RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 6

Estimated number of tutorials: 2

Tutorial S2A: Discrete Random Variables

Section A (Discussion Questions)

1 Find the mean and standard deviation of the random variable X with the following probability distribution:

x	-4	-3	-2	-1	0	1	2	3
$\mathrm{P}(X=x)$	0.04	0.16	0.24	0.16	0.15	0.1	0.1	0.05

[–0.83, 1.86]

$\mathrm{E}(X)=\sum x\,\mathrm{P}(X=x)$

$$=(-4)(0.04)+(-3)(0.16)+(-2)(0.24)+(-1)(0.16)+0+1(0.1)+2(0.1)+3(0.05)$$
$$=-0.83$$

$\mathrm{E}(X^2)=\sum x^2\,\mathrm{P}(X=x)$

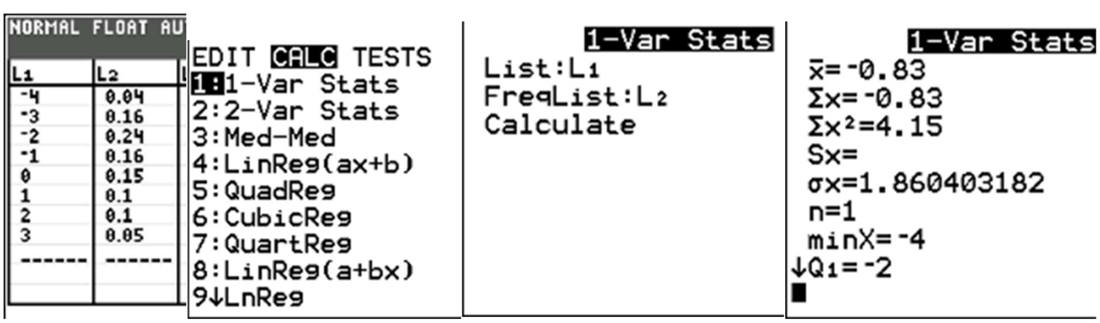
$$=4.15$$

$\mathrm{Var}(X)=\mathrm{E}(X^2)-\mathrm{E}(X)^2$

$$=4.15-(-0.83)^2$$
$$=3.4611$$

$\therefore$ Standard deviation = 1.86 (3 s.f.)

Using GC,
Enter data into 2 lists



Read off the values from “sample mean”, $\bar{x}$ and “sample standard deviation”, σx .

E(X) = -0.83
SD(X) = 1.86 (3sf)

2 9233/1980/02/Q11 (modified)

Two dice are thrown and the numbers A and B shown on each die are noted.
The score X from the throw is defined by:

$$X = \begin{cases} A+B & \text{if } A = B, \\ |A-B| & \text{if } A \neq B. \end{cases}$$

(i) Tabulate the probability distribution of X.

(ii) Evaluate the exact value of E(X) and Var(X). [**(i)** $\frac{28}{9}$ **(ii)** $\frac{1015}{162}$]

x \ d_1 / d_2	**1**	**2**	**3**	**4**	**5**	**6**
1	2	1	2	3	4	5
2	1	4	1	2	3	4
3	2	1	6	1	2	3
4	3	2	1	8	1	2
5	4	3	2	1	10	1
6	5	4	3	2	1	12

Probability distribution of X:

x	1	2	3	4	5	6	8	10	12
$\mathrm{P}(X=x)$	$\frac{10}{36}=\frac{5}{18}$	$\frac{9}{36}=\frac{1}{4}$	$\frac{6}{36}=\frac{1}{6}$	$\frac{5}{36}$	$\frac{2}{36}=\frac{1}{18}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$

$$\mathrm{E}(X) = \sum x\,\mathrm{P}(X=x)$$

$$= (1)\left(\frac{10}{36}\right)+(2)\left(\frac{9}{36}\right)+(3)\left(\frac{6}{36}\right)+(4)\left(\frac{5}{36}\right)+(5)\left(\frac{2}{36}\right)+(6)\left(\frac{1}{36}\right)+(8)\left(\frac{1}{36}\right)$$

$$+(10)\left(\frac{1}{36}\right)+(12)\left(\frac{1}{36}\right)$$

$$= \frac{28}{9}$$

$$\mathrm{E}(X^2) = \sum x^2\,\mathrm{P}(X=x)$$

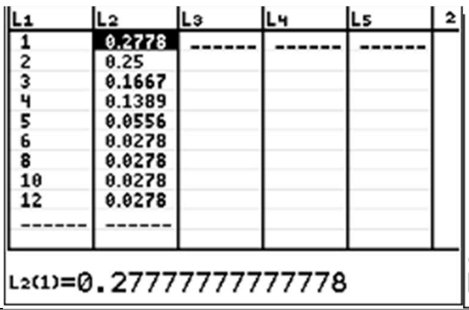
$$= (1)^2\left(\frac{10}{36}\right)+(2)^2\left(\frac{9}{36}\right)+(3)^2\left(\frac{6}{36}\right)+(4)^2\left(\frac{5}{36}\right)+(5)^2\left(\frac{2}{36}\right)+(6)^2\left(\frac{1}{36}\right)+(8)^2\left(\frac{1}{36}\right)$$

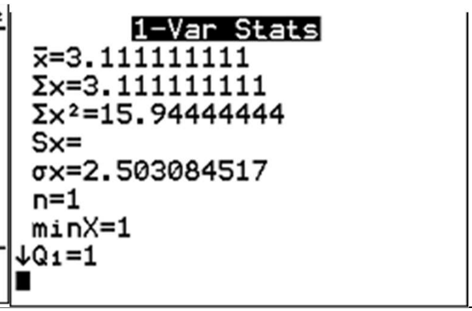
$$+(10)^2\left(\frac{1}{36}\right)+(12)^2\left(\frac{1}{36}\right)$$

$$= \frac{287}{18}$$

$$\mathrm{Var}(X) = \mathrm{E}(X^2) - \mathrm{E}(X)^2$$
$$= \frac{287}{18} - \left(\frac{28}{9}\right)^2$$
$$= \frac{1015}{162}$$

The following GC screenshots are for checking purpose only (note that question asks for exact values of E(X) and Var(X))





3 MJC Prelim 9233/2005/02/Q12

A discrete random variable X takes values 2, 3, 4, 5 with probabilities as shown in the table.

x	2	3	4	5
$\mathrm{P}(X=x)$	k	$\frac{k}{4}$	$\frac{k}{9}$	$\frac{k}{16}$

(i) Find k, leaving your answer as a fraction. [2]

(ii) Find $\mathrm{E}(|\cos X|)$, giving your answer to 3 significant figures. [3]

(iii) Find $\mathrm{P}(X_1 + X_2 = 7)$, where X_1 and X_2 are two independent observations of X. [3]

[**(i)** $k = \frac{144}{205}$ **(ii)** 0.530 **(iii)** 0.0891]

(i) $$k + \frac{k}{4} + \frac{k}{9} + \frac{k}{16} = 1$$
$$\Rightarrow k = \frac{144}{205}$$

(ii) $$\mathrm{E}\left(|\cos X|\right) = \frac{|\cos 2| \cdot 144 + |\cos 3| \cdot 36 + |\cos 4| \cdot 16 + |\cos 5| \cdot 9}{205}$$
$$= 0.530 \text{ (3 s.f.)}$$

(iii) $$\mathrm{P}(X_1 + X_2 = 7) = 2\mathrm{P}(X_1 = 2,\ X_2 = 5) + 2\mathrm{P}(X_1 = 3,\ X_2 = 4)$$
$$= 2\left(\frac{144}{205} \cdot \frac{9}{205}\right) + 2\left(\frac{36}{205} \cdot \frac{16}{205}\right)$$
$$= \frac{3744}{42025} = 0.0891 \text{ (3 s.f.)}$$

4 CJC Prelim 9233/2005/04/Q27 (modified)

A box contains 2 fair tetrahedral dice. The first die has sides labeled 1, 1, 2, and 3 and the second die has sides labeled 1, 2, 3 and 3.

A die is taken at random from the box and thrown. X, the score is defined as follows:

If the first die is picked, then it is thrown and the score is defined as two times the number which appears on the base of the first die.

If the second die is picked, then it is thrown and the score is the number which appears on the base of the second die.

Show that $\mathrm{P}(X=2)=\frac{3}{8}$, and find the probability distribution of X.

(i) Show that $\mathrm{E}(X)=\frac{23}{8}$ and find the exact value of $\mathrm{Var}(X)$.

(ii) If this experiment is performed twice, find the probability that the score is 3 for the first experiment given that the total score is 4.

(iii) Tom and Jerry take turns to perform this experiment with Jerry playing first. They stop when one of them obtains a score of 2.
What is the probability of Tom obtaining a score of 2?

[**(i)** $\frac{135}{64}$ **(ii)** $\frac{2}{13}$ **(iii)** $\frac{5}{13}$]

$\mathrm{P}(X=2)=\mathrm{P}(1^{\text{st}}$ die picked & ‘1’ obtained) + $\mathrm{P}(2^{\text{nd}}$ die picked & ‘2’ obtained)

$=\frac{1}{2}\times\frac{2}{4}+\frac{1}{2}\times\frac{1}{4}=\frac{3}{8}$ (shown)

$\mathrm{P}(X=1)=\mathrm{P}(2^{\text{nd}}$ die picked & ‘1’ obtained)$=\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$

$\mathrm{P}(X=3)=\mathrm{P}$ (2^{nd} die picked & ‘3’ obtained)$=\frac{1}{2}\times\frac{2}{4}=\frac{1}{4}$

$\mathrm{P}(X=4)=$ $\mathrm{P}(1^{\text{st}}$ die picked & ‘2’ obtained)$=\frac{2}{4}\times\frac{1}{4}=\frac{1}{8}$

$\mathrm{P}(X=6)=$ $\mathrm{P}(1^{\text{st}}$ die picked & ‘3’ obtained)$==\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$

The probability distribution of X is:

x	1	2	3	4	6
$\mathrm{P}(X=x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

(i) $\mathrm{E}(X) = 1\left(\frac{1}{8}\right) + 2\left(\frac{3}{8}\right) + 3\left(\frac{1}{4}\right) + 4\left(\frac{1}{8}\right) + 6\left(\frac{1}{8}\right)$

$= \frac{23}{8}$

$\mathrm{E}(X^2) = 1^2\frac{1}{8} + 2^2\frac{3}{8} + 3^2\frac{1}{4} + 4^2\frac{1}{8} + 6^2\frac{1}{8}$

$= \frac{83}{8}$

$\mathrm{Var}(X) = \mathrm{E}(X^2) - [\ \mathrm{E}(X)\ ]^2$

$= \frac{83}{8} - (\frac{23}{8})^2$

$= \frac{135}{64}$

(ii) $\mathrm{P}(X_1 = 3 \mid X_1 + X_2 = 4) = \frac{\mathrm{P}(X_1 = 3 \text{ and } X_2 = 1)}{\mathrm{P}(X_1 + X_2 = 4)}$

$= \frac{\mathrm{P}(X_1 = 3 \text{ and } X_2 = 1)}{2\ \mathrm{P}(X_1=1\ \&\ X_2=3) + \mathrm{P}(X_1=2\ \&\ X_2=2)}$

$= \frac{\left(\frac{1}{4}\right)\left(\frac{1}{8}\right)}{2\left(\frac{1}{8}\right)\left(\frac{1}{4}\right)+\left(\frac{3}{8}\right)\left(\frac{3}{8}\right)}$

$= \frac{2}{13}$

(iii)

Probability that Tom obtains a score of 2

$= \frac{5}{8}\left(\frac{3}{8}\right) + \left(\frac{5}{8}\right)^3\frac{3}{8} + \left(\frac{5}{8}\right)^5\frac{3}{8} + \cdots$

$= \frac{\frac{15}{64}}{1-\frac{25}{64}} = \frac{15}{39} = \frac{5}{13}$

5 9233/1993June/02/Q7(modified)

Alfred and Bertie play a game, each starting with cash amounting to £100. Two dice are thrown. If the total score is 5 or more then Alfred pays £x, where $0 \le x \le 8$, to Bertie. If the total score is 4 or less, then Bertie pays £(x + 8) to Alfred.

By finding the probability distribution of Y, where Y denotes the random variable representing Alfred's gains after one game, show that the expectation of Alfred's cash after the first game is £$\frac{1}{3}$(304 – 2x). Find the expectation of Alfred's cash after six games.

Find the value of x for the game to be fair. [Hint: For the game to be fair, expectation of Alfred's gains needs to be equal to 0.]

Given x = 3, find the variance of Alfred's cash after the first game.

[£(108 – 4x), x = 2, £$^2 \frac{245}{9}$]

The probability distribution of Y is:

y	$-x$	$x+8$
$\mathrm{P}(Y=y)$	$\mathrm{P}(\text{score} \ge 5) = \frac{5}{6}$	$\mathrm{P}(\text{score} \le 4) = \frac{6}{36} = \frac{1}{6}$

Expectation of Alfred's cash after one game

$= £\left(\mathrm{E}(100+Y)\right)$

$= £\left(100+\mathrm{E}(Y)\right)$

$= £\left(100+\frac{5}{6}(-x)+\frac{1}{6}(x+8)\right)$

$= £\frac{1}{3}(304-2x)$

$\mathrm{E}(Y_1+Y_2+...+Y_6) = 6\left(\frac{5}{6}(-x)+\frac{1}{6}(x+8)\right)$

$= (8-4x)$

$\therefore$ Expectation of Alfred's cash after 6 games is £ (108 – 4x).

For the game to be fair, we need E(Y) = 0.

$\frac{5}{6}(-x)+\frac{1}{6}(x+8)=0$

$\therefore x=2$

Given x = 3,

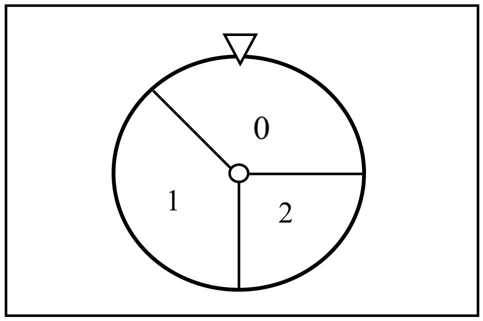
Var (Alfred's cash after one game) = Var (100 + Y)

= Var (Y)

$= \left((-3)^2\left(\frac{5}{6}\right)+11^2\left(\frac{1}{6}\right)\right)-\left[(-3)\left(\frac{5}{6}\right)+11\left(\frac{1}{6}\right)\right]^2 = \frac{245}{9}$

6 PJC Prelim 9758/2018/02/Q6 (modified)

A circular card is divided into 3 sectors scoring 0, 1, 2 and having angles $135°, 135°$ and $90°$ respectively. The card is mounted onto a wall at its centre such that it can rotate freely. A pointer is fixed on the wall just above the card, as shown in the diagram.



In a game, a participant will spin the card twice, and the random variable X is the product of the scores of the 2 spins.

(i) Tabulate the probability distribution of X. [3]

(ii) Show that $\mathrm{E}(X)=\dfrac{49}{64}$, and find $\mathrm{Var}(X)$. [2]

Dave intends to use the above setup in a fundraising carnival. For every game, the participant will be charged \$$h$ to play, and will be awarded a prize money of $\$5X$ based on the outcome of their spins. Find the least integer value of h such that on average, Dave can expect to earn at least \$1 from each participant. [2]

[**(ii)** $\dfrac{5343}{4096}$, 5]

(i) $\dfrac{90}{360}=\dfrac{1}{4}$, $\dfrac{135}{360}=\dfrac{3}{8}$

$$\mathrm{P}(X=0)=\mathrm{P}(0,0)+\mathrm{P}(0,1)\times 2!+\mathrm{P}(0,2)\times 2!$$

$$=\left(\frac{3}{8}\right)^2+\left(\frac{3}{8}\right)\left(\frac{3}{8}+\frac{1}{4}\right)2!=\frac{39}{64}$$

$$\mathrm{P}(X=1)=\mathrm{P}(1,1)=\left(\frac{3}{8}\right)^2=\frac{9}{64}$$

$$\mathrm{P}(X=2)=\mathrm{P}(1,2)\times 2!=\left(\frac{3}{8}\right)\left(\frac{1}{4}\right)\times 2!=\frac{3}{16}$$

$$\mathrm{P}(X=4)=\mathrm{P}(2,2)=\left(\frac{1}{4}\right)^2=\frac{1}{16}$$

x	0	1	2	4
$\mathrm{P}(X=x)$	$\dfrac{39}{64}$	$\dfrac{9}{64}$	$\dfrac{3}{16}$	$\dfrac{1}{16}$

(ii) $\mathrm{E}(X)=(0)\left(\frac{39}{64}\right)+(1)\left(\frac{9}{64}\right)+(2)\left(\frac{3}{16}\right)+(4)\left(\frac{1}{16}\right)=\frac{49}{64}$

$\mathrm{E}(X^2)=(0)^2\left(\frac{39}{64}\right)+(1)^2\left(\frac{9}{64}\right)+(2)^2\left(\frac{3}{16}\right)+(4)^2\left(\frac{1}{16}\right)=\frac{121}{64}$

$\mathrm{Var}(X)=\mathrm{E}(X^2)-\left[\mathrm{E}(X)\right]^2=\frac{5343}{4096}$

(iii)

$\mathrm{E}(h-5X)\geq 1$

$\mathrm{E}(h)-\mathrm{E}(5X)\geq 1$

$\mathrm{E}(h)\geq 1+5\mathrm{E}(X)$

$\mathrm{E}(h)\geq 1+5\times\frac{49}{64}$

$=4.8281$

Hence, least integer value of h is 5.

7 9758/2018/02/Q8

A bag contains $(n+5)$ numbered balls. Two of the balls are numbered 3, three of the balls are numbered 4 and n of the halls are numbered 5. Two balls are taken, at random and without replacement, from the bag. The random variable S is the sum of the numbers on the two balls taken.

(i) Determine the probability distribution of S. [4]

(ii) For the case where $n = 1$. find P($S = 10$) and explain this result. [1]

(iii) Show that E(S) = $\dfrac{10n+36}{n+5}$ and Var(S) = $\dfrac{\mathrm{g}(n)}{(n+5)^2(n+4)}$ where $g(n)$ is a quadratic polynomial to be determined. [6]

[(ii) 0**]**

	$3,3,4,4,4,\underbrace{5,5,\ldots,5}_{n\text{ terms}}$
(i)	S = sum of numbers on the 2 balls taken

s	$\mathrm{P}(S=s)$
6	$\dfrac{{}^{2}C_{2}}{{}^{n+5}C_{2}}=\dfrac{2}{(n+5)(n+4)}$ or $\dfrac{2}{n+5}\times\dfrac{1}{n+4}$
7	$\dfrac{{}^{2}C_{1}\,{}^{3}C_{1}}{{}^{n+5}C_{2}}=\dfrac{12}{(n+5)(n+4)}$ Or $\dfrac{2}{n+5}\times\dfrac{3}{n+4}\times 2!$
8	$\dfrac{{}^{3}C_{2}+{}^{2}C_{1}\,{}^{n}C_{1}}{{}^{n+5}C_{2}}=\dfrac{6+4n}{(n+5)(n+4)}$ Or $\dfrac{3}{n+5}\times\dfrac{2}{n+4}+\dfrac{2}{n+5}\times\dfrac{n}{n+4}\times 2!$
9	$\dfrac{{}^{3}C_{1}\,{}^{n}C_{1}}{{}^{n+5}C_{2}}=\dfrac{6n}{(n+5)(n+4)}$ Or $\dfrac{3}{n+5}\times\dfrac{n}{n+4}\times 2!$

		10 | $\dfrac{{}^{n}C_2}{{}^{n+5}C_2}=\dfrac{n(n-1)}{(n+5)(n+4)}$ Or $\dfrac{n}{n+5}\times\dfrac{n-1}{n+4}$
(ii)	$\mathrm{P}(S=10)=0$ This is an impossible event as there is only one ball with number 5 and it is impossible to get a sum of 10 with two 3s and/or 4s.	
(iii)	$\mathrm{E}(S)=\sum s\mathrm{P}(S=s)$ $=\dfrac{12+84+48+32n+54n+10n(n-1)}{(n+5)(n+4)}$ $=\dfrac{10n^2+76n+144}{(n+5)(n+4)}$ $=\dfrac{(10n+36)(n+4)}{(n+5)(n+4)}$ $=\dfrac{10n+36}{n+5}$	
	$\mathrm{E}(S^2)=\sum s^2\mathrm{P}(S=s)$ $=\dfrac{72+588+384+256n+486n+100n(n-1)}{(n+5)(n+4)}$ $=\dfrac{100n^2+642n+1044}{(n+5)(n+4)}$ $\mathrm{var}(S)=\dfrac{100n^2+642n+1044}{(n+5)(n+4)}-\left(\dfrac{10n+36}{n+5}\right)^2$ $=\dfrac{(100n^2+642n+1044)(n+5)-(100n^2+720n+1296)(n+4)}{(n+5)^2(n+4)}$ $=\dfrac{\begin{array}{c}100n^3+642n^2+1044n+500n^2+3210n+5220\\-(100n^3+720n^2+1296n+400n^2+2880n+5184)\end{array}}{(n+5)^2(n+4)}$ $=\dfrac{22n^2+78n+36}{(n+5)^2(n+4)}$	

8 **HCI Prelim 9758/2021/02/Q6**

Tom has a bag of wooden rectangular blocks of identical size. The bag contains 1 blue block, m red blocks and $(m-1)$ yellow blocks, where $m > 2$. Tom and Jerry play a game using Tom's bag of wooden blocks. Jerry draws 2 blocks at random, one at a time, without replacement. 3 points will be awarded if a yellow block is drawn, 2 points will be awarded if a red block is drawn, but no points will be awarded if a blue block is drawn. Jerry's final score is the product of the points awarded for the 2 blocks drawn.

Let the random variable X denotes Jerry's final score.

(i) Show that $\mathrm{P}(X=0)=\dfrac{1}{m}$ and hence, find the probability distribution of X. [4]

(ii) Find the value of m if Jerry's expected final score is 5. [2]

Tom pays Jerry \$5 if Jerry's final score is at least 5 and Jerry pays Tom \$$a$ if his final score is less than 5.

(iii) Using the value of m found in **(ii)**, find the range of values of a if Tom is expected to make a profit. [2]

[(ii) $m=6$; (iii) $a>7.70$ (to 2dp)**]**

(i)

Let X be Jerry's final score:

0, $2\times2=4$, $2\times3=6$, $3\times3=9$

$$\mathrm{P}(X=0)=\mathrm{P}(R,B)+\mathrm{P}(B,R)+\mathrm{P}(Y,B)+\mathrm{P}(B,Y)$$
$$=\frac{m(1)}{2m(2m-1)}\times2+\frac{(m-1)(1)}{2m(2m-1)}\times2$$
$$=\frac{4m-2}{2m(2m-1)}$$
$$=\frac{1}{m}$$

$$\mathrm{P}(X=4)=\mathrm{P}(R,R)$$
$$=\frac{m(m-1)}{2m(2m-1)}$$
$$=\frac{m-1}{2(2m-1)}$$

$$\mathrm{P}(X=6)=\mathrm{P}(R,Y)+\mathrm{P}(Y,R)$$
$$=\frac{m(m-1)}{2m(2m-1)}\times2$$

$$= \frac{m-1}{2m-1}$$

$$\mathrm{P}(X=9) = \mathrm{P}(Y,Y)$$

$$= \frac{(m-1)(m-2)}{2m(2m-1)}$$

x	0	4	6	9
$\mathrm{P}(X=x)$	$\frac{1}{m}$	$\frac{m-1}{2(2m-1)}$	$\frac{m-1}{2m-1}$	$\frac{(m-1)(m-2)}{2m(2m-1)}$

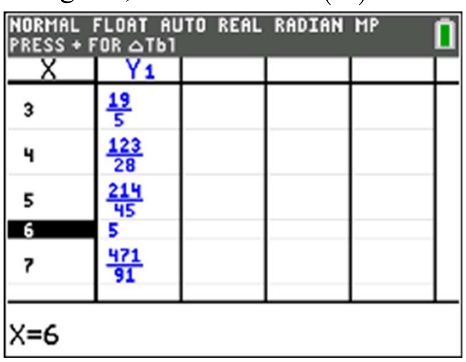
(ii)

$$\mathrm{E}(X) = \sum x\mathrm{P}(X=x)$$

$$= \frac{4(m-1)}{2(2m-1)} + \frac{6m-6}{2m-1} + \frac{9(m^2-3m+2)}{2m(2m-1)}$$

$$= \frac{25m^2-43m+18}{2m(2m-1)}$$

Using GC, $m=6$ when $\mathrm{E}(X)=5$.



(iii)

Let W be Tom's winnings (in \$).

When $m = 6$,

w	a	-5
$\mathrm{P}(W=w)$	$\mathrm{P}(X=0)+\mathrm{P}(X=4)$ $= \frac{1}{6}+\frac{5}{22}$ $= \frac{13}{33}$	$\mathrm{P}(X=6)+\mathrm{P}(X=9)$ $= \frac{5}{11}+\frac{5}{33}$ $= \frac{20}{33}$

If Tom is expected to make a profit, then

$$\frac{13}{33}a - \frac{20(5)}{33} > 0$$

$$13a > 100$$

$$a > 7.6923$$

Hence, $a > 7.70$ (to 2dp) in order for Tom to make a profit.

9 **9740/2011/02/Q11 (modified)**

A committee of 10 people is chosen at random from a group consisting of 18 women and 12 men. The number of women on the committee is denoted by R.

(i) Find the probability that $R=4$.

(ii) The most probable number of women on the committee is denoted by r. By using the fact that $\mathrm{P}(R=r)>\mathrm{P}(R=r+1)$, show that r satisfies the inequality

$$(r+1)!(17-r)!(9-r)!(r+3)!>r!(18-r)!(10-r)!(r+2)!$$

And use this inequality to find the value of r.

(iii) Use the calculator to find $\mathrm{E}(R)$ and $\mathrm{Var}(R)$.

[**(i)** 0.0941 **(ii)** 6 **(iii)** 6, 1.66]

Solution

(i) $\mathrm{P}(R=4)=\dfrac{\binom{18}{4}\binom{12}{6}}{\binom{30}{10}}\approx 0.0941$

(ii) $\mathrm{P}(R=r)>\mathrm{P}(R=r+1)$

$$\frac{\binom{18}{r}\binom{12}{10-r}}{\binom{30}{10}}>\frac{\binom{18}{r+1}\binom{12}{10-(r+1)}}{\binom{30}{10}}$$

$$\binom{18}{r}\binom{12}{10-r}>\binom{18}{r+1}\binom{12}{9-r}$$

$$\frac{18!}{r!(18-r)!}\frac{12!}{(10-r)!(2+r)!}>\frac{18!}{(r+1)!(17-r)!}\frac{12!}{(9-r)!(3+r)!}$$

$(r+1)!(17-r)!(9-r)!(r+3)!>r!(18-r)!(10-r)!(r+2)!$ (shown)

$$\frac{(r+1)!(17-r)!(9-r)!(r+3)!}{r!(18-r)!(10-r)!(r+2)!}>1$$

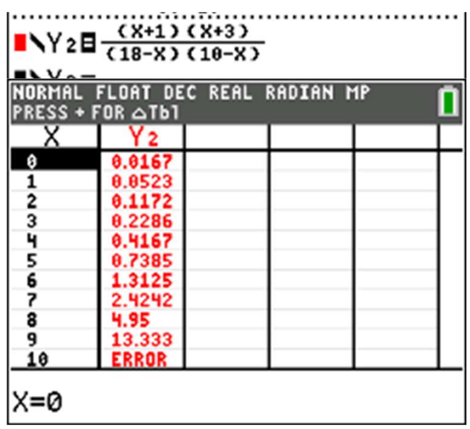
$$\frac{(r+1)(r+3)}{(18-r)(10-r)}>1$$

Using G.C, when $r=5$,

$$\frac{(r+1)(r+3)}{(18-r)(10-r)}=\frac{48}{65}<1$$

When $r=6$,

$$\frac{(r+1)(r+3)}{(18-r)(10-r)}=\frac{21}{16}>1$$



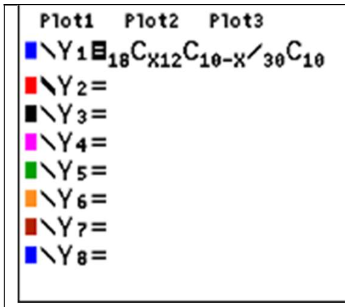
this implies that

$\mathrm{P}(R=6)>\mathrm{P}(R=7)>\mathrm{P}(R=8)>\ldots>\mathrm{P}(R=10)$

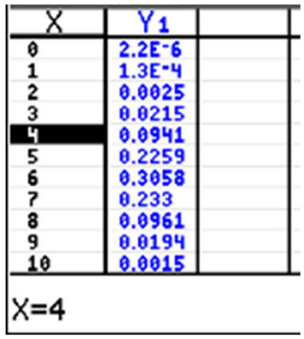
Similarly, it can be observed that for $\mathrm{P}(R=r)>\mathrm{P}(R=r+1)$ for $r\le 5$.

$\therefore\ \mathrm{P}(R=0)<\ldots<\mathrm{P}(R=4)<\mathrm{P}(R=5)<\mathrm{P}(R=6)$

Therefore, $r=6$.



Probability Distribution for R

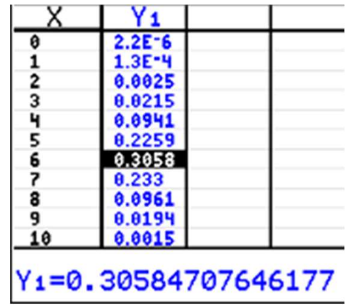


X	Y_1
0	2.2E-6
1	1.3E-4
2	0.0025
3	0.0215
4	0.0941
5	0.2259
6	0.3058
7	0.233
8	0.0961
9	0.0194
10	0.0015

X=4

Answer to (i) 0.0941

Successive values for Y increase in value till 0.3058 then decrease.

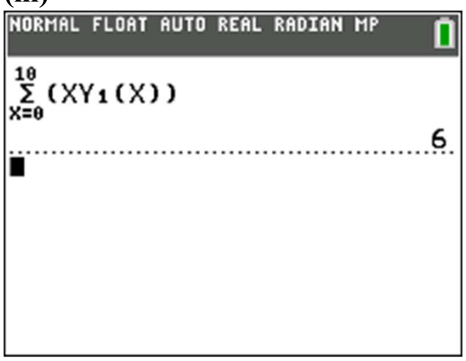


X	Y_1
0	2.2E-6
1	1.3E-4
2	0.0025
3	0.0215
4	0.0941
5	0.2259
6	0.3058
7	0.233
8	0.0961
9	0.0194
10	0.0015

Y_1=0.30584707646177

Mode when R = 6

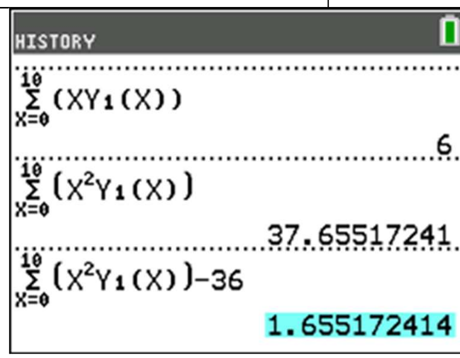
(iii)



$\mathrm{E}(R)=6$

In general, the mean of a hypergeometric distribution is $n\frac{K}{N}$, where the probability mass function is $\frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$. For this question,

$\mathrm{E}(R)=10\left(\frac{18}{30}\right)=6$.



$\mathrm{Var}(R)=\mathrm{E}(R^2)-\mathrm{E}(R)^2=1.66$

The variance of a hypergeometric distribution is

$n\frac{K}{N}\frac{(N-K)}{N}\frac{N-n}{N-1}=10\frac{18}{30}\frac{12}{30}\frac{20}{29}=\frac{48}{29}$