H3 Thermal Physics

Temperature

1 The measurement of temperature on the thermodynamic (kelvin) scale may be made using a special type of thermometer called a constant-volume gas thermometer. (To answer this data analysis question, you do not need to know anything about the working of this thermometer.)

The pressure p_T of a fixed mass of gas at constant volume is compared with the pressure p_{tr} at a reference temperature (the triple point of water). The thermodynamic temperature T is given by

$$T = 273.16 \lim_{p_{tr} \to 0} \left(\frac{p_T}{p_{tr}} \right),$$

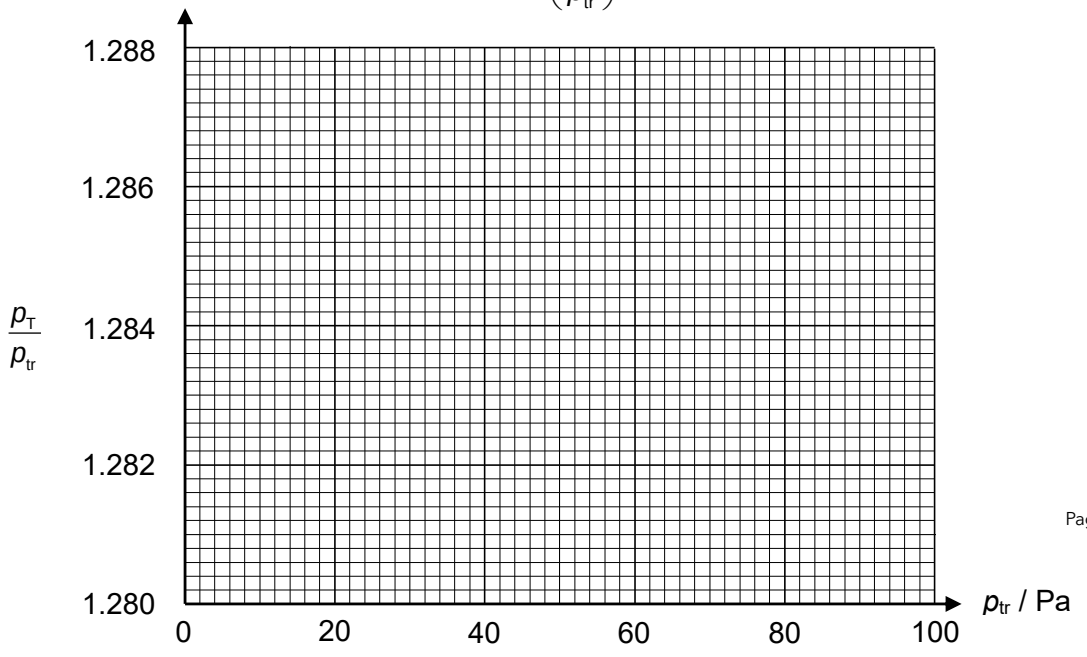
where the notation '$\lim_{p_{tr} \to 0} \left(\frac{p_T}{p_{tr}} \right)$' means 'the value of $\frac{p_T}{p_{tr}}$ for a vanishingly small value of p_{tr}'.

A constant volume gas thermometer is used to measure the thermodynamic temperature T of a bath containing a boiling liquid. The following readings are taken.

p_T /kPa	17.64	55.00	74.42	93.59
p_{tr} /kPa	13.72	42.83	58.00	73.01
p_T / p_{tr}				

(a) Calculate, to an appropriate number of significant figures, the corresponding values of $\frac{p_T}{p_{tr}}$. Fill up the blanks in the table above.

(b) Use a graphical method to obtain $\lim_{p_{tr} \to 0} \left(\frac{p_T}{p_{tr}} \right)$.



(c) Hence calculate the thermodynamic temperature T of the boiling liquid.

(d) Suggest why the value of T does not depend on the gas used in the thermometer.

Ideal Gas

2 **(a)** What do you understand by an *ideal gas*?

The gas constant *R* in your data sheet is sometimes called the *universal* gas constant, because its value is the same for all gases (when they are treated as ideal). Use this fact to show that equal volumes of different ideal gases, under the same conditions of temperature and pressure, contain equal numbers of molecules.

(b) Fig. 2.1 shows a closed cylinder 1.00 m long, which contains an ideal gas on each side of a freely-moving but gas-tight piston. The piston is a perfect thermal insulator.

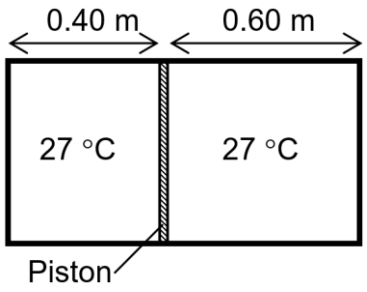


Fig. 2.1

Initially the gas on both sides of the piston is 27 °C. The equilibrium position of the piston is then 0.40 m from the left-hand end of the cylinder, as shown in Fig. 2.1.

(i) For this initial state, find the ratio of the internal energy of the gas in the left-hand compartment to the internal energy of the gas in the right-hand compartment.

The gas in the left-hand compartment is then heated very slowly to 177 °C, the gas in the right-hand compartment being maintained at 27 °C.

(ii) Describe the motion of the piston. How far is its new equilibrium position from the original?

(iii) For this final state, find the ratio of the internal energy of the gas in the left-hand compartment to that of the gas in the right-hand compartment.

(iv) Discuss whether it is correct to describe the final state of the system as an equilibrium state.

3 **(a)** In the simple kinetic theory of an ideal gas, it is assumed that the gas contains a very large number of molecules, that they move at random, and that they make perfectly elastic collisions with the walls of the container.

Explain why these assumptions are vital to the theory.

(b) Use kinetic theory ideas to show that the number x of gas molecules striking an area A of the wall of the container in unit time is given by

$$x = \frac{1}{6} nA\langle c \rangle$$

where n is the number density of molecules in the container (the number per unit volume) and $\langle c \rangle$ is their average speed.

(c) Experiments on the properties of solid surfaces are often hampered by the presence of a layer of gas molecules sticking to the surface. To avoid this, the experiment may be carried out in a vacuum chamber in which the number density of gas molecules is relatively small.

Such a chamber is maintained at a pressure of 5.0×10^{-8} Pa. The gas in the chamber is oxygen at 20 °C. A scientist wants to do an experiment on the surface properties of a semiconductor crystal in the chamber. A clean surface of area 1.0×10^{-5} m^2 can be generated by cutting the crystal, but when oxygen molecules strike the test surface they stick to it instead of rebounding elastically. When more than two oxygen molecules stick for every ten semiconductor atoms in the clean surface, the surface properties are significantly changed.

(i) Estimate the number density of oxygen molecules in the chamber and their root mean square speed. Any kinetic theory equation required may be quoted without proof. [mass of oxygen molecule $= 5.3 \times 10^{-26}$ kg]

(ii) From your knowledge of the approximate size of atoms, make an order-of-magnitude estimate of the number of semiconductor atoms in the uppermost layer of the clean test surface.

(iii) Estimate the time that the scientist has available to carry out the experiment before the test surface becomes significantly contaminated. Make use of your answers in (i) and (ii), and assume that the average speed is equal to the root-mean-square speed.

First Law of Thermodynamics

4 (a) (i) Write down an equation representing the First Law of Thermodynamics. Define the symbols you use.

(ii) "The internal energy is determined by the state of the system." Explain what this means, and discuss whether heating and work, like internal energy, are functions of state.

(b) A compressor takes in air at atmospheric pressure of 1.0×10^5 Pa and density 1.2 kg m^{-3}. The compressed air is delivered at a pressure of 1.0×10^6 Pa and at a flow rate of 0.015 m^3 s^{-1}. Assume that the compression takes place at constant temperature, and that air behaves as an ideal gas.

(i) Find the work required to compress 1.0 kg of air in this compressor.

(ii) Assuming that there are no frictional losses in the compressor, calculate the power needed to drive it.

5 A fixed mass of an ideal gas is taken through the following cycle of operation, starting at pressure p_A, volume V_A and temperature T_A. This is state A of the gas.

1. Expansion at constant pressure to p_B, V_B, T_B (state B).
2. Expansion at constant temperature to p_C, V_C, T_C (state C).
3. Reduction in volume at constant pressure to p_D, V_D, T_D (state D).
4. Reduction in volume at constant temperature to p_A, V_A, T_A (back to state A).

(a) Show this cycle by sketching graphs of

1. pressure p against volume V,
2. pressure p against temperature T,
3. volume V against temperature T,

On each of your three graphs, label the points corresponding to states A, B, C and D.

(b) Shade any area on any of your three graphs in (i) that represents the net work done during the cycle.

(c) In each part of the cycle, heat must either be taken into the gas, or be removed. On your graph of p against V, identify these parts of the cycle by the labels Q_{in} for heat taken in and Q_{out} for heat removed. Making reference to the principle of conservation of energy, suggest why heat must be taken in, or removed, in these parts of the cycle.

6 Consider two different methods of converting 1.0 kg of water at a temperature of 25 °C into water vapour at the same temperature:

Method 1	water at 25 °C to water vapour at 25 °C	
Method 2	**2.1**	water at 25 °C to water at 100 °C
	2.2	water at 100°C to water vapour at 100 °C
	2.3	water vapour at 100°C to water vapour at 25 °C

In each method, atmospheric pressure remains constant at 1.0×10^5 Pa.

(a) **(i)** Explain how any change of internal energy in method 1 is related to any overall change of internal energy in method 2.

(ii) Explain how any work done on or by the system in method 1 is related to the overall work done on or by the system in method 2.

(iii) Explain how the heating of the system in method 1 is related to the overall heating in method 2.

(b) **(i)** Make use of the following data to deduce the specific latent heat of vapourization of water at 25 °C.

specific heat capacity of water between 25 °C and 100 °C is 4.2×10^3 J kg^{-1} K^{-1}
specific heat capacity of water vapour between 25 °C and 100 °C (measured at constant pressure) is 2.0×10^3 J kg^{-1} K^{-1}
specific latent heat of vapourization of water at 100 °C and 1.0×10^5 Pa is 2.26×10^6 J kg^{-1}.

(ii) Explain why your answer to **(b)(i)** is not exactly the same as the specific latent heat of vapourization of water at 100 °C.

Answer Key
1(c) 351.53 K
2(b)(i) 0.67 **(b)(ii)** to the right by 0.10 m **(b)(iii)** $U_L = U_R$
3(c)(i) 480 m s^{-1} **(c)(ii)** 10^{14} **(c)(iii)** 0.90hr
4(b)(i) 1.92×10^5 J **(b)(ii)** 3.45 kW
6(b)(i) 2.43×10^6 J kg^{-1}

H3 Thermal Physics

Temperature

1 The measurement of temperature on the thermodynamic (kelvin) scale may be made using a special type of thermometer called a constant-volume gas thermometer. (To answer this data analysis question, you do not need to know anything about the working of this thermometer.)

The pressure p_T of a fixed mass of gas at constant volume is compared with the pressure p_{tr} at a reference temperature (the triple point of water). The thermodynamic temperature T is given by

$$T = 273.16 \lim_{p_{tr} \to 0} \left(\frac{p_T}{p_{tr}} \right),$$

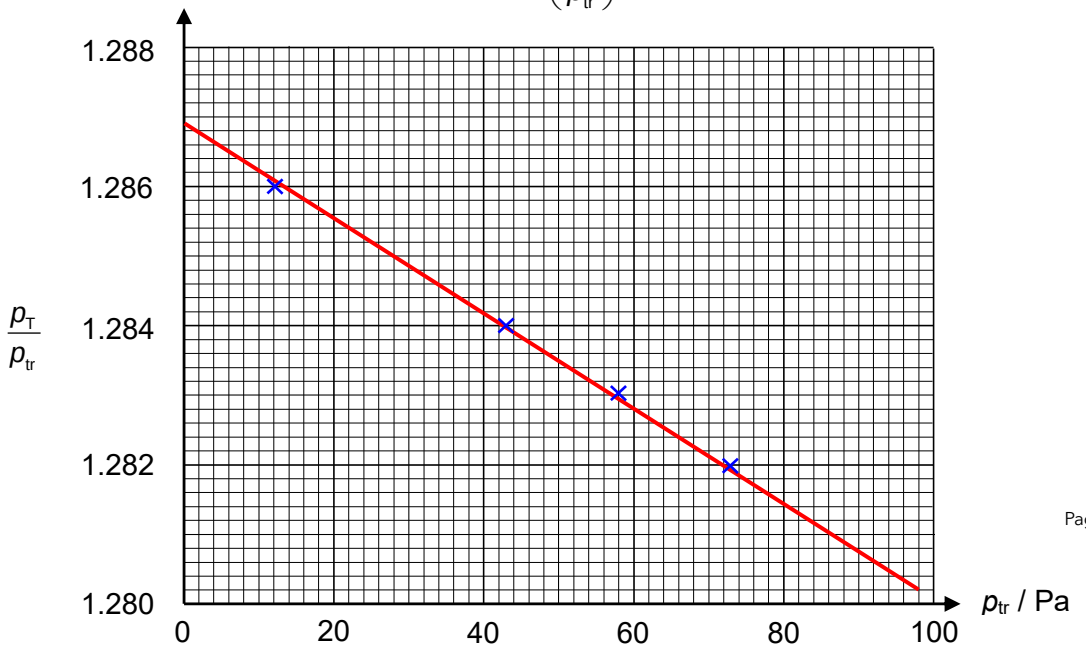
where the notation '$\lim_{p_{tr} \to 0} \left(\frac{p_T}{p_{tr}} \right)$' means 'the value of $\frac{p_T}{p_{tr}}$ for a vanishingly small value of p_{tr}'.

A constant volume gas thermometer is used to measure the thermodynamic temperature T of a bath containing a boiling liquid. The following readings are taken.

p_T /kPa	17.64	55.00	74.42	93.59
p_{tr} /kPa	13.72	42.83	58.00	73.01
p_T / p_{tr}	1.286	1.284	1.283	1.282

(a) Calculate, to an appropriate number of significant figures, the corresponding values of $\frac{p_T}{p_{tr}}$. Fill up the blanks in the table above.

(b) Use a graphical method to obtain $\lim_{p_{tr} \to 0} \left(\frac{p_T}{p_{tr}} \right)$.



(c) Hence calculate the thermodynamic temperature T of the boiling liquid.

The thermodynamic temperature of the boiling liquid is
$T = 273.16(1.2869) = 351.53$ K

(d) Suggest why the value of T does not depend on the gas used in the thermometer.

As the pressure p_{tr} approaches a vanishingly small value, all gases behave like an ideal gas. Hence they have the same p_T / p_{tr} ratio, and give the same value of T.

Ideal Gas

2 **(a)** What do you understand by an *ideal gas*?

The gas constant *R* in your data sheet is sometimes called the *universal* gas constant, because its value is the same for all gases (when they are treated as ideal). Use this fact to show that equal volumes of different ideal gases, under the same conditions of temperature and pressure, contain equal numbers of molecules.

An ideal gas is one that obeys the ideal gas equation $pV = nRT$ where p is its pressure, V its volume, n is the amount of gas in moles, R the molar gas constant and T its thermodynamic temperature.

By manipulating the ideal gas equation, $n = \frac{pV}{RT}$.

For two gases A and B, $n_A = \frac{p_A V_A}{RT_A}$ and $n_B = \frac{p_B V_B}{RT_B}$.

Taking ratio, $\frac{n_A}{n_B} = \frac{p_A V_A RT_B}{p_B V_B RT_A} = 1$.

Since pressure, volume and temperatures of A and B are the same and that R is a universal constant. ∴ gases A and B have equal number of molecules.

(b) Fig. 2.1 shows a closed cylinder 1.00 m long, which contains an ideal gas on each side of a freely-moving but gas-tight piston. The piston is a perfect thermal insulator.

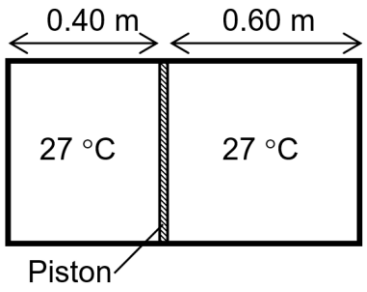


Fig. 2.1

Initially the gas on both sides of the piston is 27 °C. The equilibrium position of the piston is then 0.40 m from the left-hand end of the cylinder, as shown in Fig. 2.1.

(i) For this initial state, find the ratio of the internal energy of the gas in the left-hand compartment to the internal energy of the gas in the right-hand compartment.

Internal energy of an ideal gas is $U = \frac{3}{2}nRT = \frac{3}{2}pV$.

The ratio of U on the left (L) to that of the gas on the right (R):

$$\frac{U_L}{U_R} = \frac{p_L V_L}{p_R V_R} = \frac{0.40A}{0.60A} = 0.67$$

since at equilibrium $p_L = p_R$ (as piston is free moving which allows for the equilization of pressure in both compartments). A is the cross-sectional area of the piston.

The gas in the left-hand compartment is then heated very slowly to 177 °C, the gas in the right-hand compartment being maintained at 27 °C.

(ii) Describe the motion of the piston. How far is its new equilibrium position from the original?

The gas in the left compartment expands, pushing the piston to the right, until a new equilibrium is reached.

Initially, both gases have the same pressure and temperature.

Since $n = \frac{pV}{RT} \Rightarrow n \propto V$, $\therefore \frac{n_L}{n_R} = \frac{V_L}{V_R} = 0.67$.

As the piston is gas-tight, this ratio remains the same even after the left compartment is heated. After heating, $T_L = 450$ K and $T_R = 300$ K, $n \propto \frac{V}{T}$ and the ratio of n is

$$\frac{n_L}{n_R} = \frac{V_L T_R}{V_R T_L} = 0.67,$$

as pressure eventually equalizes again.

$$\therefore \frac{V_L(300)}{V_R(450)} = 0.67 \Rightarrow \frac{V_L}{V_R} = 1$$

Since the volume of gas in the left and right compartments are now the same, the piston is now at the centre of the cylinder. Hence the piston has moved to the right by 0.10 m.

(iii) For this final state, find the ratio of the internal energy of the gas in the left-hand compartment to that of the gas in the right-hand compartment.

Internal energy $U = \frac{3}{2}nRT = \frac{3}{2}pV$. Since both sides now have the same p and the same V, $\therefore U_L = U_R$.

(iv) Discuss whether it is correct to describe the final state of the system as an equilibrium state.

As long as the right compartment is maintained at 300 K and the left compartment at 450 K, the piston will remain at the centre of the piston. Mechanically, therefore the system is in a state of equilibrium.

Thermally, however, the system is not in equilibrium since the left and right compartment are at different thermodynamic temperatures. Although there is no heat transfer between the compartments, this is attributed to the insulation of the piston rather than the compartments being in thermal equilibrium.

According to the Zeroth Law of Thermodynamics, two systems are in thermal equilibrium only if they can be in thermal equilibrium with a third system and this is not possible for the two compartments which are at two different temperatures.

3 (a) In the simple kinetic theory of an ideal gas, it is assumed that the gas contains a very large number of molecules, that they move at random, and that they make perfectly elastic collisions with the walls of the container.

Explain why these assumptions are vital to the theory.

It is vital to assume the gas contains a large number of molecules so as to give a statistical treatment to the behaviour of the gas which is characterised by their randomness in terms of motion. This ensures that macroscopic values such as the root-mean-square speed will not be biased by the erratic behaviour of a small number of molecules.

Random motion of gas molecules is assumed so that, on average, the gas pressure can be assumed to exert equally in all directions.

Collisions are assumed to be perfectly elastic so that the molecules can be assumed to retain a constant average energy so long as there is no heat transfer into or out of the system or work done by or on the system.

(b) Use kinetic theory ideas to show that the number x of gas molecules striking an area A of the wall of the container in unit time is given by

$$x = \frac{1}{6} nA\langle c \rangle$$

where n is the number density of molecules in the container (the number per unit volume) and $\langle c \rangle$ is their average speed.

Assume a cubical container having six walls. A gas molecule would have an equal probability of travelling in one of the six possible directions i.e. $+x$, $-x$, $+y$, $-y$, $+z$ and $-z$ directions.

In a unit time interval, 1/6th of the gas molecules will be heading towards one particular face of the container. Given a molecular mean speed of $\langle c \rangle$, then the volume of gas striking the surface per unit time will be $A \times \langle c \rangle \times 1 = A\langle c \rangle$ where A is the area of the wall.

Since only 1/6th of the molecules in this volume will be heading towards the surface, the number density of molecules heading that way is (1/6)n.

Hence the number of gas molecules x striking area A in unit time is $x = \frac{1}{6} nA\langle c \rangle$.

(c) Experiments on the properties of solid surfaces are often hampered by the presence of a layer of gas molecules sticking to the surface. To avoid this, the experiment may be carried out in a vacuum chamber in which the number density of gas molecules is relatively small.

Such a chamber is maintained at a pressure of 5.0×10^{-8} Pa. The gas in the chamber is oxygen at 20 °C. A scientist wants to do an experiment on the surface properties of a semiconductor crystal in the chamber. A clean surface of area 1.0×10^{-5} m^2 can be generated by cutting the crystal, but when oxygen molecules strike the test surface they stick to it instead of rebounding elastically. When more than two oxygen molecules stick for every ten semiconductor atoms in the clean surface, the surface properties are significantly changed.

(i) Estimate the number density of oxygen molecules in the chamber and their root mean square speed. Any kinetic theory equation required may be quoted without proof. [mass of oxygen molecule = 5.3×10^{-26} kg]

From $pV = NkT$, number density $n = \frac{N}{V} = \frac{p}{kT}$.

$\therefore\ n = \frac{N}{V} = \frac{5.0 \times 10^{-8}}{1.38 \times 10^{-23} \times 293} = 1.237 \times 10^{13}\ \text{m}^{-3}$.

Root-mean-square speed $c_{rms} = \sqrt{3kT/m}$

$= \sqrt{\left(3 \times 1.38 \times 10^{-23} \times 293\right) / \left(5.3 \times 10^{-26}\right)}$

$= 478.4\ \text{m s}^{-1} \approx 480\ \text{m s}^{-1}$

(ii) From your knowledge of the approximate size of atoms, make an order-of-magnitude estimate of the number of semiconductor atoms in the uppermost layer of the clean test surface.

Average diameter of an atom $d = 2.5 \times 10^{-10}$ m. Hence the planar area occupied by an atom is a square of side length d:

$A = d^2 = \left(2.5 \times 10^{-10}\right)^2 = 6.25 \times 10^{-20}\ \text{m}^2$.

On an area of 1.0×10^{-5} m^2, number of semiconductor atoms is $\frac{1.0 \times 10^{-5}}{6.25 \times 10^{-20}} = 1.6 \times 10^{14}$.

Hence the order of magnitude is 10^{14}.

(iii) Estimate the time that the scientist has available to carry out the experiment before the test surface becomes significantly contaminated. Make use of your answers in (i) and (ii), and assume that the average speed is equal to the root-mean-square speed.

The rate at which molecules hit the surface $x = \frac{1}{6} nA\langle c \rangle$.

$\Rightarrow x = \frac{1}{6}\left(1.237 \times 10^{13}\right)\left(1.0 \times 10^{-5}\right)\left(478.4\right) = 9.869 \times 10^{9}\ \text{s}^{-1}$.

The surface is significantly contaminated if (2/10) of $1.6 \times 10^{14} = 3.2 \times 10^{13}$ oxygen molecules stick onto the surface.

$\therefore$ the time taken would be $(3.2 \times 10^{13})/(9.869 \times 10^{9}) = 3242\ \text{s} = 0.90\ \text{hr}$

First Law of Thermodynamics

4 (a) (i) Write down an equation representing the First Law of Thermodynamics. Define the symbols you use.

The First Law of Thermodynamics is given by $\Delta U = W + Q$ where ΔU is the increase in internal energy of the system, Q is heat absorbed by the system and W is the work done on the system.

(ii) "The internal energy is determined by the state of the system." Explain what this means, and discuss whether heating and work, like internal energy, are functions of state.

A system is said to be in a definite state when it is in equilibrium under a set of conditions. At a particular thermodynamic state, the properties of a system have a specific value corresponding to that state. These properties are its pressure (p), thermodynamic temperature (T), volume (V) and number of moles n of the gas. They are related by the ideal gas equation $pV = nRT$. Given any three quantities, the fourth can be determined.

The values of these properties are a function of the state of the system and are independent of the path by which the system arrived at that state. Function of state refers to the property of a system which depends only on its current state and not the process.

Only internal energy is a function of state and does not depend on how that state is arrived. If a system undergoes a series of changes from one state to another on the pV curve, its change in internal energy is the same, regardless of which path it has taken.

Work and heat, however, are not functions of state. Work done depends on the process or the path on the p vs V graph. Since heat $Q = \Delta U - W$, and there can be work done (W) even without an increase in internal energy (ΔU), heat Q is also not a function of state.

(b) A compressor takes in air at atmospheric pressure of 1.0×10^5 Pa and density 1.2 kg m^{-3}. The compressed air is delivered at a pressure of 1.0×10^6 Pa and at a flow rate of 0.015 m^3 s^{-1}. Assume that the compression takes place at constant temperature, and that air behaves as an ideal gas.

(i) Find the work required to compress 1.0 kg of air in this compressor.

If input density $\rho_1 = 1.2$ kg m^{-3} the output density $\rho_2 = 12$ kg m^{-3} as the pressure at the output is increased by ten times.

1 kg of air at the input has a volume $V_1 = \frac{1}{1.2} = 0.8333$ m^{-3} whereas 1 kg of air at the output has a volume of $V_2 = \frac{1}{12} = 0.08333$ m^{-3}.

Since it's an isothermal process (constant temperature), $pV = nRT = \text{constant}$.

$\therefore pV = 1.0 \times 10^5 \times 0.8333 = 1.0 \times 10^6 \times 0.08333 = 8.333 \times 10^4$ J.

Work done to compress the gas $W = -\int p\,dV = -\int \frac{nRT}{V}\,dV$

$$= -\int_{0.8333}^{0.08333} \frac{8.333 \times 10^4}{V}\,dV = -8.333 \times 10^4 \left[\ln V\right]_{0.8333}^{0.08333}$$
$$= -8.333 \times 10^4 \left[-\ln 10\right]$$
$$= 1.92 \times 10^5 \text{ J}$$

(ii) Assuming that there are no frictional losses in the compressor, calculate the power needed to drive it.

The output mass flow rate $m_o = 0.015 \times 1.2 = 0.018$ kg s^{-1}.
The work required to compress 1 kg of air is 1.92×10^5 J.
Hence the work required to compress 0.18 kg is $0.018 \times 1.919 \times 10^5 = 3453.9$ J.
This work occurs over 1 s. $\therefore$ the power needed to drive the compressor is 3.45 kW.

5 A fixed mass of an ideal gas is taken through the following cycle of operation, starting at pressure p_A, volume V_A and temperature T_A. This is state A of the gas.

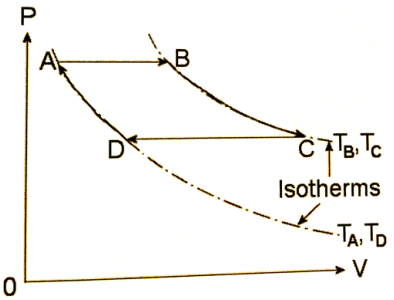
1. Expansion at constant pressure to p_B, V_B, T_B (state B).
2. Expansion at constant temperature to p_C, V_C, T_C (state C).
3. Reduction in volume at constant pressure to p_D, V_D, T_D (state D).
4. Reduction in volume at constant temperature to p_A, V_A, T_A (back to state A).

(a) Show this cycle by sketching graphs of

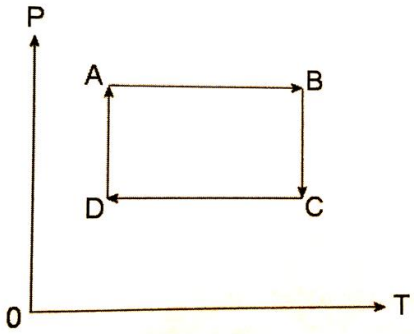
1. pressure p against volume V,
2. pressure p against temperature T,
3. volume V against temperature T,

On each of your three graphs, label the points corresponding to states A, B, C and D.

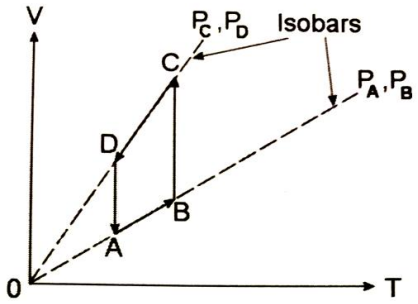
1. Pressure p against volume V,



2. Pressure p against temperature T,

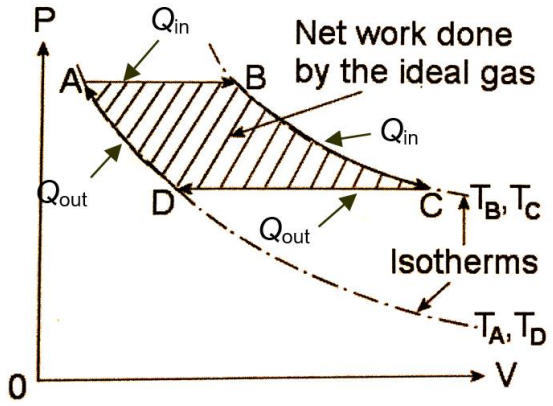


3. Volume V against temperature T.



Work done is area under the $p - V$ graph. Hence net work done by the gas is area enclosed by the loop in the $p - V$ graph. This is shown in the graph above.

(b) Shade any area on any of your three graphs in (i) that represents the net work done during the cycle.



Work done is area under the *p* – *V* graph. Hence net work done by the gas is area enclosed by the loop in the *p* – *V* graph. This is shown in the graph above.

(c) In each part of the cycle, heat must either be taken into the gas, or be removed. On your graph of p against V, identify these parts of the cycle by the labels Q_{in} for heat taken in and Q_{out} for heat removed. Making reference to the principle of conservation of energy, suggest why heat must be taken in, or removed, in these parts of the cycle.

For process A to B, heat has to be taken in (Q_{in}). From the First Law of Thermodynamics,

$$\Delta U = Q + W$$

ΔU is positive since there is an increase in temperature from isotherm T_A to isotherm T_B, and $\Delta U = \frac{3}{2} nR\Delta T$. At the same time the work done on the gas (W) is negative, since the gas expands and work is actually done by the gas. Thus,

$$\Delta U = Q - W$$
$$\therefore Q = \Delta U + W > 0$$

For process B to C, heat has to be taken in (Q_{in}). From FLT,

$$\Delta U = Q + W$$

ΔU is zero since there is no change in temperature along an isotherm. At the same time the work done on the gas (W) is negative, since the gas expands and work is actually done by the gas. Thus,

$$0 = Q - W$$
$$\therefore Q = W > 0$$

For process C to D, heat has to be removed (Q_{out}). From the First Law of Thermodynamics (FLT),

$$\Delta U = Q + W$$

ΔU is negative since there is a decrease in temperature from isotherm T_C to isotherm T_D, and $\Delta U = \frac{3}{2} nR\Delta T$. At the same time the work done on the gas (W) is positive, since the gas is compressed. Thus,

$$-\Delta U = Q + W$$
$$Q = -\Delta U - W < 0$$

For process D to A, heat has to be removed (Q_{out}). From FLT,

$$\Delta U = Q + W$$

ΔU is zero since there is no change in temperature along an isotherm. At the same time the work done on the gas (W) is positive, since the gas is compressed. Thus,

$$0 = Q + W$$
$$\therefore Q = -W < 0$$

6 Consider two different methods of converting 1.0 kg of water at a temperature of 25 °C into water vapour at the same temperature:

Method 1	water at 25 °C to water vapour at 25 °C	
Method 2	**2.1**	water at 25 °C to water at 100 °C
	2.2	water at 100°C to water vapour at 100 °C
	2.3	water vapour at 100°C to water vapour at 25 °C

In each method, atmospheric pressure remains constant at 1.0×10^5 Pa.

(a) **(i)** Explain how any change of internal energy in method 1 is related to any overall change of internal energy in method 2.

The change in internal energy in method 1 is the same as the change in internal energy in method 2. Internal energy is a function of state of the ideal gas. As long as the system's initial and final states (*p*, *V* and *T*) are the same, the change in internal energy will be the same.

(ii) Explain how any work done on or by the system in method 1 is related to the overall work done on or by the system in method 2.

The work done by the system in method 1 is the same as the work done by the system in method 2. Since pressure is constant, work done $= p\Delta V$. The change in volume in method 1 = change in volume of method 2.1 + change in volume of method 2.2 – change in volume in method 2.3

$$\Delta V_1 = \Delta V_{2.1} + \Delta V_{2.2} - \Delta V_{2.3}$$

Since pressure remains constant all this while (atmospheric pressure),

$$p\Delta V_1 = p\Delta V_{2.1} + p\Delta V_{2.2} - p\Delta V_{2.3}$$

$$W_1 = W_{2.1} + W_{2.2} - W_{2.3}$$

(iii) Explain how the heating of the system in method 1 is related to the overall heating in method 2.

$\Delta U = Q + W$ thus $Q = \Delta U - W$. Since ΔU and W are the same in both methods, the heating in method 1 is also the same as in method 2.

(b) **(i)** Make use of the following data to deduce the specific latent heat of vapourization of water at 25 °C.

specific heat capacity of water between 25 °C and 100 °C is 4.2×10^3 J kg^{-1} K^{-1}
specific heat capacity of water vapour between 25 °C and 100 °C (measured at constant pressure) is 2.0×10^3 J kg^{-1} K^{-1}
specific latent heat of vapourization of water at 100 °C and 1.0×10^5 Pa is 2.26×10^6 J kg^{-1}.

Specific latent heat of vaporization of water at 25 °C
= heat absorbed to convert unit mass of water at 25 °C to vapour at 25 °C
= heat absorbed to raise temperature of unit mass of water from 25 °C to 100 °C + heat absorbed to convert unit mass of water at 100 °C to vapour at 100 °C – heat released when unit mass of vapour cools from 100 °C to 25 °C

$= (1)(4200)(75) + (1)(2.26 \times 10^6) – (1)(2000)(75)$
$= 2.43 \times 10^6$ J kg^{-1}

(ii) Explain why your answer to **(b)(i)** is not exactly the same as the specific latent heat of vapourization of water at 100 °C.

At lower temperatures, water molecules have lower speeds, and therefore lower mean KE. Thus, they do not have sufficient energy to overcome their intermolecular forces of attraction. Therefore, more heat energy needs to be supplied to separate the molecules at 25 °C than at 100 °C.

Answer Key
1(c) 351.53 K
2(b)(i) 0.67 **(b)(ii)** to the right by 0.10 m **(b)(iii)** $U_L = U_R$
3(c)(i) 480 m s^{-1} **(c)(ii)** 10^{14} **(c)(iii)** 0.90hr
4(b)(i) 1.92×10^5 J **(b)(ii)** 3.45 kW
6(b)(i) 2.43×10^6 J kg^{-1}