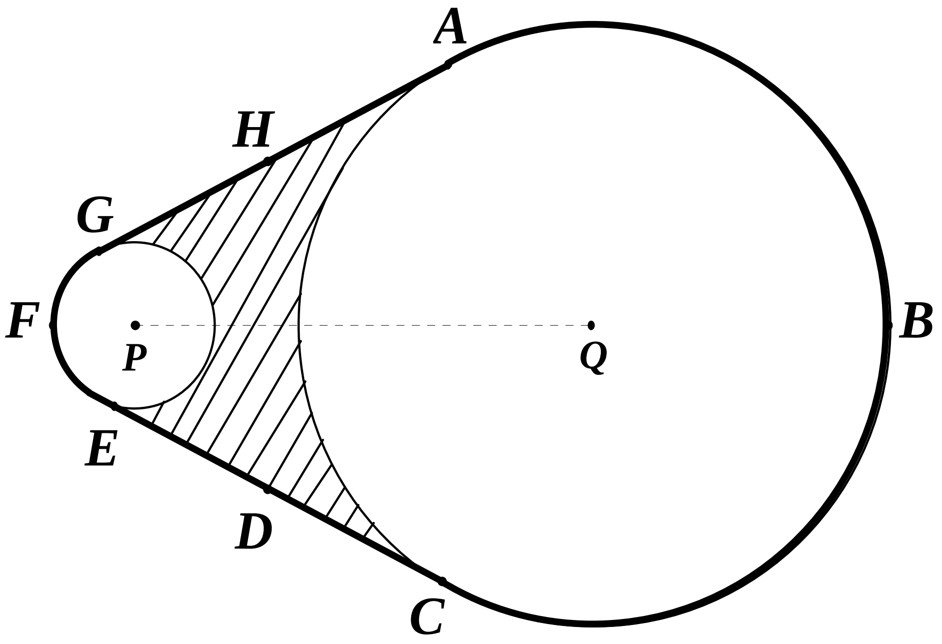
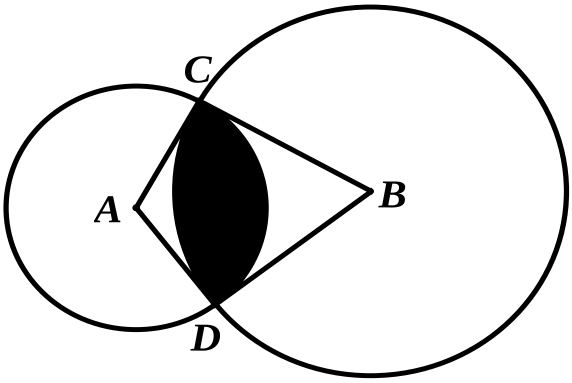
Topic: Circular Measure

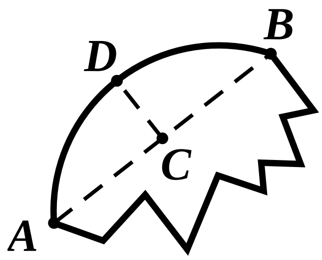
Question 1 [2009 S4 EOY, Q21]	marks
A pulley system in a car comprises two circular discs with circumferences 28π cm and 8π cm respectively. The two discs are bound tightly by a belt ***ABCDEFGHA*** and their centres *P* and *Q* are 20 cm apart.  (i) Show that angle *AQP* is $\frac{\pi}{3}$ radians. **Solution:** $\cos \angle AQP = \frac{10}{20} = \frac{1}{2}$ $\therefore \angle AQP = \frac{\pi}{3} (shown)$	[2]
(ii) Find the length of the belt ***ABC*** in contact with the bigger disc. **Solution:** $AQ = \frac{28\pi}{2\pi} = 14cm$	[3]

$\angle AQC = 2\pi - 2\angle AQP = \frac{4\pi}{3}$ $Arc\ \ ABC = 14 \times \frac{4\pi}{3} = 58.6$	
(iii) Find the area of the shaded region. **Solution:** $GA = \sqrt{20^2 - 10^2} = \sqrt{300}$ $\angle GPQ = \pi - \angle AQP = \frac{2\pi}{3}$ Shaded area = trapezium *AQPG* – area of sectors $= 2\left[\frac{1}{2}\sqrt{300}(4+14) - \frac{1}{2}(14)^2\left(\frac{\pi}{3}\right) - \frac{1}{2}(4)^2\left(\frac{2\pi}{3}\right)\right]$ $\approx 73.0\ \ cm^2\ \ (3s.f.)$	[4]
Question 2 [2010 S4 EOY, Q16]	marks
The diagram shows two circles with centres *A* and *B*, intersecting at points *C* and *D*. The radii of the circles are 8 cm and 12 cm respectively, and angle *ACB* = angle *ADB* = $\frac{\pi}{2}$ radians.  (i) Show that angle *CBD* is approximately 1.18 radians. **Solution:** $(i)\ \ \angle CBD = 2\tan^{-1}\left(\frac{8}{12}\right) \approx 1.176 \approx 1.18\ \ rad$	[2]

The shaded region is bounded by minor arcs *CD* with centres *A* and *B* respectively. (ii) Find the perimeter of the shaded region. **Solution:** $(ii)\ arc\ CD\ centre\ A + arc\ CD\ centre\ B$ $= 8(\pi - 1.18) + 12(1.18)$ $\approx 29.9\ cm$	[3]
(iii) Find the area of the shaded region. **Solution:** (*iii*) Area of segment given by $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$. $area\ of\ segment\ centre\ A$ $= \frac{1}{2}8^2\left[(\pi - 1.18) - \sin(\pi - 1.18)\right] \approx 33.184$ [Accept 33.360 if students use 1.176] $area\ of\ segment\ centre\ B$ $= \frac{1}{2}12^2\left[1.18 - \sin 1.18\right] \approx 18.388$ Area of shaded region = $51.6\ cm^2$.	[5]

Question 3 [2011 S4 EOY, Q8]	marks

During a school field trip, Jane discovered a broken circular plate, as shown in the diagram below, from a deserted historical site. She measured the length of the chord *AB* as 14 cm and from *C*, the mid-point of *AB*, she measured the length *CD* as 1 cm where *D* is the point on the circumference nearest to *C*.

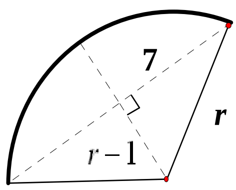


Find

(i)

the radius of the plate, [3]

Solution:



$r^2 = (r-1)^2 + 7^2$

$r = 25$ A1

(ii)

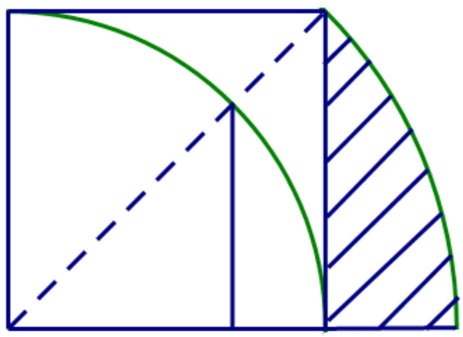
the area of the segment *ADB*. [4]

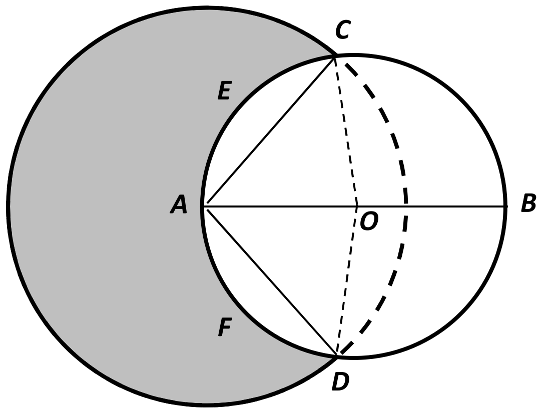
Solution:

$$\angle AOB = 2\tan^{-1}\left(\frac{AC}{OC}\right) = 2\tan^{-1}\left(\frac{7}{24}\right)$$

$$Area\ of\ \ sector\ OAB$$

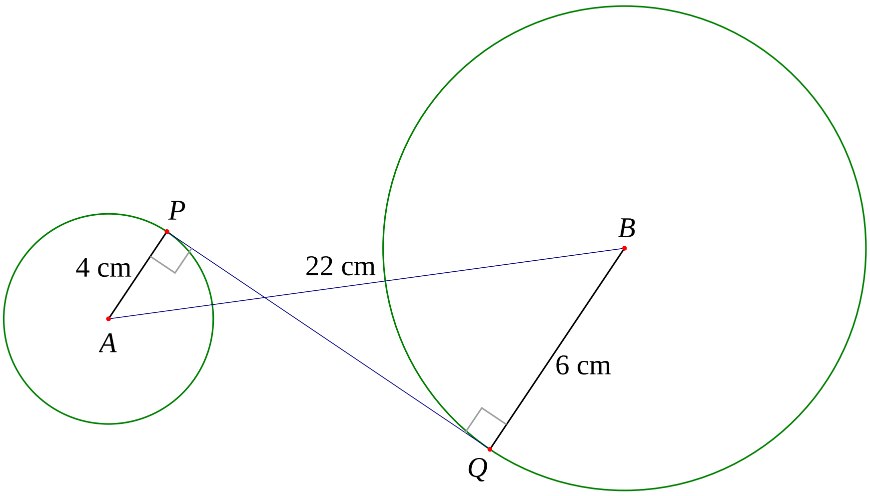
$$= \frac{1}{2}(25)^2\left[2\tan^{-1}\left(\frac{7}{24}\right)\right]$$

Area of ΔOAB $= \frac{1}{2}(25)^2 \sin\left[2\tan^{-1}\left(\frac{7}{24}\right)\right]$ *Area of segment* $= Area\ of\ sector\ OAB - Area\ of\ \Delta OAB$ $= \frac{1}{2}(25)^2\left[2\tan^{-1}\left(\frac{7}{24}\right) - \sin\left[2\tan^{-1}\left(\frac{7}{24}\right)\right]\right]$ $\approx 9.37\ cm^2$	
Question 4 [2012 S4 EOY, Q9]	marks
The diagram shows a quadrant inscribed in the square *ABCD* of sides 6 cm. *ACE* is a sector centred at *A*. *F* is the point of intersection of the diagonal *AC* and the arc *DB*. *FG* is perpendicular to *AE*.  Find (i) the area of the shaded region *BCE*, **<u>Solution:</u>** Area of shaded region *BCE* $= \frac{1}{2}(AC)^2(\frac{\pi}{4}) - \frac{1}{2}(6)^2$ $= \frac{\pi}{8}\left(6^2+6^2\right) - 18$ $= 9\pi - 18 \approx 10.3\ \text{cm}^2$	[3]

(ii) the perimeter of *FGBECF*. **Solution:** Perimeter of *FGBEC* $= GE + CE + FC + FG$ $= AC - AG + CE + FC + AG$ $= \sqrt{72} + \frac{\pi}{4}\sqrt{72} + \sqrt{72} - 6$ ≈ 17.6 cm	[4]
Question 5 [2013 S4 EOY, Q15]	marks
Two circles are such that *A* is the centre of the bigger circle and *AB* is the diameter of the smaller circle. The radius of the bigger circle is 7 cm and the diameter *AB* is of length 10 cm. Points *C* and *D* are the intersection of the two circles and *O* is the centre of the smaller circle.  (i) Show that $\cos\angle CAO$ is $\frac{7}{10}$. **Solution:** $AC = 7$ cm and $AO = 5$ cm Since triangle OAC is isosceles with $OA = OC$ $\cos\measuredangle CAO = \frac{7}{2} \div 5 = \frac{7}{10}$	[2]

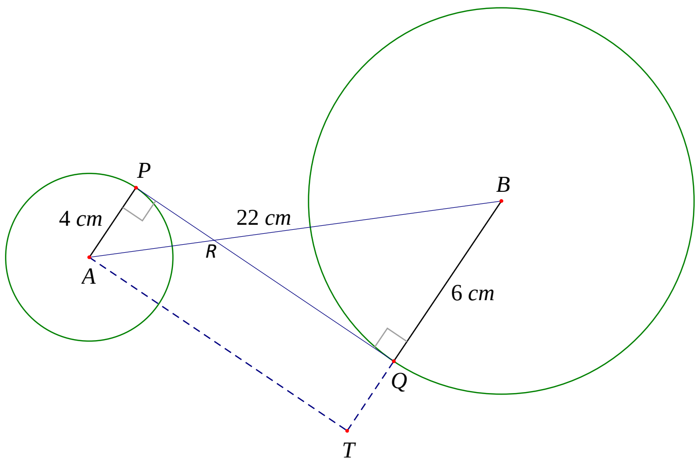
(ii) Find the area of the minor segment *ACE*. **<u>Solution:</u>** $\angle AOC = \cos^{-1}\dfrac{5^2+5^2-7^2}{2(5)(5)}$ $= \cos^{-1}\dfrac{1}{50}$ Hence area of minor segment *ACE* $= \dfrac{1}{2}(5)^2\left[\angle AOC - \sin\angle AOC\right]\text{ cm}^2$ $= 6.89\text{ cm}^2$ (3 d.p.)	[3]
(iii) Find the area of the shaded crescent. **<u>Solution:</u>** Area of sector *ACD* $= \dfrac{1}{2}(7)^2 2\cos^{-1}\dfrac{7}{10}\text{ cm}^2$ $= 38.975\text{ cm}^2$ (3 d.p.) Hence area of shaded crescent $= \pi(7)^2 - 2(6.887) - 38.975\text{ cm}^2$ $= 101\text{ cm}^2$ (3 s.f.)	[3]

Question 6 [2014 S4 EOY, Q6]	marks
The diagram shows two circles, circle *A*, with radius $AP = 4$ cm and circle *B*, with radius $BQ = 6$ cm having a common internal tangent PQ. If the distance between A and B is 22 cm, find PQ.	[3]



Solution:

Extend line *BQ*. Construct line parallel to *PQ* passing through *A* to meet extended line *BQ* at *T*.



$BT = 6 + 4 = 10$ cm

$AT^2 + 10^2 = 22^2$

$AT = \sqrt{22^2 - 10^2}$

$= \sqrt{384}$

$= 19.6$ cm (3s.f.)

PQ = *AT* = 19.6 cm

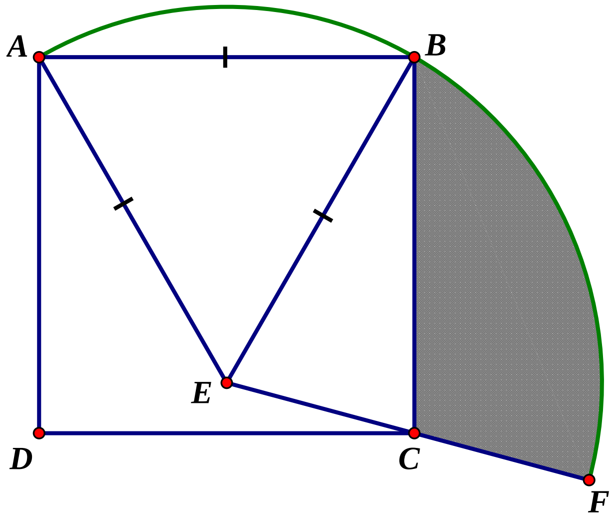
Using similar triangles,

Let *AR* = *x* and *RB* = *(22-x)*

$\frac{4}{6} = \frac{x}{22 - x}$

$x = 8.8$

$PQ = \sqrt{8.8^2 - 4^2} + \sqrt{13.2^2 - 6^2}$

Question 7 [2015 S4 EOY, Q9]	marks
The diagram shows a square *ABCD* of length 2 cm and an equilateral triangle *ABE*. An arc *ABF* is drawn with radius *AE* and centre at *E*. Points *E*, *C* and *F* lie on a straight line.  (i) Show that $\angle BEC = \frac{5}{12}\pi$ radians. **<u>Solution:</u>** $\angle EBC = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$ Since *BE* = *BC*, $\angle BEC = \frac{1}{2}\left(\pi - \frac{\pi}{6}\right)$ $= \frac{5}{12}\pi$ radians (shown)	[2]
(ii) Find the length of line segment *CE*, **<u>Solution:</u>** Using cosine rule, $CE = \sqrt{2^2 + 2^2 - 2(2)(2)\cos\frac{\pi}{6}} = 1.04$ cm (3 s.f.)	[2]

(iii) Find the perimeter of the shaded region, **Solution:** Length of arc $BF = (2)\left(\frac{5}{12}\pi\right) = \frac{5}{6}\pi$ cm Perimeter of shaded region $= \frac{5}{6}\pi + 2 + (2 - 1.035) = 5.58$ cm (3 s.f.)	[2]
(iv) Find the area of the shaded region, leave your answer in the form $a\pi + b$, where *a* and *b* are constants to be determined. **Solution:** Area of shaded region = Area of sector *EBF* − Area of triangle *EBC* $= \frac{1}{2}(2)^2\left(\frac{5}{12}\pi\right) - \frac{1}{2}(2)(2)\sin\frac{\pi}{6}$ $= \left(\frac{5}{6}\pi - 1\right)$ cm^2	[3]