1$, showing the equations of any asymptotes and the coordinates of any intersections with the axes.

**8** Solve the equation $4\cos 2x+\operatorname{cosec} x-4=0$ for $0\leq x\leq 2\pi$.

$\left[\frac{\pi}{6} \text{ or } \frac{5\pi}{6}\right]$

**9** Find all the angles between 0 and 2π exclusive which satisfy the equation $2\sin^3 x=3\cos x\sin x$.

$\left[\frac{\pi}{3},\pi,\frac{5\pi}{3}\right]$

**10** Relative to an origin *O*, the position vectors of the points *A*, *B*, *C* and *D* are

$$\overrightarrow{OA}=\begin{pmatrix}6\\-5\end{pmatrix},\ \overrightarrow{OB}=\begin{pmatrix}10\\3\end{pmatrix},\ \overrightarrow{OC}=\begin{pmatrix}x\\y\end{pmatrix} \text{ and } \overrightarrow{OD}=\begin{pmatrix}12\\7\end{pmatrix}.$$

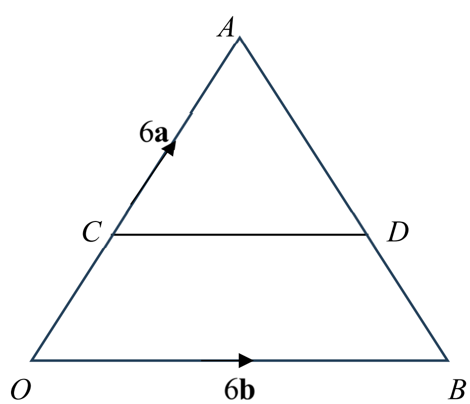
**(a)** Find the unit vector in the direction of $\overrightarrow{AB}$.

**(b)** The point *A* is the mid-point of *BC*. Find the value of *x* and of *y*.

**(c)** The point *E* lies on *OD* such that $OE:OD=1:1+\lambda$. Find the value of $\lambda$ such that $\overrightarrow{BE}$ is parallel to the *x*-axis.

$\left[\textbf{(a)}\frac{1}{\sqrt{5}}\begin{pmatrix}1\\2\end{pmatrix}\ \textbf{(b)}\ x=2, y=-13\ \ \textbf{(c)}\frac{4}{3}\right]$

**11**




In the above diagram, $\overrightarrow{OA}=6\mathbf{a}$ and $\overrightarrow{OB}=6\mathbf{b}$. The points *C* and *D* lie on *OA* and *BA* respectively such that $OC:OA=BD:BA=1:3$.

**(a)** Express as simply as possible, in terms of $\mathbf{a}$ and/or $\mathbf{b}$,

**(i)** $\overrightarrow{BD}$, **(ii)** $\overrightarrow{CD}$.

**(b)** What type of quadrilateral is *OBDC*?

**(c)** State the ratio of the area of triangle *AOB* to area of triangle *ACD*.

[**(a)(i)** $2(\mathbf{a}-\mathbf{b})$ **(ii)** $4\mathbf{b}$ **(c)** 9:4]

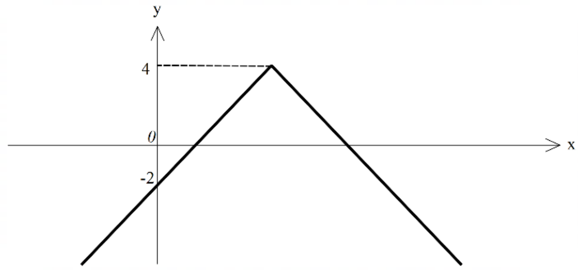
**12** Solve the following equations.

**(a)** $\left|x^2-2\right|=1$ **(b)** $|2x-3|=|x+5|$ **(c)** $2|x-1|=|4-4x|-6$

[**(a)** $x=\pm\sqrt{3}$ or $\pm 1$ **(b)** $x=-\dfrac{2}{3}$ or $8$ **(c)** $x=-2$ or $4$ ]

**13** **2020/Nan Hua High School/4047/02/Q5(i)-(iv)**

The diagram below shows the graph of $y=a-2|x-b|$, where $a$ and $b$ are constants. It is given that the $y$-intercept of the graph is $-2$ and the maximum value of $y$ is 4.




**(i)** State the value of $a$. [1]

**(ii)** Find the value of $b$. [2]

**(iii)** State the value of $k$ for which the equation $a-2|x-b|=k$ has only one solution. [1]

**(iv)** State the range of values of $m$ for which the equation $a-2|x-b|=mx-1$ has exactly two distinct solutions. [2]

[**(i)** 4 **(ii)** 3 **(iii)** 4 **(iv)** $-2<m<2$ ]