Hwa Chong Institution 华侨中学
Mathematics (SMTP)

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MA304.5.1 – Simple Trigonometric Equations

At the end of this unit, you should be able to

- **solve simple trigonometric equations**

E1. Determine which possible quadrants the angles A, B, C, D and E can lie such that

(a) $\sin A = 0.4$

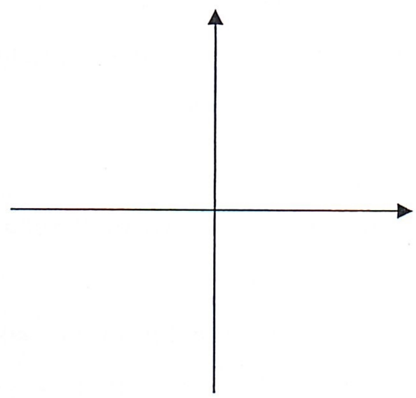
first, second

(b) $\tan B = -3.5$

second, fourth

(c) $\cos C = 0.9$

first, ,third



(d) $\cos D, \tan D > 0$

first

(e) $\sin E < 0, \cos E > 0$

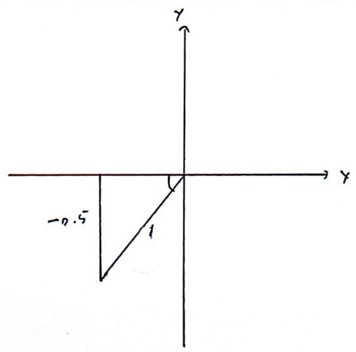
fourth

P1. Find the basic angle α for the following equations. You **do not** need the calculator for this.

(a) $\sin A = -0.5$

$\sin \alpha = 0.5$

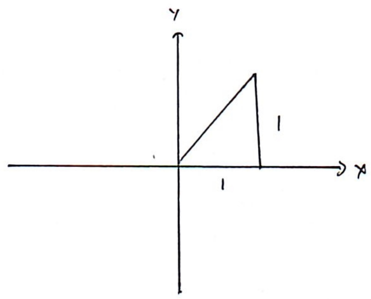
$\alpha = 30°$



(b) $\tan A = 1$

$\tan \alpha = 1$

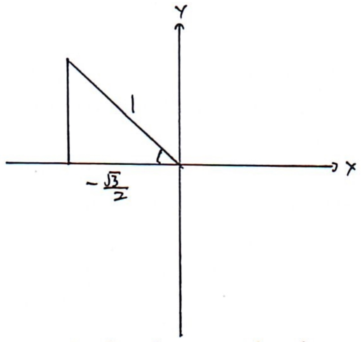
$\alpha = 45°$



(c) $\cos A = -\frac{\sqrt{3}}{2}$

$\cos \alpha = \frac{\sqrt{3}}{2}$

$\alpha = 30°$

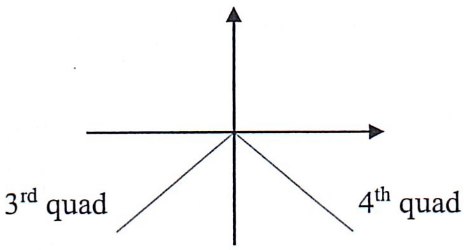


Basic trigonometric equations are of the following three forms: $\sin\theta = k$, $\cos\theta = k$ and $\tan\theta = k$. As the angle θ could be in any quadrant, depending on the sign of k, we have to rely on the *basic angle* α to help us determine the possible values of θ. We will illustrate this via an example.

E2. Solve for all values of θ such that $\sin\theta = -0.6$ for $0° \le \theta < 360°$

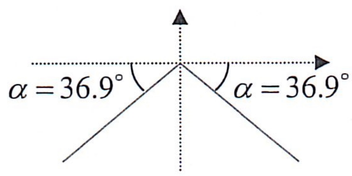
Step 1: Locate the possible quadrants of θ and draw out a diagram.

In this case, since sine is negative, the possible quadrants (based on ASTC) are the 3rd and 4th quadrants.



Step 2: Find the basic angle.

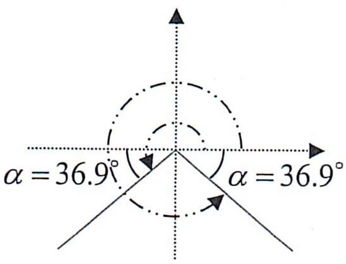
After locating the quadrants which θ can lie, find the basic angle α by solving $\sin\alpha = 0.6$, i.e. $\alpha = \sin^{-1} 0.6 = 36.9°$.



Note that the basic angle is always positive and acute.

Step 3: Determine from the domain all the possible angles with the basic angle α.

The given domain is $0^\circ \le \theta < 360^\circ$, which is one complete round anticlockwise starting from the positive x-axis. Hence there are two such angles for θ:

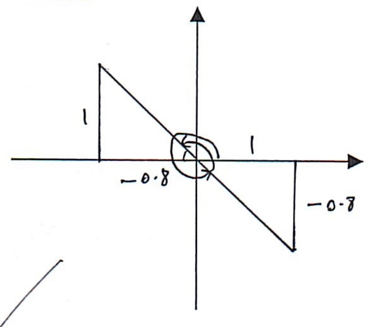


Step 4: Evaluate θ from diagram.

From the diagram, it is clear that θ can either be $180^\circ + \alpha$ or $360^\circ - \alpha$, i.e. 216.9° or 323.1°.

P2. Solve $\tan A = -0.8$ for $0 < A \le 2\pi$.

Step 1: Draw a diagram and locate the possible quadrants (See right diagram)



Step 2: Find the basic angle

$\alpha = \tan^{-1}(0.8)$

$= 0.675$ (3s.f.)

Step 3: Determine all possible angles

A lies in the second and fourth quadrant

Step 4: Evaluate all possible angles

$A = \pi - 0.675$ or $2\pi - 0.675$

$= 2.47$ (3s.f.) or 5.61 (3s.f.)

P3. Solve $\dfrac{8\cos x + 1}{2 - \cos x} = 3$ for $-180^\circ < x \le 180^\circ$.

$8\cos x + 1 = 6 - 3\cos x$

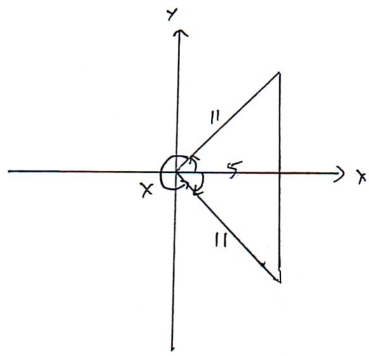
$11\cos x = 5$

$\cos x = \frac{5}{11}$

$\alpha = \cos^{-1}\left(\frac{5}{11}\right)$

$= 63.0^\circ$ (1d.p.)

$x = 63.0^\circ$ (1dp) or -63.0° (1d.p.)



P4. Solve the following equations for x.

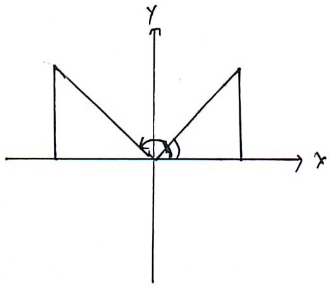
(a) $\sin^2 x = 0.5$, $0 < x < \pi$

$\sin x = \pm \frac{1}{\sqrt{2}}$

$\alpha = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{\pi}{4}$

$x = \frac{\pi}{4}$ or $\frac{3\pi}{4}$



(b) $\cos^3 x = -\frac{1}{8}$, $180° < x < 360°$

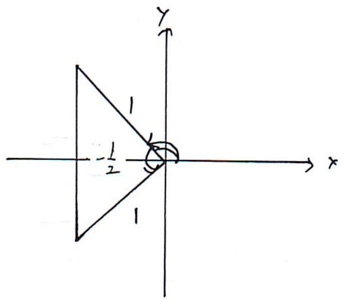
$\cos x = -\frac{1}{2}$

$\alpha = \cos^{-1}\left(\frac{1}{2}\right)$

$= 60°$

$x = 180° + 60°$ or $180° - 60°$

$= 240°$ or $120°$ (reject)

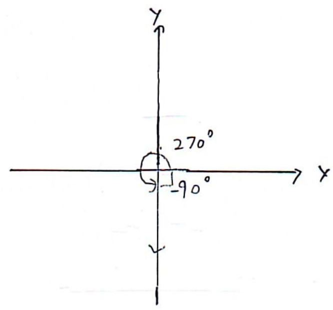


(c) $\sin(-x) = 1$, $0° < x < 180°$

$\sin x = -1$

$x = -90°$ (reject)

No solution for x.



P5. Factorise $4\sin x\cos x + 10\sin x + 2\cos x + 5$. Solve $4\sin x\cos x + 10\sin x + 2\cos x + 5 = 0$ for $0 < x < 2\pi$.

$4\sin x\cos x + 10\sin x + 2\cos x + 5 = (2\sin x + 1)(2\cos x + 5)$

$2\sin x + 1 = 0$ or $2\cos x + 5 = 0$

$\sin x = -\frac{1}{2}$ or $\cos x = -\frac{5}{2}$ (no solution)

Let α be the basic angle.

$\alpha = \sin^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{6}$

$x = \pi + \frac{\pi}{6}$ or $2\pi - \frac{\pi}{6}$

$= \frac{7\pi}{6}$ or $\frac{11\pi}{6}$