Hwa Chong Institution 华侨中学
Mathematics (SMTP)

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MA302.5.2 – Matrix Operations

At the end of this unit, you should be able to

- **perform arithmetic operations on matrices**

Matrix addition and subtraction

P1. Given matrices $\mathbf{A}=\begin{pmatrix}3 & 4\\ 0 & -1\end{pmatrix}$, $\mathbf{B}=\begin{pmatrix}3 & 5 & -2\\ -2 & -1 & 6\end{pmatrix}$, $\mathbf{C}=\begin{pmatrix}-9 & 5 & 2\\ 5 & -3 & 4\end{pmatrix}$ and $\mathbf{D}=\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}$, find, where possible,

(a) $2\mathbf{B}-4\mathbf{C}$,

$$= \begin{bmatrix}3 & 5 & -2\\ -2 & -1 & 6\end{bmatrix} \times 2 - \begin{bmatrix}-9 & 5 & 2\\ 5 & -3 & 4\end{bmatrix} \times 4$$

$$= \begin{bmatrix}6 & 10 & -4\\ 4 & -2 & 12\end{bmatrix} - \begin{bmatrix}-36 & 20 & 8\\ 20 & -12 & 16\end{bmatrix}$$

$$= \begin{bmatrix}42 & -10 & -12\\ -16 & 10 & -4\end{bmatrix}$$

(b) $(\mathbf{A}+2\mathbf{D})-2\mathbf{B}$,

$$= \left(\begin{bmatrix}3 & 4\\ 0 & -1\end{bmatrix} + \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix} \times 2\right) - \begin{bmatrix}3 & 5 & -2\\ -2 & -1 & 6\end{bmatrix}$$

= No solution due to different order.

(c) another matrix $\mathbf{A}'$ such that $\mathbf{A}+\mathbf{A}'=\mathbf{D}$.

$A + A' = D$

$A' = D - A$

$$= \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix} - \begin{bmatrix}3 & 4\\ 0 & -1\end{bmatrix}$$

$$= \begin{bmatrix}-4 & -4\\ 0 & 0\end{bmatrix}$$

P2. An electronics store has two warehouses. The following table shows the stocks on 1st July.

	Refrigerators	Televisions
Jurong	260	547
Pasir Ris	155	483

During the month, 180 refrigerators and 250 televisions were added to the stock at its Jurong warehouse while 200 refrigerators and 320 televisions were added to the stock at its Pasir Ris warehouse.

On 30th July, the stocks were found to be:

	Refrigerators	Televisions
Jurong	283	388
Pasir Ris	142	295

It is given that the matrix $\mathbf{B}=\begin{pmatrix}260 & 547\\ 155 & 483\end{pmatrix}$ represents the stocks on 1st July.

(a) Represent the stocks on 30th July as a matrix **E**.

$$E = \begin{bmatrix} 283 & 388 \\ 142 & 295 \end{bmatrix}$$

(b) Write down the matrix **A** to represent the stocks added during the month.

$$A = \begin{bmatrix} 180 & 250 \\ 200 & 320 \end{bmatrix}$$

(c) Find $\mathbf{B}+\mathbf{A}-\mathbf{E}$ and explain what the numbers obtained represent.

$$B + A - E = \begin{bmatrix} 260 & 547 \\ 155 & 483 \end{bmatrix} + \begin{bmatrix} 180 & 250 \\ 200 & 320 \end{bmatrix} - \begin{bmatrix} 283 & 388 \\ 142 & 295 \end{bmatrix}$$

$$= \begin{bmatrix} 157 & 409 \\ 213 & 508 \end{bmatrix}$$

It represent the amount of products sold during July.

Matrix multiplication

Recall earlier where there were three stalls selling four different types of cameras, and both Linda and Simon have to buy different quantities of each type of camera. We can represent the quantities and costs in the different stalls by the following two matrices

$$\mathbf{D}=\begin{pmatrix} 3 & 0 & 0 & 2 \\ 2 & 2 & 1 & 3 \end{pmatrix} \text{ and } \mathbf{C}=\begin{pmatrix} 450 & 460 & 450 \\ 800 & 780 & 810 \\ 220 & 250 & 260 \\ 350 & 350 & 320 \end{pmatrix}.$$

The first and second rows of **D** represent the quantities of each type of camera Linda and Simon bought, respectively. Recall that each column in **C** represents the cost of each type of camera in the various shops.

P3. Fill in the following table for the above scenario.

Total Cost ($) if bought from Stall…	*…X*	*…Y*	*…Z*
Linda	$3\times450+$ $0\times800+$ $0\times220+$ $2\times350=$ 2050	3×460 + 0×780 + 0×250 + 2×350 = 2080	3×450+ 0×810+ 0×260+ 2×320 = 1990
Simon	$2\times450+$ $2\times800+$ $1\times220+$ $3\times350=$ 3770	2×460+ 2×780+ 1×250+ 3×350= 3780	2×450+ 2×810 + 1×260+ 3×320 = 3740

(a) Write down the matrix which represents the above table. What does this matrix represent? How useful is this new matrix?

Total cost of cameras : $\begin{bmatrix} 2050 & 2080 & 1990 \\ 3770 & 3780 & 3740 \end{bmatrix}$

This represented the cost of each store's cameras.

(m, n)

the (mth row, nth column) value of product = (mth row of first matrice) (nth column of second matrice)

The above illustrates matrix multiplication. When two matrices are multiplied, the resultant matrix can be obtained by summing the multiplication of corresponding row and column entries.

$$3\times 450+0\times 800+0\times 220+2\times 350=\mathbf{2050}$$

$$\mathbf{DC}=\begin{pmatrix}\boxed{3\quad 0\quad 0\quad 2}\\ 2\quad 2\quad 1\quad 3\end{pmatrix}\times\begin{pmatrix}\boxed{\begin{matrix}450\\800\\220\\350\end{matrix}} & \begin{matrix}460\\780\\250\\350\end{matrix} & \begin{matrix}450\\810\\260\\320\end{matrix}\end{pmatrix}=\begin{pmatrix}\boxed{\mathbf{2050}} & \cdots & \cdots\\ \mathbf{3770} & \cdots & \cdots\end{pmatrix}$$

(b) Write down the expression which gives **3770** in the bottom left entry of the product matrix.

$2\times 450+2\times 800+1\times 220+3\times 350=3770$

$\begin{bmatrix}3770\end{bmatrix}=\begin{bmatrix}2 & 2 & 1 & 3\end{bmatrix}\times\begin{bmatrix}450\\800\\220\\350\end{bmatrix}$

P4. The following table shows the win-loss record for four particular teams in a soccer competition over two months. All 4 teams played a total of 6 games each.

	Win	Draw	Loss
Team A	2	2	2
Team B	3	0	3
Team C	4	1	1
Team D	0	6	0

The tournament management wanted to determine whether different scoring systems will affect the overall ranking of the teams. Two proposals were proposed, and the scorings are as shown in the table below.

	Proposal 1	Proposal 2
Win	3	1
Draw	1	0
Loss	0	-2

(a) Write down two matrices **R** and **S** which represents the results and scoring systems respectively. Write down also the order of the two matrices.

$$R = \begin{bmatrix} 2 & 2 & 2 \\ 3 & 0 & 3 \\ 4 & 1 & 1 \\ 0 & 6 & 0 \end{bmatrix}$$ Order : 4×3

$$S = \begin{bmatrix} 3 & 1 \\ 1 & 0 \\ 0 & -2 \end{bmatrix}$$ Order : 3×2

(b) By multiplying the two matrices **RS**, determine the scores of each team with respect to the two different scoring systems.

$$RS = \begin{bmatrix} 2 & 2 & 2 \\ 3 & 0 & 3 \\ 4 & 1 & 1 \\ 0 & 6 & 0 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ 1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -2 \\ 9 & -3 \\ 13 & 2 \\ 6 & 0 \end{bmatrix}$$

(c) Describe the results. Is the ranking of the teams affected by the proposals?

Proposal 1: $C > B > A > D$

Proposal 2: $C > D > A > B$

Yes the ranking is affected

P5. State the condition for two matrices **R** and **S** to be multiplied together such that

(a) **RS** exists.

$(m \times n) \cdot (n \times k)$

Num of column in R = Num of rows in S.

$R \cdot S$
$m \cdot n \quad p \cdot q$
$n = p$ for RS to exist
$(m \times q)$ is order of RS.

(b) **SR** exists.

$(m \times n) \cdot (n \cdot k)$

Num of column in S = Num of rows in R.

For each case, show a counter-example when the product does *not* exist.

(a) $\begin{bmatrix} 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix}$

P6. Given that $\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -3 & 9 \\ 5 & 2 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 4 & 0 & 5 \\ -2 & 3 & 1 \end{pmatrix}$, find the product of the following matrices if they exist:

(a) **AB**

= does not exist.

(b) **BA**

$= \begin{bmatrix} -3 & 9 \\ 5 & 2 \end{bmatrix} \times \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$= \begin{bmatrix} 21 \\ 16 \end{bmatrix}$

(c) **CA**

= does not exist

(d) **BC**

$= \begin{bmatrix} -3 & 9 \\ 5 & 2 \end{bmatrix} \times \begin{bmatrix} 4 & 0 & 5 \\ -2 & 3 & 1 \end{bmatrix}$

$= \begin{bmatrix} -30 & 27 & -6 \\ 16 & 6 & 27 \end{bmatrix}$

(e) **CB**

= does not exist.

P7. Matrices **A** and **B** are such that both **AB** and **BA** exist. What can you deduce about the order of both matrices?

The order is the same. or the order are inverse of each other. $\begin{bmatrix} A \cdot B \\ (n \times m) \times (m \times n) \end{bmatrix}$

$\begin{bmatrix} A \cdot B \\ (m \times n) \times (n \times m) \end{bmatrix}$

It is given that $\mathbf{A} = \begin{pmatrix} -1 & 3 \\ 2 & -2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 0 & -4 \\ 1 & -3 \end{pmatrix}$.

(a) Find **AB**.

$= \begin{bmatrix} -1 & 3 \\ 2 & -2 \end{bmatrix} \times \begin{bmatrix} 0 & -4 \\ 1 & -3 \end{bmatrix}$

$= \begin{bmatrix} 3 & -5 \\ -2 & -2 \end{bmatrix}$

(b) Find **BA**.

$= \begin{bmatrix} 0 & -4 \\ 1 & -3 \end{bmatrix} \times \begin{bmatrix} -1 & 3 \\ 2 & -2 \end{bmatrix}$

$= \begin{bmatrix} -8 & 8 \\ -7 & 9 \end{bmatrix}$

Is $\mathbf{AB} = \mathbf{BA}$ (commutative)?

No.

It is also given that $\mathbf{C} = \begin{pmatrix} -2 & 1 \\ 3 & 1 \end{pmatrix}$. Show that $\mathbf{A(BC)} = \mathbf{(AB)C}$, i.e. multiplication over matrices is associative.

$$A(BC) = \begin{bmatrix} -1 & 3 \\ 2 & -2 \end{bmatrix} \times \left(\begin{bmatrix} 0 & -4 \\ 1 & -3 \end{bmatrix} \times \begin{bmatrix} -2 & 1 \\ 3 & 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} -1 & 3 \\ 2 & -2 \end{bmatrix} \times \begin{bmatrix} -12 & -4 \\ -11 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -21 & -2 \\ -2 & -4 \end{bmatrix}$$

$$(AB)C = \left(\begin{bmatrix} -1 & 3 \\ 2 & -2 \end{bmatrix} \times \begin{bmatrix} 0 & -4 \\ 1 & -3 \end{bmatrix} \right) \times \begin{bmatrix} -2 & 1 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -5 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -21 & -2 \\ -2 & -4 \end{bmatrix} \text{ (shown)}$$

$\therefore A(BC) = (AB)C$ (shown)

P8. A restaurant sells two types of burgers – chicken and bacon, each in small and large portions. The cost of a small portion of either burger is \$2 and the cost of a large portion is \$3. During a period of ten minutes the following numbers of burgers were sold.

	Small	Large
Chicken	6	2
Bacon	5	1

Given that $\mathbf{P} = \begin{pmatrix} 6 & 2 \\ 5 & 1 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$,

(a) find **PQ**,

$$PQ = \begin{bmatrix} 6 & 2 \\ 5 & 1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 18 \\ 13 \end{bmatrix}$$

(b) explain what the numbers given in your answer to (a) represent.

18 represents the revenue made from Chicken burger.
13 represents the revenue made from Bacon burgers.