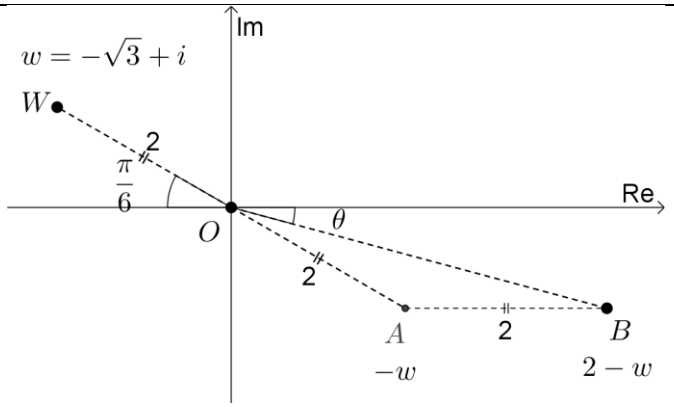
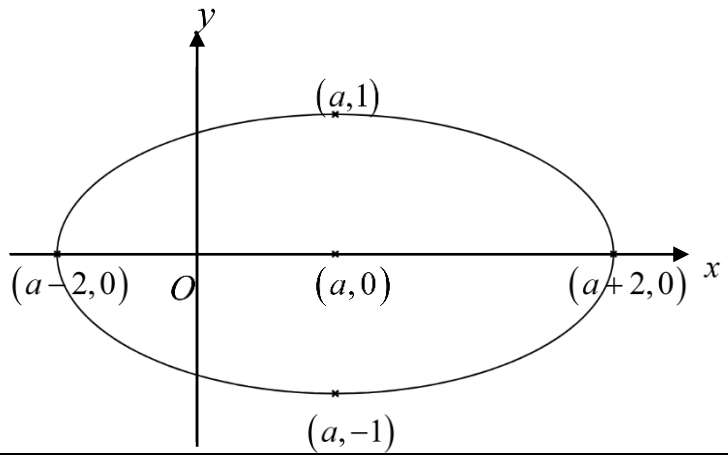
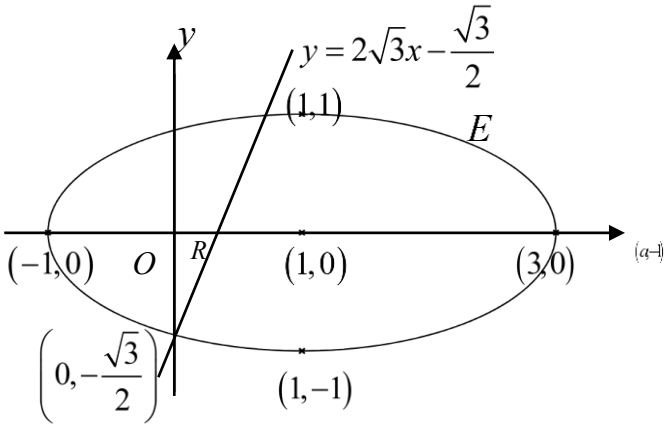
2025 NJC Preliminary Examination H2 Mathematics Paper 2 Solutions

1	Suggested Solution
(i)	$\lvert w\rvert = \lvert \mathrm{i} - \sqrt{3}\rvert$ $= \sqrt{\left(-\sqrt{3}\right)^2 + 1^2}$ $= 2$ $\arg(w) = \pi - \tan^{-1}\dfrac{1}{\sqrt{3}}$ $= \pi - \dfrac{\pi}{6}$ $= \dfrac{5\pi}{6}$
(ii)	 The point W represents w and the point B represents $2 - w$.
(iii)	Since $\lvert w\rvert = 2$, the triangle OAB is an isosceles triangle. AB is parallel to the real-axis, so $\angle AOB = \angle ABO = \theta$. $2\theta = \dfrac{\pi}{6}$ $\theta = \dfrac{\pi}{12}$ $\arg(2 - w) = -\dfrac{\pi}{12}$.

2	Suggested Solutions
(i)	$(x-a)^2+4y^2=4$ $\frac{(x-a)^2}{2^2}+y^2=1$ 
(ii)	Given $(x-1)^2+4y^2=4$ When $x=0$, $(0-1)^2+4y^2=4$ $y=\pm\frac{\sqrt{3}}{2}$ Differentiate w.r.t. x, $(x-1)^2+4y^2=4$ $2(x-1)+8y\frac{dy}{dx}=0$ $\frac{dy}{dx}=-\frac{x-1}{4y}$ grad. of normal $=\frac{4y}{x-1}$ Since gradient of $y=2\sqrt{3}x-\frac{\sqrt{3}}{2}$ is positive, from the diagram, the normal should have the y-intercept below the O i.e. $-\frac{\sqrt{3}}{2}$. For the point $\left(0,-\frac{\sqrt{3}}{2}\right)$, grad of normal $=\frac{4\left(-\frac{\sqrt{3}}{2}\right)}{0-1}=2\sqrt{3}$. Equation of line: $y=2\sqrt{3}x-\frac{\sqrt{3}}{2}$ (since y-intercept is $-\frac{\sqrt{3}}{2}$)

2	Suggested Solutions
(iii)	Equation of normal: $y = 2\sqrt{3}x - \frac{\sqrt{3}}{2}$

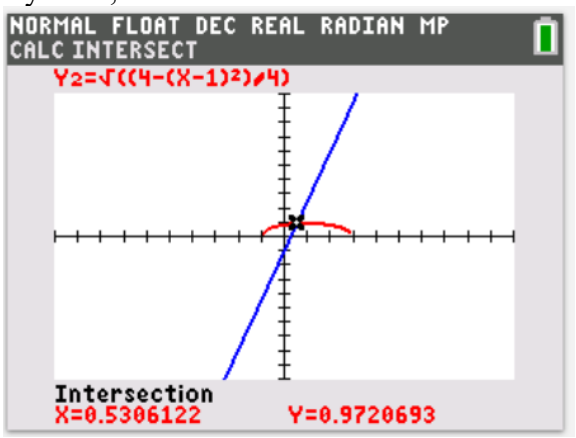


To find the intersection between normal and E, consider the positive square root of y.

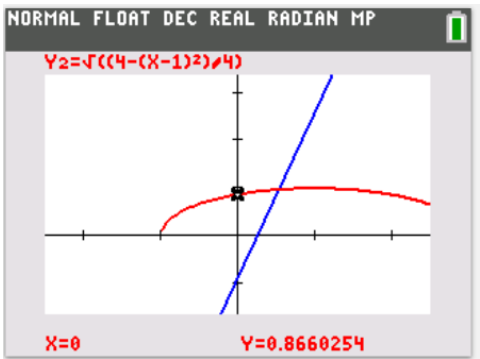
$(x-1)^2 + 4y^2 = 4$

$y = \sqrt{\frac{4-(x-1)^2}{4}}$

By G.C.,



The coordinates of point of intersection are $(0.530612, 0.972069)$.



The y-intercept of E is 0.866025.

2	Suggested Solutions
	To find solid R, make x the subject of the formula, $x = 1 \pm \sqrt{4 - 4y^2}$ Volume of solid R $= \frac{1}{3}\pi(0.530612)^2\left(\frac{\sqrt{3}}{2} + 0.972069\right)$ $-\pi\int_{0.8660254}^{0.972069}\left(1 - \sqrt{4 - 4y^2}\right)^2 \mathrm{d}y$ $= 0.517 \text{ units}^3$ (3 s.f.)

3	Suggested Solutions
(i)	$\mathbf{p} = \begin{pmatrix} \cos t \\ \sin t \\ -1 \end{pmatrix}, \mathbf{q} = \begin{pmatrix} \cos 2t \\ -\sin 2t \\ \alpha \end{pmatrix}$ $\mathbf{p} \times \mathbf{q}$ $= \begin{pmatrix} \cos t \\ \sin t \\ -1 \end{pmatrix} \times \begin{pmatrix} \cos 2t \\ -\sin 2t \\ \alpha \end{pmatrix}$ $= \begin{pmatrix} \alpha \sin t - \sin 2t \\ -\alpha \cos t - \cos 2t \\ -\sin 2t \cos t - \cos 2t \sin t \end{pmatrix}$ $= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$
	Method 1: Equate **k** -component to 0 $-\sin 2t \cos t - \cos 2t \sin t = 0$ $-(\sin 2t \cos t + \cos 2t \sin t) = 0$ $-\sin 3t = 0$ For $-\sin 3t = 0 \Rightarrow t = \frac{\pi}{3}, \frac{2\pi}{3}$. When $t = \frac{\pi}{3}$: $-\left(\alpha \cos\left(\frac{\pi}{3}\right) + \cos 2\left(\frac{\pi}{3}\right)\right) = 0$ $\left(\alpha\left(\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\right) = 0$ $\alpha = 1$ When $t = \frac{2\pi}{3}$ $-\left(\alpha \cos\left(\frac{2\pi}{3}\right) + \cos 2\left(\frac{2\pi}{3}\right)\right) = 0$ $\left(\alpha\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\right) = 0$ $\alpha = -1$ Final answers: $t = \frac{\pi}{3}, \alpha = 1$; $t = \frac{2\pi}{3}, \alpha = -1$

Method 2: **q** is a scalar multiple of **p**

$$\mathbf{p} = k\mathbf{q} \Rightarrow \begin{pmatrix} \cos 2t \\ -\sin 2t \\ \alpha \end{pmatrix} = k \begin{pmatrix} \cos t \\ \sin t \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} \cos 2t \\ -\sin 2t \\ \alpha \end{pmatrix} = \begin{pmatrix} k\cos t \\ k\sin t \\ -k \end{pmatrix}$$

From **j**-component:

$-\sin 2t = k\sin t$

$-2\cos t \sin t = k\sin t$

$-2\cos t \sin t - k\sin t = 0$

$\sin t(-2\cos t - k) = 0$

$\sin t = 0$ or $-2\cos t - k = 0$

$t = 0$ (rejected $\because 0 < t < \pi$) or $\cos t = -\frac{k}{2}$

From **i**-component:

$\cos 2t = k\cos t$

$2\cos^2 t - 1 - k\cos t = 0$

$2\cos^2 t - k\cos t - 1 = 0$

Sub $\cos t = -\frac{k}{2}$ into equation $2\cos^2 t - k\cos t - 1 = 0$:

$$2\left(-\frac{k}{2}\right)^2 - k\left(-\frac{k}{2}\right) - 1 = 0$$

$$\frac{k^2}{2} + \frac{k^2}{2} = 1$$

$k = \pm 1$

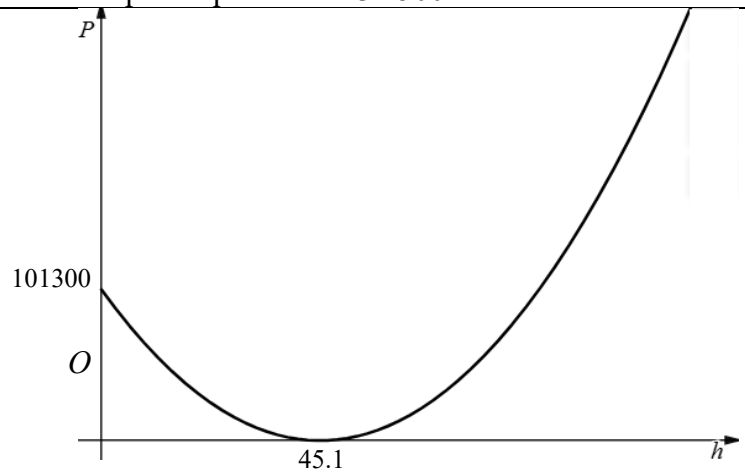
Since $\alpha = k$, $\alpha = -1$ or $\alpha = 1$.

When $\alpha = k = 1$, $t = \cos^{-1}\left(-\frac{k}{2}\right) = \frac{2\pi}{3}$

When $\alpha = k = -1$, $t = \cos^{-1}\left(-\frac{k}{2}\right) = \frac{\pi}{3}$

(ii)	$\mathbf{p}\cdot\mathbf{q}$ $= \begin{pmatrix}\cos t\\ \sin t\\ -1\end{pmatrix}\cdot\begin{pmatrix}\cos 2t\\ -\sin 2t\\ 0.5\end{pmatrix}$ $= \cos t\cos 2t - \sin t\sin 2t - 0.5$ $= \cos(2t+t) - 0.5$ $= \cos 3t - 0.5$ $\dfrac{\lvert\mathbf{p}\cdot\mathbf{q}\rvert}{\lvert\mathbf{p}\rvert}$ $= \dfrac{\lvert\cos 3t - 0.5\rvert}{\sqrt{2}}$ $= \dfrac{1}{\sqrt{2}}\lvert\cos 3t - 0.5\rvert$ Greatest length $= \dfrac{1}{\sqrt{2}}\lvert -1 - 0.5\rvert$ $= \dfrac{3}{2\sqrt{2}}$ or 1.06 (3 s.f.)

4	Suggested Solution
(i)	$\dfrac{\mathrm{d}P}{\mathrm{d}h} = -k\left(\dfrac{P}{T}\right)$ $T = 293 - 6.5h$ $\dfrac{\mathrm{d}P}{\mathrm{d}h} = \dfrac{-kP}{293 - 6.5\,h}$, where $k > 0$ or $\dfrac{\mathrm{d}P}{\mathrm{d}h} = \dfrac{kP}{293 - 6.5\,h}$, where $k < 0$
(ii)	$\dfrac{1}{P}\dfrac{\mathrm{d}P}{\mathrm{d}h} = -\dfrac{k}{293 - 6.5\,h}$ $\displaystyle\int \frac{1}{P}\ \mathrm{d}P = \int -\frac{k}{293 - 6.5\,h}\ \mathrm{d}h$ Since $P > 0$, $\ln P = \dfrac{k}{6.5}\ln\lvert 293 - 6.5\,h\rvert + C$ $P = \mathrm{e}^{\frac{k}{6.5}\ln\lvert 293-6.5\,h\rvert + C}$ $= A\left[\mathrm{e}^{\ln\lvert 293-6.5\,h\rvert}\right]^{\frac{k}{6.5}}$, where $A = \mathrm{e}^{C}$ $= A\lvert 293 - 6.5h\rvert^{\frac{k}{6.5}}$ $= A\lvert 293 - 6.5h\rvert^{b}$ Since $k > 0$, $b = \dfrac{k}{6.5} > 0$.
(iii)	When $h = 0$, $P = 101300$, $101300 = A\left(293^{b}\right)$ ...(1) When $h = 2$, $P = 80000$, $80000 = A\left(293 - 6.5\times 2\right)^{b}$ $80000 = A\left(280^{b}\right)$ ...(2) (1) / (2), $\dfrac{101300}{80000} = \left(\dfrac{293}{280}\right)^{b}$ $b = \dfrac{\ln\frac{101300}{80000}}{\ln\frac{293}{280}}$ $= 5.2015$ (5 s.f.)

	$A = \dfrac{101300}{293^{5.2015}}$ $= 1.4935 \times 10^{-8}$ $P = 1.4935 \times 10^{-8} \lvert 293 - 6.5h \rvert^{5.2015}$ When $h = 8.848$, $P = 1.4935 \times 10^{-8} \lvert 293 - 6.5(8.848) \rvert^{5.2015}$ $= 32500$ (to 3 s.f.) The atmospheric pressure is 32 500 Pascals.
(iv)	
	Either The model suggests that when altitude is greater than 45.1 km, atompheric pressure increases (to infinity) as altitude increases, which is not possible. **Or** As the temperature T measured in Kelvin can only be positive, the model is invalid if $T = 293 - 6.5h < 0$.

5	Solutions

$$\frac{\mathrm{P}(Y=k)}{\mathrm{P}(Y=k-1)}$$

$$=\frac{\binom{12}{k}p^{k}(1-p)^{12-k}}{\binom{12}{k-1}p^{k-1}(1-p)^{12-k+1}}$$

$$=\frac{\frac{12!}{k!(12-k)!}(p)}{\frac{12!}{(k-1)!(13-k)!}(1-p)}$$

$$=\frac{\frac{1}{k(k-1)!(12-k)!}(p)}{\frac{1}{(k-1)!(13-k)(12-k)!}(1-p)}$$

$$=\frac{(13-k)p}{k(1-p)} \quad (\text{Shown})$$

Given mode is 4,

$$\mathrm{P}(Y=4)>\mathrm{P}(Y=3)$$

$$\frac{\mathrm{P}(Y=4)}{\mathrm{P}(Y=3)}>1$$

$$\frac{9p}{4(1-p)}>1$$

$$9p>4-4p$$

$$p>\frac{4}{13}$$

and

$$\mathrm{P}(Y=4)>\mathrm{P}(Y=5)$$

$$\frac{\mathrm{P}(Y=5)}{\mathrm{P}(Y=4)}<1$$

$$\frac{8p}{5(1-p)}<1$$

$$8p<5-5p$$

$$p<\frac{5}{13}$$

Hence $\frac{4}{13}<p<\frac{5}{13}$.

6

(a)

$$\mathrm{P}(T=3)$$
$$=\mathrm{P}(\text{getting a 2, 4 and 5 from three tosses})$$
$$+\mathrm{P}(\text{getting a 3, 3 and 5 from three tosses})$$
$$+\mathrm{P}(\text{getting a 3, 4 and 4 from three tosses})$$
$$=\left(\frac{2}{6}\right)^2\left(\frac{1}{6}\right)\times(3!-2)+\left(\frac{1}{6}\right)^3\times\left(\frac{3!}{2!}-1\right)+\left(\frac{1}{6}\right)\left(\frac{2}{6}\right)^2\times\frac{3!}{2!}$$
$$=\frac{5}{36}\ (\text{Shown})$$

$$\mathrm{P}(T=1)=\mathrm{P}(\text{getting a 5 from one toss})$$
$$=\frac{1}{6}$$

$$\mathrm{P}(T=2)=\mathrm{P}(\text{getting a 2 and 3 from two tosses})$$
$$=\left(\frac{2}{6}\right)\left(\frac{1}{6}\right)\times 2!$$
$$=\frac{1}{9}$$
$$\mathrm{P}(T\geq 4)=1-\frac{1}{6}-\frac{1}{9}-\frac{5}{36}$$
$$=\frac{7}{12}$$

(b)

s	5	10	15	0
$\mathrm{P}(S=s)$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{7}{12}$

$$\mathrm{E}(S)=5\left(\frac{1}{6}\right)+10\left(\frac{1}{9}\right)+15\left(\frac{5}{36}\right)$$
$$=\frac{145}{36}\text{ or }4.03\ (3\text{ s.f.})$$

$$\mathrm{Var}(S)=\mathrm{E}(S^2)-\left[\mathrm{E}(S)\right]^2$$
$$=5^2\left(\frac{1}{6}\right)+10^2\left(\frac{1}{9}\right)+15^2\left(\frac{5}{36}\right)-\left(\frac{145}{36}\right)^2$$
$$=\frac{39275}{1296}\text{ or }30.3\ (3\text{ s.f.})$$

7	
(a)	**Method 1** Number of ways with 1 engine at the front $= \dfrac{8!}{3!3!2!}$ $= 560$ Number of ways with 2 engines at the back $= \dfrac{7!}{2!3!2!}$ $= 210$ Number of ways with 1 engine at the front and 2 engine at the back $= \dfrac{6!}{3!2!}$ $= 60$ Total number of ways to form the rail train $= 560 + 210 - 60$ $= 710$
	Method 2 Number of cases where the first unit is *E* and the last two units are not *EE (E_ _ _ _ _ _ _E'E')* $= \binom{6}{3}\binom{5}{3} + \binom{6}{2}\binom{5}{3}$ *(E_ _ _ _ _ _ _E'E')* *(E_ _ _ _ _ _ _E'E)* $+ \binom{6}{2}\binom{5}{3}$ *(E_ _ _ _ _ _ _EE')* $= 500$ Number of cases where the first unit is not *E* and the last two units are *EE (E'_ _ _ _ _ _ _EE)* $= \binom{6}{2}\binom{5}{3}$ $= 150$ Number of cases where the first unit is *E* and the last two units are *EE (E_ _ _ _ _ _ _EE)* $= \binom{6}{1}\binom{5}{2}$ $= 60$ Total number of ways $= 500 + 150 + 60 = 710$

(b)

Method 1

Let E, C and W represent a engine, carriage and wagon respectively.

Consider the arrangement:
E_C_C_C_W_W_

Number of ways to have the remaining 3 engines together in one of the six slots
$= 6$

Number of ways to have the remaining 3 engines separated into E and EE
$= \binom{6}{2} \times 2$
$= 30$

Number of ways to have the remaining 3 engines all separated
$= \binom{6}{3}$
$= 20$

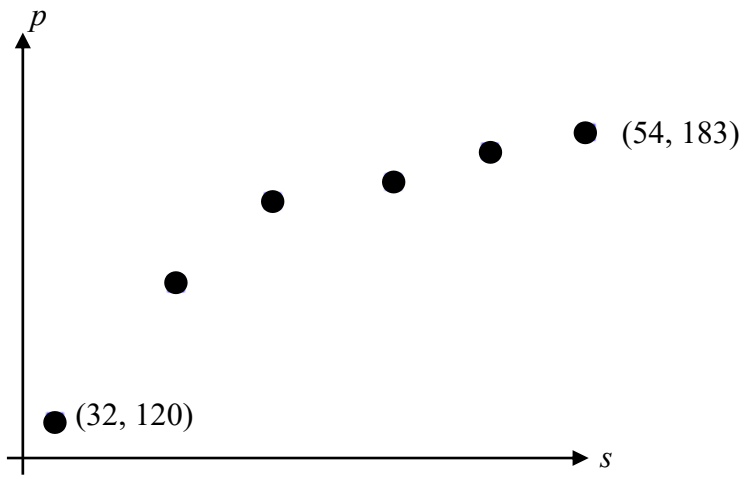
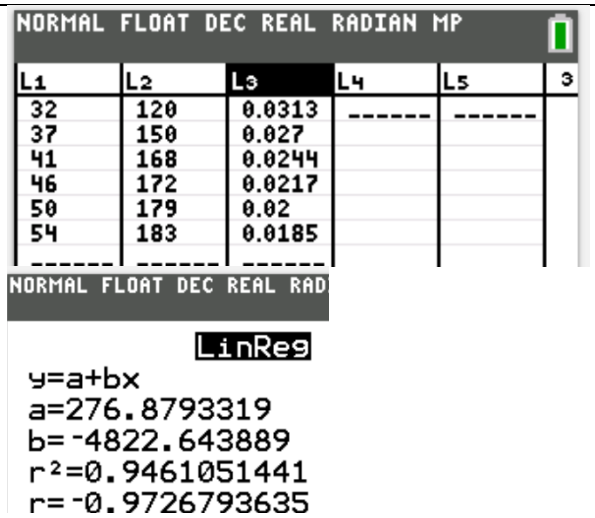
Total number of ways to form the toy train
$= 6 + 30 + 20$
$= 56$

Method 2

No. of ways to choose 5 slots out of 8 for three carriages and two wagons $= \binom{8}{5}$

No of ways to order three carriages and two wagons
$= \binom{8}{5} \times 1$

Total number of ways
$= \binom{8}{5} \times 1 \times \binom{3}{3}$ (remaining three slots are to have three
$= 56$
engines)

8	Suggested Solutions
(i)	
(ii)	The equation (B) $p = c + \frac{d}{s}$ is a better model because from the scatter diagram, as s increases, p increases at a decreasing rate.
(iii)	 $p = 276.88 - \frac{4822.6}{s}$ (5 s.f.) $p = 277 - \frac{4820}{s}$ (3 s.f.) product moment correlation coefficient, $r = -0.973$ (3 s.f.)
(iv)	when $s = 45$, $p = 276.88 - \frac{4822.6}{45}$ $p = 169.71$ (5 s.f.) $p = 170$ (3 s.f.) Since $s = 45$ lies within the range of values of s [32,54] and the product moment correlation coefficient, $r = -0.973$ is close to -1, which indicates a strong negative linear correlation between p and $\frac{1}{s}$, this estimate is reliable.

9	Suggested Solutions
(i)	$\mathrm{P}(\text{exactly one picture of A})$ $= \mathrm{P}(AA'A') + \mathrm{P}(A'AA') + \mathrm{P}(A'A'A)$ $= (0.1)(1-0.1)^2 + (1-0.1)(0.1)(1-0.1) + (1-0.1)^2(0.1)$ $= 0.243$
(ii)	**Method 1** $\mathrm{P}\begin{pmatrix}\text{the other two characters are } B \text{ and } C \\ \vert\text{exactly one character of } A\end{pmatrix}$ $= \dfrac{\mathrm{P}(\text{exactly one character of } A, B \text{ and } C \text{ respectively})}{\mathrm{P}(\text{exactly one character of } A)}$ $\mathrm{P}(\text{exactly one character of } A, B \text{ and } C \text{ respectively})$ $= \mathrm{P}(ABC) + \mathrm{P}(ACB) + \mathrm{P}(BAC)$ $+\mathrm{P}(BCA) + \mathrm{P}(CAB) + \mathrm{P}(CBA)$ $= 6(0.1)^3$ $= 0.006$ $\dfrac{\mathrm{P}(\text{exactly one character of } A, B \text{ and } C \text{ respectively})}{\mathrm{P}(\text{exactly one character of } A)}$ $= \dfrac{0.006}{0.243}$ $= \dfrac{2}{81}$ $= 0.024691$ **Method 2: Counting** Required probability $= \dfrac{n(A, B, C \text{ out of three coupons})}{n(\text{exactly one 'A' out of three coupons})}$ $= \dfrac{3(2)}{3(1\times 9\times 9)}$ $= \dfrac{2}{81}$ $= 0.024691$
(iii)	$p_1 = \dfrac{3}{10} \quad p_n = \dfrac{7}{10} p_{n-1}$

(iv)	**Method 1: Geometric Series** $\frac{3}{10}+\left(\frac{7}{10}\right)\frac{3}{10}+\left(\frac{7}{10}\right)^2\frac{3}{10}+\ldots+\left(\frac{7}{10}\right)^{n-1}\frac{3}{10}$ $=\frac{\frac{3}{10}\left(1-\left(\frac{7}{10}\right)^n\right)}{1-\frac{7}{10}}=1-\left(\frac{7}{10}\right)^n$ **Method 2: Considering Complement** $\mathrm{P}(\text{not getting final character})=0.7$ After n additional purchases, $\mathrm{P}(\text{not getting final character})=(0.7)^n$ After n additional purchases, $\mathrm{P}(\text{getting final character})=1-(0.7)^n$	

10	Suggested Solution
(i)	Each lunch box produced on that day has an equal chance (the same probability) of being selected. The event of a lunch box being selected is independent of another lunch box being selected.
(ii)	As the distribution of the heights of the lunch boxes is unknown, a large sample size (of at least 30) is needed to apply the Central Limit Theorem so that the sample mean follows a normal distribution approximately.
(iii)	Unbiased estimate of population mean, $\bar{x} = 13 + \frac{-3.6}{48} = 12.925$ Unbiased estimate of population variance, $s^2 = \frac{1}{47}\left(12.02 - \frac{(-3.6)^2}{48}\right) = 0.25$
(iv)	Let the population mean height of the lunch boxes be μ cm. To test: $H_0 : \mu = 12.8$ against $H_1 : \mu \neq 12.8$ at 10% level of significance Under H_0, $\overline{X} \sim N\left(12.8, \frac{0.25}{48}\right)$ approximately by the Central Limit Theorem, since the sample size $n = 48 > 30$ is large. Hence $Z = \frac{\overline{X} - 12.8}{\sqrt{\frac{0.25}{48}}} \sim N(0,1)$. By GC, p-value =0.0833 < 0.10 (or $\lvert z_{stat} \rvert = 1.732 < 1.645 = z_{crit}$) We reject H_0 and conclude that there is sufficient evidence at the 10% significance level to claim that the population mean height of the lunch boxes is not 12.8 cm.

(v)

To test:

$H_0 : \mu = 12.8$ against

$H_1 : \mu > 12.8$ at 2% level of significance

Under H_0,

$\overline{X} \sim N\left(12.8, \frac{0.04}{n}\right)$ approximately by the Central Limit Theorem since the sample size *n* is large

Hence $Z = \frac{\overline{X} - 12.8}{\sqrt{\frac{0.04}{n}}} \sim N(0,1)$.

Let the critical region be $\bar{x} \geq a$.

$$P\left(\overline{X} \geq a\right) = 0.02$$

$$P\left(\frac{\overline{X} - 12.8}{\sqrt{\frac{0.04}{n}}} \geq \frac{a - 12.8}{\sqrt{\frac{0.04}{n}}}\right) = 0.02$$

$$P\left(Z \geq \frac{a - 12.8}{\frac{0.2}{\sqrt{n}}}\right) = 0.02$$

$$\frac{(a - 12.8)\sqrt{n}}{0.2} = 2.0537489$$

$$a = 12.8 + \frac{0.411}{\sqrt{n}}$$

The critical region is $\bar{x} \geq 12.8 + \frac{0.411}{\sqrt{n}}$

11	Suggested Solutions
(a)	Let X and Y be the random variable denoting the fasting glucose concentration of a man and woman in mmol/L respectively. $X-Y \sim \text{N}\left(5.4-5.0, 0.5^2+0.3^2\right)$ $X-Y \sim \text{N}\left(0.4, 0.34\right)$ $\text{P}\left(X>Y\right)$ $=\text{P}\left(X-Y>0\right)$ $=0.754\left(3\text{ s.f.}\right)$ The assumption is that the fasting glucose concentration of a man is independent of that of a woman. OR The assumption is that the fasting glucose concentration of ALL men and women are independent.
(b)	$X \sim \text{N}\left(5.4, 0.5^2\right)$ $\text{P}\left(X-5.4 \geq c\right) \leq 0.03$ $\text{P}\left(\frac{X-5.4}{0.5} \geq \frac{c}{0.5}\right) \leq 0.03$ $\text{P}\left(Z \geq \frac{c}{0.5}\right) \leq 0.03$ $\frac{c}{0.5} \geq 1.880794$ $c \geq 0.940$ (3 s.f.)

(c)

$X \sim \mathrm{N}\left(5.4, 0.5^2\right)$

$\overline{Y} = \dfrac{Y_1 + Y_2 + Y_3 + Y_4}{4} \sim \mathrm{N}\left(5.0, \dfrac{0.3^2}{4}\right)$

Method 1

$p_1 = \mathrm{P}\left(X > k\right) = \mathrm{P}\left(Z > \dfrac{k - 5.4}{0.5}\right)$

$p_2 = \mathrm{P}\left(\overline{Y} > k\right)$

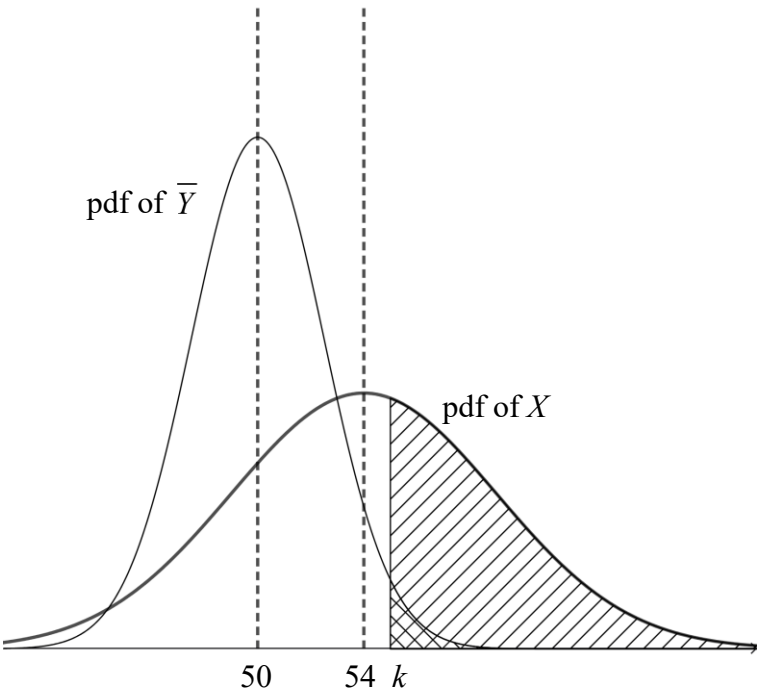
$= \mathrm{P}\left(Z > \dfrac{k - 5.0}{\frac{0.3}{\sqrt{2}}}\right)$

$= \mathrm{P}\left(Z > \dfrac{k - 5.0}{0.15}\right)$

For $k > 5.4$, $k - 5.4 < k - 5.0$ and $0.5 > 0.15 > 0$,
we have $\dfrac{k - 5.0}{0.15} > \dfrac{k - 5.4}{0.5}$.

Hence, $p_1 > p_2$.

Method 2



By comparing the areas of the shaded region in the diagram, $p_1 > p_2$..

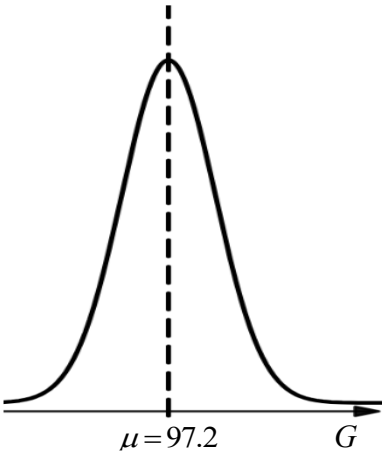
(d)

$X \sim \mathrm{N}\left(5.4, 0.5^2\right)$

$\mathrm{E}(G) = \mathrm{E}(18.0X) = 18.0\mathrm{E}(X) = 18.0 \times 5.4 = 97.2$

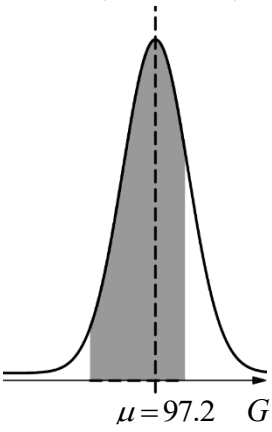
$\mathrm{Var}(G) = \mathrm{Var}(18.0X) = 18.0^2 \times 0.5^2 = 81.0$

$G \sim \mathrm{N}(97.2, 81.0)$



(e)

$G \sim \mathrm{N}(97.2, 81.0)$



$\mathrm{P}(79 < G < 106) = 0.814$ (3 s.f)