TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2022
Secondary 4

CANDIDATE NAME			
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS 4049/01

Paper 1

Thursday 11 August 2022

2 hours 15 minutes

Candidates answer on the Question Paper.
No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid / tape.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots.(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

[Turn over for Question 1]

1 Find the set of values of m, for which the line $y = 2m - x$ cuts the curve $x^2 - 3xy + y^2 + 1 = 0$ at two points and represent this set on a number line. [5]

2 A curve has equation $y = x^3 + ax^2 + bx + c$.

(i) Find $\dfrac{dy}{dx}$. [1]

(ii) It is given that y is a decreasing function for $-1 < x < 3$.
Show that $a = -3$ and $b = -9$. [3]

3 (a) Given that $\left[\left(\frac{100}{6}\right)^{-2} \times \sqrt{5^3}\right] \div \frac{5}{2} = 2^a 3^b 5^c$, find the value of each of *a*, *b* and *c*. [3]

(**b**) A man invested $50 000 of his savings at the beginning of 2010. The money he invested grew by 5% each year. Find the amount of money he will have in his savings, at the beginning of 2022. [2]

4 Express $\dfrac{2x^2+x+1}{x^2-x-2}$ in partial fractions. [5]

5 The graph of $y = p\sin 3x + q$, where p and q are positive, has a maximum value of 3 and passes through the point $(\pi, 1)$.

(i) Show that p = 2 and q = 1. [2]

(ii) Sketch the graph of $y = p\sin 3x + q$ for $0 \le x \le \pi$. [2]

(iii) Sketch on the same axes, the graph of $y = \frac{2}{\pi}x - 1$.

Explain why there are 5 solutions for $p\sin 3x + q = \frac{2}{\pi}x - 1$ when $0 \le x \le 2\pi$. [2]

(iv) State the maximum value of the graph of $y = -(p\sin 3x + q)^2$. [1]

6 Find the coordinates of the stationary points of the curve $y=(x+2)(x-1)^2$ and determine the nature of each stationary point. [6]

7 **(i)** Show that $\dfrac{\cot x - \tan x}{\cot x + \tan x} = \cos 2x$. [3]

(ii) Hence solve the equation $\dfrac{\cot x - \tan x}{\cot x + \tan x} = \cos x$ for $0° \le x \le 360°$. [4]

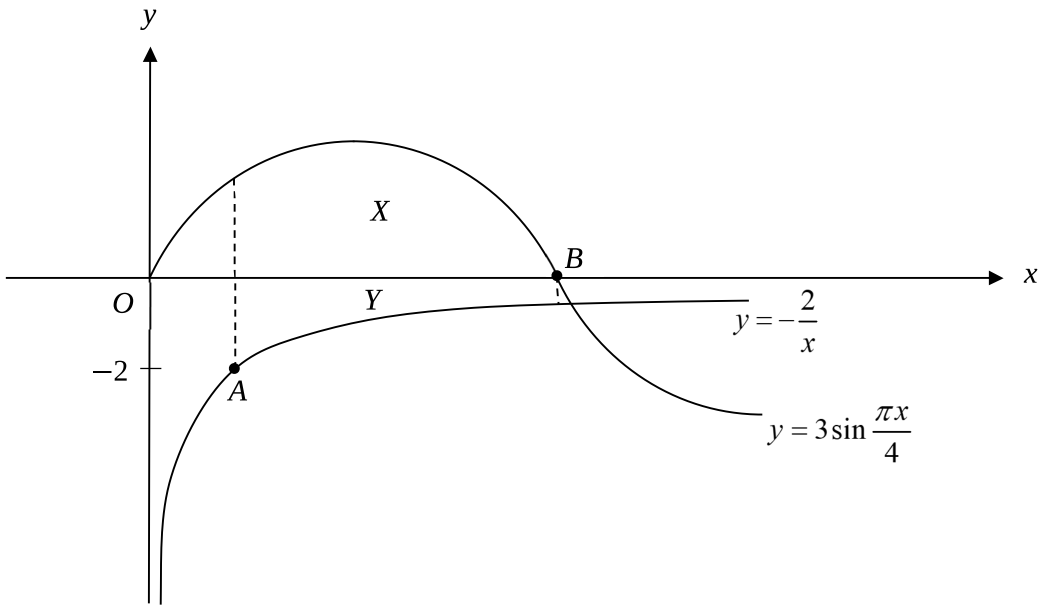
8 A polynomial, P, is $2x^3 - 9x^2 + 3x + m$, where m is a constant.

(i) Given that P and $x^2 + 4x - 5$ have a common factor, find the possible values of m. [3]

(ii) In the case where $m = 4$, solve the equation $P = 0$ completely. [4]

(iii) Using your answer in part **(ii)**, solve $2-9y^2+3y^4+4y^6=0$. [2]

9 The diagram shows parts of the curves $y = -\frac{2}{x}$ and $y = 3\sin\frac{\pi x}{4}$.



The curve $y = -\frac{2}{x}$ passes through point A where the y-coordinate is -2.

The curve $y = 3\sin\frac{\pi x}{4}$ crosses the x-axis at the origin and point B.

(i) Find the coordinates of A and of B. [2]

X is the area bounded by the curve $y = 3\sin\frac{\pi x}{4}$, the x-axis and the vertical lines passing through A and B.

(ii) Show that Area $X = \frac{6}{\pi}\left(2+\sqrt{2}\right)$ square units. [3]

Y is the area bounded by the x-axis, the curve $y = -\frac{2}{x}$, and the vertical lines passing through A and B.

(iii) Calculate the value of $\frac{\text{Area } X}{\text{Area } Y}$. [4]

10 A particle, travelling in a straight line, passes through a point A and 7 seconds later, it comes to an instantaneous rest at point B. The velocity of the particle is given by $v = 3t^2 + kt - 7$, where t is the time t seconds after passing A and k is a constant.

(i) Find the value of k. [2]

(ii) Find the acceleration of the particle when $t = 4$. [2]

(iii) Find the value of t when the particle returns to A. [3]

(iv) Find the distance travelled by the particle in the first 8 seconds. [3]

11 **(a)** Express $16-6x-x^2$ in the form $a(x+b)^2+c$.
Hence, explain why the graph of $y=16-6x-x^2$ and the line $y=30$ will not intersect. [4]

(b) Angles A and B are both acute and $\cos(A+B)=\dfrac{5}{13}$.

(i) What can be deduced about the size of angle $(A + B)$? [1]

(ii) Find the exact value of $\sin(A + B)$. [1]

(iii) Given that $\sin A=\dfrac{4}{5}$, find the exact value of $\cos B$. [5]

12 The equation of a circle C_1 is $x^2 + y^2 - 14x + 2y + 46 = 0$.

(i) Find the radius of circle C_1 and the coordinates of its centre. [4]

Circle C_1 is reflected about the line $x = 4$ to form another circle C_2 with centre M.

(ii) Find the coordinates of M . [1]

(iii) Find equation of circle C_2. [1]

A line, l passing through M, forms an angle θ with the positive x-axis such that $\tan\theta = 1$.

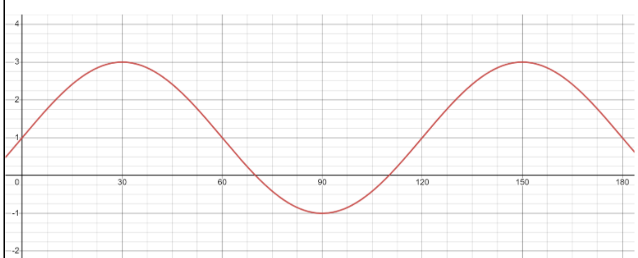
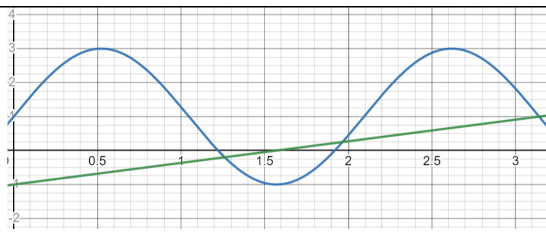
(iv) Find equation of line l. [1]

The tangent to circle C_1 at the point (8.2, 0.6) meets line l at Q.

(v) Find the coordinates of Q. [5]

End of Paper

ANSWER KEY

1	m < −1 or m > 1
2(i)	$y = x^3 + ax^2 + bx + c$ $\frac{dy}{dx} = 3x^2 + 2ax + b$
(ii)	a= −3 and b = −9
3a	a = -1, b = 2, c = -3.5
3b	$89792.82
4	$\frac{2x^2+x+1}{x^2-x-2} = 2 + \frac{11}{3(x-2)} - \frac{2}{3(x+1)}$
5(i)	$1 = p\sin 3(\pi) + q$ $1 = q$ Since y max = 3, p = 3-1 = 2
(ii)	
(iii)	 Between $0 \le x \le 2\pi$, there will be 3 cycles of the graph $y = p\sin 3x + q$. For line $y = \frac{2}{\pi}x - 1$, when $x = 2\pi, y = 3$ For graph $y = p\sin 3x + q$, when $x = 2\pi, y = 1$ The line will intersect the graph at 5 points within the domain.
(iv)	0
6	(1,0) is a minimum point (-1, 4) is a maximum point
7(ii)	$x = 0°, 120°, 240°, 360°$
8(i)	m = 4, 490
(ii)	X = 1, 4, -0.5

(iii)	$2-9y^2+3y^4+4y^6=0$ $x=\frac{1}{y^2}$ $y=\pm1,\pm\frac{1}{2}$
9(i)	A = (1, -2), B = (4, 0)
(iii)	2.35
10(i)	K = -20
(ii)	4
(iii)	10.66
(iv)	208m
11(a)	$16-6x-x^2$ $=-(x^2+6x-16)$ $=-(x+6x+3^2-16-3^2)$ $=-(x+3)^2+25$
11b(i)	(A+B) is an acute angle
(ii)	$\sin(A+B)=\frac{12}{13}$
(iii)	63/65
12(i)	$x^2+y^2-14x+2y+\ 46=0$ $centre=(7,-1)$ $radius=\sqrt{7^2+(-1)^2-(46)}=2units$
(ii)	$M=(1,\ -1)$
(iii)	Eqn of C_2: $(x-1)^2+(y+1)^2=4$ $x^2+y^2-2x+2y-2=0$
(iv)	Equation of line l: $y-(-1)=1(x-1)$ $y=x-2$
(v)	Q = (5, 3)

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