Solution to Common Test Practice 2

1 The equation of a curve is $y = mx^2 + (c - m)x - (c + 3)$. Given that $m > 0$, show that the curve cuts the x-axis at two distinct points. [3]

Let $mx^2 + (c - m)x - (c + 3) = 0$

$$\begin{aligned}\text{Discriminant } &= (c - m)^2 - 4m(-c - 3)\\ &= c^2 - 2cm + m^2 + 4mc + 12m\\ &= c^2 + 2cm + m^2 + 12m\\ &= (c + m)^2 + 12m\end{aligned}$$

Since $m > 0$ and $(c + m)^2 \geq 0$, the discriminant > 0.

Hence, the curve cuts the x-axis at two distinct points.

2 **(a)** Find the exact value of x if $8^{-2x+1} = \sqrt{32}$. [3]

(b) Simplify $\dfrac{1}{5+\sqrt{3}} + \dfrac{1}{5-\sqrt{3}}$. [2]

(c) Find the area of a right angled triangle ABC, given that angle ABC is a right angle,

$AB = \left(5 + 3\sqrt{2}\right)$ cm and $BC = \left(4 - \dfrac{5}{\sqrt{2}}\right)$ cm. Leave your answer in the form $a + b\sqrt{2}$.

[3]

(a)

$$\begin{aligned}8^{-2x+1} &= \sqrt{32}\\ (2^3)^{-2x+1} &= 2^{\frac{5}{2}}\\ -6x + 3 &= \frac{5}{2}\\ x &= \frac{1}{12}\end{aligned}$$

(b)

$$\begin{aligned}\frac{1}{5+\sqrt{3}} + \frac{1}{5-\sqrt{3}} &= \frac{5-\sqrt{3}+5+\sqrt{3}}{\left(5+\sqrt{3}\right)\left(5-\sqrt{3}\right)}\\ &= \frac{10}{25-3}\\ &= \frac{10}{22}\\ &= \frac{5}{11}\end{aligned}$$

(c) Area of $\Delta ABC = \frac{1}{2}(5+3\sqrt{2})\left(4-\frac{5}{\sqrt{2}}\right)$

$$= \frac{1}{2}\left(20-\frac{25}{\sqrt{2}}+12\sqrt{2}-15\right)$$

$$= \frac{1}{2}\left(20-\frac{25\sqrt{2}}{2}+12\sqrt{2}-15\right)$$

$$= \frac{1}{2}\left(5-\frac{\sqrt{2}}{2}\right)$$

$$= \left(\frac{5}{2}-\frac{\sqrt{2}}{4}\right) \text{ cm}^2$$

3 **(a)** Find the range of values of x for which $\frac{x}{2}(2x+1) \geq 25+\frac{x}{2}$. [3]

(b) Given that the curve whose equation is $y = p-(x-q)^2$ crosses the x-axis at the points $(1,0)$ and $(3,0)$,

(i) find the value of p and of q, [4]

(ii) the maximum value of y. [1]

(a) $\frac{x}{2}(2x+1) \geq 25+\frac{x}{2}$

$$x^2+\frac{x}{2} \geq 25+\frac{x}{2}$$

$$x^2 \geq 25$$

$$(x+5)(x-5) \geq 0$$

$$\therefore x \leq -5 \text{ or } x \geq 5$$

(b) **(i)** At $(1,0)$, $0 = p-(1-q)^2$

$$p = (1-q)^2$$

At $(3,0)$, $0 = p-(3-q)^2$

$$p = (3-q)^2$$

$$(1-q)^2 = (3-q)^2$$

$$1-2q+q^2 = 9-6q+q^2$$

$$4q = 8 \Rightarrow q = 2$$

When $q = 2$, $p = (1-q)^2$

$$= (1-2)^2$$

$$= 1$$

$$\therefore p = 1, \quad q = 2$$

(b) (ii) Since $y = 1-(x-2)^2$, the maximum value of y is 1.

4 Find the values of q for which the line $y = qx - 8$ is a tangent to the curve $x^2 = 4y$.

Leave your answer in **exact** values. [4]

$y = qx - 8$ .... (1) and $x^2 = 4y$ .... (2)

Substitute (1) into (2),

$$x^2 = 4(qx - 8)$$

$$x^2 - 4qx + 32 = 0$$

Since $y = qx - 8$ is a tangent to curve, discriminant $= 0$

$$(-4q)^2 - 4(32) = 0$$

$$16q^2 - 128 = 0$$

$$q^2 - 8 = 0$$

$$\therefore q = \pm\sqrt{8}$$

$$= \pm 2\sqrt{2}$$

5 Solve the simultaneous equations:

$$27^x \div 3^y = 9,$$

$$2^{2x} \times 4^{1-y} = 64.$$

[4]

$27^x \div 3^y = 9$

$3^{3x} \div 3^y = 3^2$

$3x - y = 2$

$y = 3x - 2$ .... (1)

$2^{2x} \times 4^{1-y} = 64$

$2^{2x} \times 2^{2-2y} = 2^6$

$2x + 2 - 2y = 6$

$2x - 2y = 4$

$x - y = 2$ .... (2)

Substitute (1) into (2).

$x - (3x - 2) = 2$

$-2x + 2 = 2$

$x = 0$

Substitute $x = 0$ into (1).

$y = 3(0) - 2$

$= -2$

$\therefore x = 0,\ y = -2$.

6 Sketch the graph of $y=3x^2-8x+9$, showing clearly the coordinates of the turning point and the y-intercept. [5]

$y=3x^2-8x+9$

$=3\left[x^2-\frac{8}{3}x+3\right]$

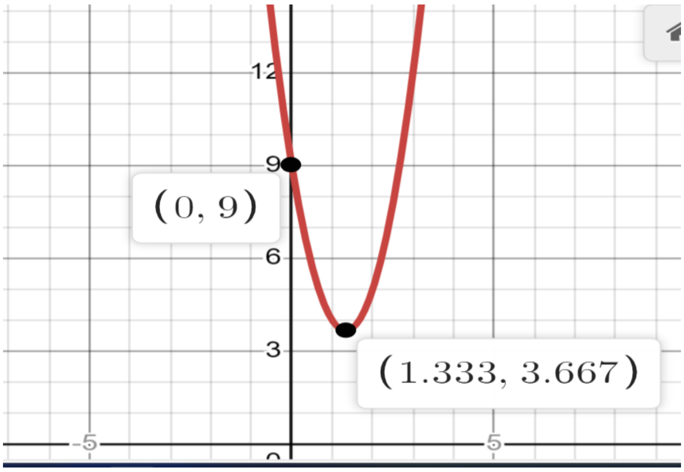
$=3\left[x^2-\frac{8}{3}x+\left(-\frac{4}{3}\right)^2-\left(-\frac{4}{3}\right)^2+3\right]$

$=3\left[\left(x-\frac{4}{3}\right)^2-\frac{16}{9}+3\right]$

$=3\left(x-\frac{4}{3}\right)^2+\frac{11}{3}$

Turning point is $\left(\frac{4}{3},\frac{11}{3}\right)$.

y-intercept is $(0,9)$.



7 Using a suitable substitution, solve the equation $2^{x+1}+2^{-x}=3$. [4]

$2^{x+1}+2^{-x}=3$

$2^x\times 2^1+\frac{1}{2^x}=3$

Let $u=2^x$

$2u+\frac{1}{u}=3$

$2u^2-3u+1=0$

$(2u-1)(u-1)=0$

$u=\frac{1}{2}$ or $u=1$

$2^x=\frac{1}{2}$ or $2^x=1$

$x=-1$ or $x=0$

8 Solve the equation : $x-3\sqrt{x}-4=0$. [4]

$x-3\sqrt{x}-4=0$

$x-4=3\sqrt{x}$ isolate surd

$(x-4)^2=\left(3\sqrt{x}\right)^2$

$x^2-8x+16=9x$

$x^2-17x+16=0$

$(x-16)(x-1)=0$

$x=16$ or $x=1$ (n.a.)

Check:

When $x=1$, LHS $=1-3\sqrt{1}-4=-6\neq$ RHS

When $x=16$, LHS $=16-3\sqrt{16}-4=0=$ RHS

$\therefore x=16$

If there are 2 answers, must check the validity of both asnwers!