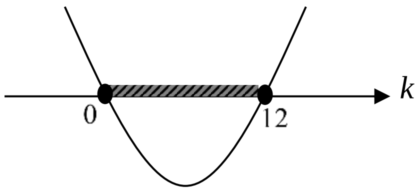
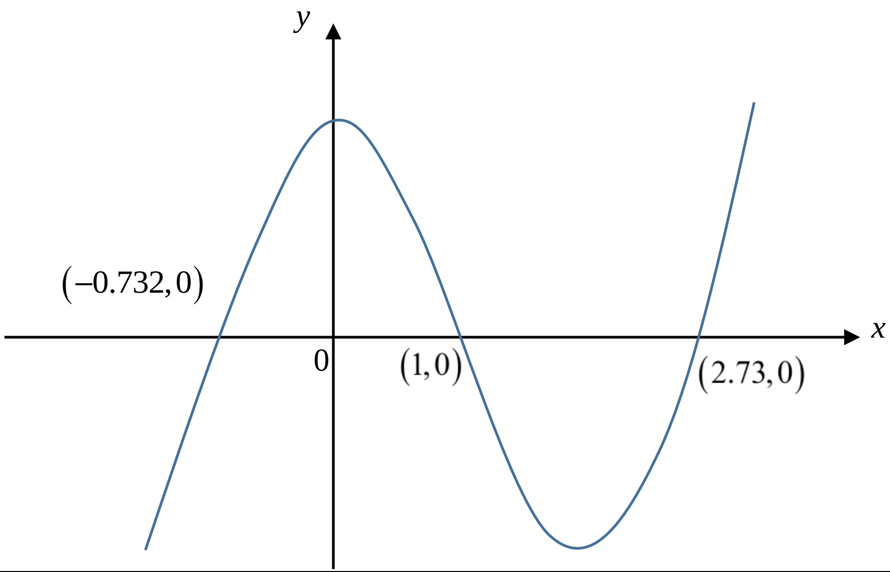
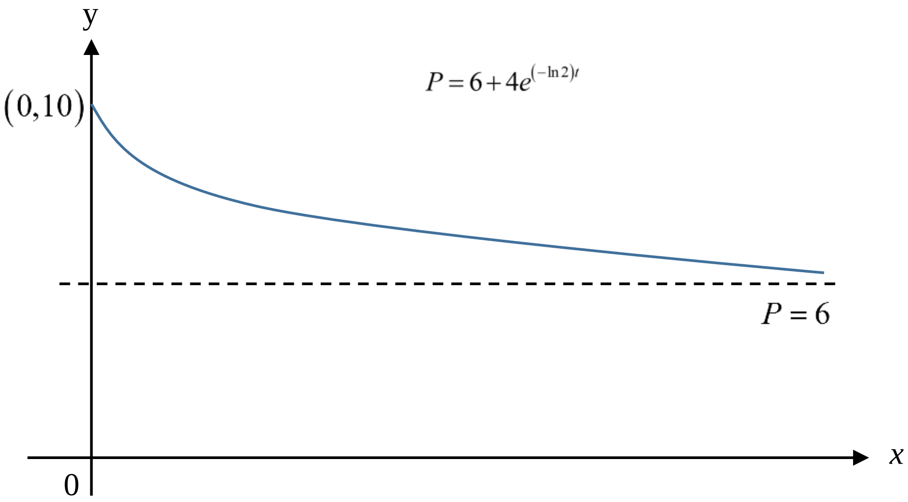
Question	Suggested Solutions
1	Since $x^2+(k-2)x+2k+1>0$, Discriminant < 0 $(k-2)^2-4(1)(2k+1)<0$ $k^2-4k+4-8k-4<0$ $k^2-12k<0$ $k(k-12)<0$  $0<k<12$ Therefore the set of values of k is $\{k\in\mathbb{R}\mid 0<k<12\}$
2(ia)	$\dfrac{\mathrm{d}}{\mathrm{d}x}e^{3x+2}=3e^{3x+2}$
2(ib)	$\dfrac{\mathrm{d}}{\mathrm{d}x}3\ln\left(4+2x^3\right)=3\left(\dfrac{6x^2}{4+2x^3}\right)$ $=\dfrac{9x^2}{2+x^3}$

2(ii)	$\int \frac{5}{(3-4x)^2}\,\mathrm{d}x = 5\int (3-4x)^{-2}\,\mathrm{d}x$ $= 5\frac{(3-4x)^{-1}}{(-1)(-4)} + C$ $= \frac{5}{4(3-4x)} + C,$ where C is an arbitrary constant
3(i)	Let \$$x$, \$$y$ and \$$z$ be the cost of mobile phone, a laptop and a television respectively. $y = 2x \Rightarrow 2x - y = 0 - (1)$ $3x + 2y + 8z = 24120 - (2)$ $y = x + 2z - 640 \Rightarrow x - y + 2z = 640 - (3)$ Solving (1), (2) and (3) using GC, $x = 1960$ $y = 3920$ $z = 1300$ $\therefore$ The cost of a mobile phone is \$1960.
3(ii)	Total amount Mia has to pay $= \$0.9\left[(0.2)(80)(1960) + (0.2)(80)(3920) + (0.6)(80)(1300)\right]$ $= \$140832$

4(i)	$y = x^3 - 3x^2 + 2$ 
4(ii)	$y = x^3 - 3x^2 + 2$ $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 6x$ When $x = 0.5$, $y = (0.5)^3 - 3(0.5) + 2 = \frac{11}{8}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 3(0.5)^2 - 6(0.5) = -\frac{9}{4}$ Equation of tangent when $x = 0.5$,

	$y-\frac{11}{8}=-\frac{9}{4}\left(x-\frac{1}{2}\right)$ $y-\frac{11}{8}=-\frac{9}{4}x+\frac{9}{8}$ $8y-11=-18x+9$ $8y+18x-20=0$ $4y+9x-10=0$ or $-4y-9x+10=0$
4(iii)	Area of region $=\int_3^4 x^3-3x^2+2\ \mathrm{d}x$ $=\left[\frac{x^4}{4}-\frac{3x^3}{3}+2x\right]_3^4$ $=\left[\frac{4^4}{4}-4^3+2(4)\right]-\left[\frac{3^4}{4}-3^3+2(3)\right]$ $=8.75$ units^2
5(i)	$P=6+ce^{-at}$ When $t=0$, $P=10$ $10=6+c\Rightarrow c=4$ When $t=1$, $P=8$

	$8=6+4e^{-a} \Rightarrow c=4$ $e^{-a}=\frac{1}{2}$ $-a=\ln\frac{1}{2}$ $a=-\ln\frac{1}{2}$ $=-(\ln 1-\ln 2)$ $=\ln 2$
5(ii)	$P=6+4e^{-(\ln 2)t}$ From GC, When t = 2, $\frac{\mathrm{d}P}{\mathrm{d}t}=-0.693$ **<u>ALT</u>** $\frac{\mathrm{d}P}{\mathrm{d}t}=4(-\ln 2)e^{-(\ln 2)t}$ $=-4\ln 2e^{-(\ln 2)t}$ When t = 2, $\frac{\mathrm{d}P}{\mathrm{d}t}=4(-\ln 2)e^{-2(\ln 2)}$ $=-4\ln 2e^{-(\ln 2)t}$

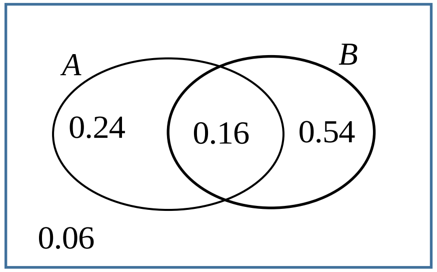
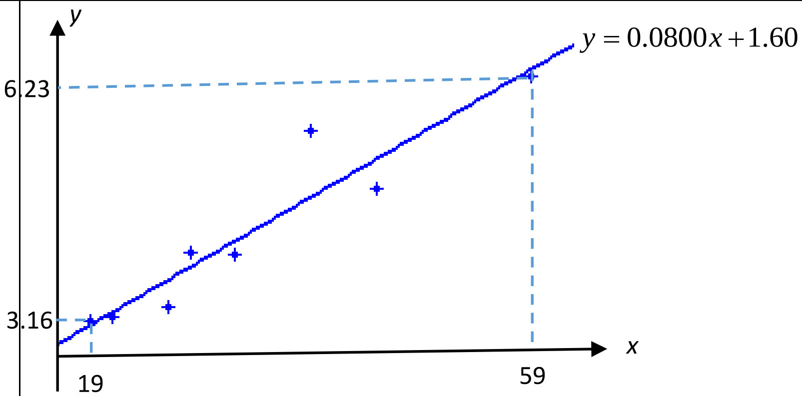
5(iii)	
5(iv)	As $t \to \infty, P \to 6$ since as $t \to \infty, e^{-\ln 2} = \frac{1}{e^{\ln 2}} \to 0$. Therefore the approximate size of the population is 6 million.

5(v)	$Q=t^{\frac{1}{2}}+5t^{-\frac{1}{2}}$ $\frac{\mathrm{dQ}}{\mathrm{d}t}=\frac{1}{2}t^{-\frac{1}{2}}-\frac{5}{2}t^{-\frac{3}{2}}$ When Q has a stationary point, $\frac{\mathrm{dQ}}{\mathrm{d}t}=0$ $\frac{1}{2}t^{-\frac{1}{2}}-\frac{5}{2}t^{-\frac{3}{2}}=0$ $\frac{1}{t^{\frac{1}{2}}}-\frac{5}{2t\left(t^{\frac{1}{2}}\right)}=0$ $\frac{1}{t^{\frac{1}{2}}}\left(1-\frac{5}{t}\right)=0$ $\frac{5}{t}=1$ $t=5$ when $t=5$, $Q=\sqrt{5}+\frac{5}{\sqrt{5}}=2\sqrt{5}$ $\therefore$ coordinates of stationary point is $(5,\ 2\sqrt{5})$.
5(vi)	$P=6+4e^{(-\ln 2)t}=\sqrt{t}+\frac{5}{\sqrt{t}}$ Using GC, $t=0.340, P=Q=9.16$ million

6(i)	Required probability = $\frac{80}{1600}=\frac{1}{20}$
(ii)	Require probability = $\frac{1600-(256+64)}{1600}=\frac{4}{5}$
(iii)	$\mathrm{P}(\text{student in year 1}\mid\text{student travels on foot})$ $=\frac{\mathrm{P}(\text{student in year 1}\cap\text{travels on foot})}{\mathrm{P}(\text{student travels on foot})}$ $=\frac{400}{400+112}$ $=\frac{25}{32}$
(iv)	Let A be the event that 'the student travels by car', and B be the event that 'the student is in Year 1'. $\mathrm{P}(A\cap B)=\frac{432}{1600}=\frac{27}{100}$ $\mathrm{P}(A)\times\mathrm{P}(B)=\frac{432+144}{1600}\times\frac{1200}{1600}=\frac{27}{100}$ Since $\mathrm{P}(A\cap B)=\mathrm{P}(A)\times\mathrm{P}(B)$, therefore events A and B are independent.
7(i)	Let X be the random variable denoting number of oranges that is not ripe out of 12 oranges. $X\sim \mathrm{B}(12, 0.15)$ $\mathrm{P}(X=1)=0.30122=0.301\,(\text{to 3 s.f.})$
(ii)	$\mathrm{P}(X>3)=1-\mathrm{P}(X\le 3)=0.092206=0.0922$ (to 3 s.f.)

(iii)	Required probability $=\mathrm{P}(X>3)\times\mathrm{P}(X>3)$ $=0.0085020$ $=0.00850\left(\text{to 3 s.f.}\right)$
(iv)	Let Y be the random variable denoting the number of bags that were sold at the reduced price out of 20 bags. $X\sim\mathrm{B}(20,0.092206)$ $\mathrm{P}(Y<4)=\mathrm{P}(Y\leq 3)=0.89366=0.894\left(\text{to 3 s.f.}\right)$
8(i)	Number of ways $= {}^{12}C_5$ $=792$
(ii)	<u>Case 1: 5 Girls and no Boy chosen</u> ${}^7C_5\times{}^5C_0=21$ <u>Case 2: 4 Girls and 1 Boy chosen</u> ${}^7C_4\times{}^5C_1=175$ <u>Case 3: 3 Girls and 2 Boys chosen</u> ${}^7C_3\times{}^5C_2=350$ Total number of ways $=21+175+350$ $=546$

(iii)	$\boxed{\underline{B_1}\ \underline{B_2}}$ _ _ _ Group B_1B_2 as 1 unit, then total permutation of 4 units = 4! Permutate the unit consisting of B_1B_2 = 2! $\therefore$ Number of ways = 4! $\times$ $2! = 48$
9(i)	It means the probability of event A occurring given that event B does not occur.
(ii)	$\mathrm{P}(A \mid B') = 0.8$ $\dfrac{\mathrm{P}(A \cap B')}{\mathrm{P}(B')} = 0.8$ $\dfrac{\mathrm{P}(A \cap B')}{1-0.7} = 0.8$ $\mathrm{P}(A \cap B') = 0.8 \times 0.3 = 0.24$
(iii)	$\mathrm{P}(A \cap B) = \mathrm{P}(A) - \mathrm{P}(A \cap B')$ $= 0.4 - 0.24$ $= 0.16$ Required probability $\mathrm{P}(A \cup B)$ $= \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ $= 0.4 + 0.7 - 0.16$ $= 0.94$

	<u>Alt</u> $\mathrm{P}(A\cup B)=\mathrm{P}(B)+\mathrm{P}(A\cap B')$ $=0.7+0.24$ $=0.94$
(iv)	$\mathrm{P}(B\cap A')=\mathrm{P}(B)-\mathrm{P}(A\cap B)$ $=0.7-0.16$ $=0.54$ $\mathrm{P}((A\cup B)')$ $=1-\mathrm{P}(A\cup B)$ $=1-0.94$ $=0.06$ 
10(i)	
(ii)	Using GC, $r=0.94203=0.942$ (to 3 s.f.) It shows that there is a strong positive linear correlation between *x* (the ages of the employees at this particular company) and *y* (the monthly earnings of the employees).

(iii)	Using GC, $y = 0.079964x + 1.6025$ $y = 0.0800x + 1.60$, where $m = 0.0800$ and $c = 1.60$ [*For the sketch of the regression line on the scatter diagram, please refer to the scatter diagram in (i)*]
(iv)	When $x = 24$, $y = 0.079964(24) + 1.6025$ $= 3.521636$ $= 3.52$ (to 3 s.f.) Thus, monthly earnings is $3.52 \times 1000 = \$3520$. Since $x = 24$ is within the range of x $(19 \le x \le 59)$ given in the table, the estimate is an interpolation. Also, the value of product moment correlation coefficient r is 0.942, which is close to 1. This suggests that there is a strong positive linear correlation between the ages of the employees and their monthly earnings. Therefore, we would expect this to be a reliable estimate.
(v)	The value of m will be the same as that found in part (iii). Because when the monthly earnings of all the employees are increased by a fixed number of dollars, all the data points on the scatter diagram will be moved upwards by the same margin. Therefore the new regression line of y on x will still be parallel to the original regression line of y on x in part (i), as the new regression line will just be translated upwards by the same margin in the positive y direction. Hence the gradient of the new regression line will be the same as the gradient of the original regression line.
11(i)	Let C be the time taken (in minutes) by Jai to service a car. $C \sim \mathrm{N}(100, 8^2)$ $P(90 < C < 120)$ $= 0.88814$ $= 0.888$ (to 3 s.f.)

(ii)	Let V be the time taken (in minutes) by Jai to service a van. $V \sim \mathrm{N}\left(150, 10^2\right)$ $\mathrm{P}\left(V \leq t\right) = 0.90$ Using GC, $t = 162.8155$ ≈ 163 (to 3 s.f.)
(iii)	Let $T = C_1 + C_2 + C_3 + V_1 + V_2$ $\mathrm{E}\left(T\right) = 3\mathrm{E}\left(C\right) + 2\mathrm{E}\left(V\right) = 3\left(100\right) + 2\left(150\right) = 600$ $\mathrm{Var}\left(T\right) = 3\mathrm{Var}\left(C\right) + 2\mathrm{Var}\left(V\right) = 3\left(8^2\right) + 2\left(10^2\right) = 392$ $\therefore T \sim \mathrm{N}\left(600, 392\right)$ $P\left(T < 630\right)$ $= 0.93514$ $= 0.935$ (to 3 s.f.)
(iv)	Let $L = 45\left[\dfrac{C_1 + C_2 + C_3}{60}\right] + 90\left[\dfrac{V_1 + V_2}{60}\right]$

	$\mathrm{E}(L)=\frac{45}{60}\times 3\mathrm{E}(C)+\frac{90}{60}\times 2\mathrm{E}(V)$ $=\frac{45}{60}\times 3(100)+\frac{90}{60}\times 2(150)$ $=675$ $\mathrm{Var}(L)=\left(\frac{45}{60}\right)^2\times 3\mathrm{Var}(C)+\left(\frac{90}{60}\right)^2\times 2\mathrm{Var}(V)$ $=\left(\frac{45}{60}\right)^2\times 3(8^2)+\left(\frac{90}{60}\right)^2\times 2(10^2)$ $=558$ $\therefore L\sim \mathrm{N}(675,558)$ $P(L>700)$ $=0.14495$ $=0.145$ (to 3 s.f.)
12(i)	Let X be the random variable denoting the height of a plant in region A (in cm) and μ be the population mean height of the plants. $\mathrm{H}_0:\mu=28.5$ $\mathrm{H}_1:\mu\neq 28.5$
12(ii)	Under H_0, since $n=40$ is large, by Central Limit Theorem, $\bar{X}\sim \mathrm{N}\left(28.5,\frac{6.6}{40}\right)$ approximately Using a 2 tailed test at 5% significance level, $\bar{x}=27.7$ gives $z_{\text{calc}}=-1.97$ and p-value $=0.0489<0.05$ Therefore we reject H_0 and conclude that there is sufficient evidence, at 5% significance level, that the mean height of plants is not 28.5 cm.

12(iii)	Unbiased estimate of the population mean is $\bar{x}$ $=\frac{\sum x}{n}=\frac{1650}{60}=27.5$ Unbiased estimate of the population variance is s^2 $=\frac{1}{n-1}\left[\sum x^2-\frac{\left(\sum x\right)^2}{n}\right]$ $=\frac{1}{60-1}\left[46170.1-\frac{1650^2}{60}\right]$ $=\frac{7951}{590}$ or $13.476\approx 13.5$
(iv)	Let Y be the random variable denoting the height of a plant in region B (in cm) and μ be the population mean height of the plants. $\mathrm{H}_0:\mu=28.5$ $\mathrm{H}_1:\mu\neq 28.5$ Under H_0, since $n=60$ is large, by Central Limit Theorem, $\bar{X}\sim \mathrm{N}\left(28.5,\frac{7951}{35400}\right)$ approximately Using a 2 tailed test at α % significance level, $\bar{x}=27.5$ gives $z_{\text{calc}}=-2.11$ and $p\text{-value}=0.0349$ Since H_0 is not rejected, $p\text{-value}>\frac{\alpha}{100}$ $\alpha<100\left(p\text{-value}\right)$ $\alpha<100\left(0.0349\right)$ $\alpha<3.49$ The set of values of α is $\left\{\alpha\in\mathbb{R}^{+}:\alpha<3.49\right\}$