Anglo-Chinese School

(Independent)

THE BEST IS YET TO BE

FINAL EXAMINATION 2021

YEAR 5 IB DIPLOMA PROGRAMME

MATHEMATICS

HIGHER LEVEL

PAPER 1

MONDAY 20 SEP 2021

2 hours

Candidate Session Number

0	0	2	3	2	9				

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the writing paper provided. Fill in your session number on each answer sheet, and attach them to this examination paper using the string provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics Analysis and Approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**
- Questions with an asterisk (*) are common to both HL and SL papers

Section A (55 Marks)		Section B (55 Marks)	
Question	**Marks**	**Question**	**Marks**
1		10	
2			
3			
4		11	
5			
6			
7		12	
8			
9			
Subtotal		**Subtotal**	
TOTAL	/ 110		

This question paper consists of 12 printed pages including this cover page.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1.* [Maximum Mark: 4]

Solve the following equation $2^{x+1} = \dfrac{\sqrt{10^x + 3}}{5^x}$.

2.* **[Maximum Mark: 6]**

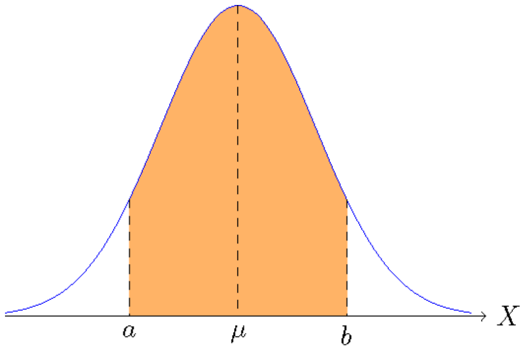
Solve the following equations.

(a) $\log_3 x - \log_3 (x+4) + 2 = 0$ [3]

(b) $\log_x 5 + \log_5 x = 2$ [3]

3.* **[Maximum Mark: 4]**

X is normally distributed and has a mean μ. It is also given that *a* and *b* are real numbers.



(a) If $P(X < a) + P(X \leq b) = 1$, prove that $P(X \geq a) = P(X \leq b)$. [2]

(b) If $a + b = 100$, find μ. [2]

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4. [Maximum Mark: 5]

(a) Expand $(1+4x)^{\frac{1}{2}}$ in descending powers of x, up to and including the first three non-zero terms. [4]

(b) State the values of x for which the expansion is valid. [1]

5.* **[Maximum Mark: 8]**

(a) In an arithmetic progression with first term a and common difference d, we are given that $u_{10} = S_{10} = 10$, where u_{10} is the tenth term and S_{10} is the sum of the first 10 terms of the progression.

(i) Find the values of a and d.

(ii) Hence find the sum of the first 20 terms of this arithmetic progression, S_{20}. [5]

(b) The sum of the first n terms of another series is given by $S_n = 3n^2 - 2n$. Determine if this series is an arithmetic series, justifying your reasons. [3]

6. [Maximum Mark: 5]

Consider the following system of linear equations:

$$\begin{aligned} x - y - z &= 1 \\ 3x + y + z &= 3 \\ 2x - 2y + kz &= 1 \end{aligned}$$

Discuss the value(s) of $k,$ where $k \in \mathbb{Z},$ for which the system has

(a) unique solutions; and [4]

(b) no solution. [1]

7. [Maximum Mark: 9]

(a) An arc PQ of a circle subtends an angle θ radians at the centre O of the circle. Given that the ratio of the area of the shaded region, segment POQ, to the area of sector POQ is equals to $1:4$, find the ratio of $\theta : \sin\theta$. [4]

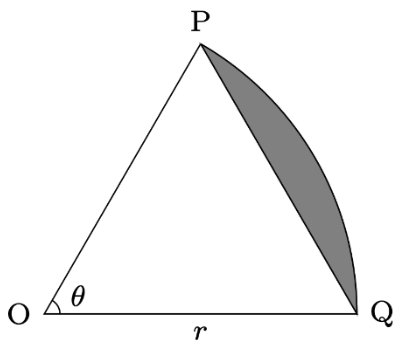


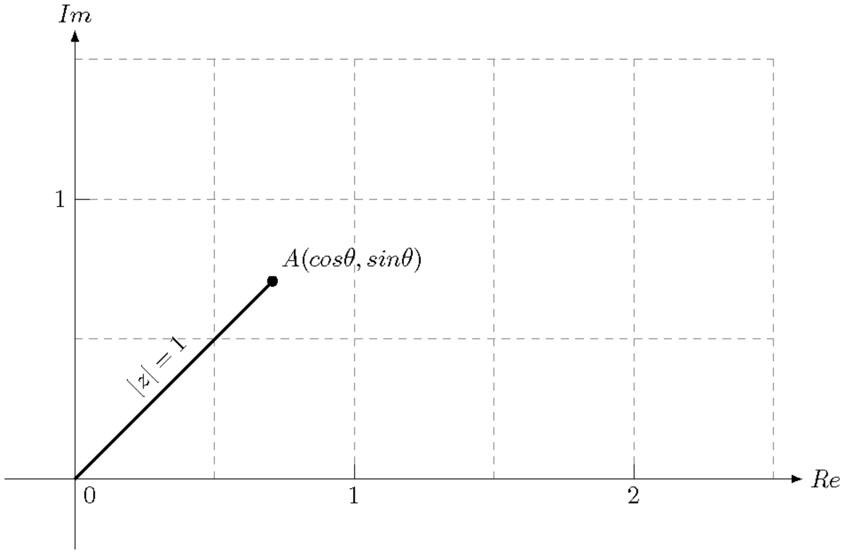
Diagram not to scale

(b) Prove the following trigonometric identity.

$$\frac{\cos 3\theta}{\sin 2\theta} \equiv \frac{1}{2}\operatorname{cosec}\theta - 2\sin\theta$$ [5]

8. [Maximum Mark: 7]

The complex number $z = \cos\theta + i\sin\theta,\ 0 < \theta \le \frac{\pi}{2}$ is represented by the point A in the Argand diagram below.



(a) Sketch the point B representing the complex number $z + 1$ on the same Argand diagram above. [1]

(b) Hence, by considering the triangle OAB or otherwise, find the modulus and argument of the complex number $z + 1$ in terms of $\frac{\theta}{2}$. [6]

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9. [Maximum Mark: 7]

The recurrence relation is defined by $x_1 = \frac{5}{3}$, $x_{n+1} = \sqrt[3]{\left(\frac{11}{8} + x_n\right)}$, for positive integers n. Prove by mathematical induction that $x_n < 2$ for all positive integers n.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer sheets provided. Please start each question on a new page

10.* [Maximum Mark: 19]

Consider the function defined by $f(x)=\dfrac{2x-5}{x-2}, x\neq k,$ where k is a constant.

(a) State the equation of the horizontal asymptote on the graph of $y=f(x)$. [1]

(b) Sketch the graph of $y=f(x),$ stating clearly the equations of any asymptote and the coordinates of any axial intercept. [4]

(c) Show that the function $f(x)$ is self-inverse. [2]

(d) Sketch, on a separate diagram for each function,

(i) the graph of $y=|f(x)|$ and;

(ii) $y=\dfrac{1}{|f(x)|},$ stating clearly the equation of any asymptotes and coordinates of any axial intercepts. [6]

(e) The graph of $y=f(x)$ undergoes two transformations and the image of $f(x)$ after the transformation is $g(x)=f(2x)-1.$

(i) Describe the two transformations in the correct order. [3]

(ii) Write down the expression of $g(x)$ in the form $\dfrac{2x+b}{cx+d},$ where b, c and d are constants. [3]

11. [Maximum Mark: 19]

Let $\omega=\cos\left(\dfrac{2\pi}{7}\right)+i\sin\left(\dfrac{2\pi}{7}\right).$

(a) Verify that ω is a root of the equation $z^7-1=0, z\in\mathbb{C}.$ [2]

(b) Show that $1+\omega+\omega^2+\omega^3+\omega^4+\omega^5+\omega^6=0.$ [3]

(c) Given that $1,\ \omega,\ \omega^2,\ \dots,\ \omega^6$ are the seven roots of the equation $z^7=1,$ sketch the position of these roots on an Argand diagram. [3]

Consider the quadratic equation $z^2+bz+c=0,$ where $z\in\mathbb{C}, b, c\in\mathbb{R},$ where the roots of the equation are α and α^*. α^* is the complex conjugate of α.

(d) Write down an expression of b and of c in terms of α and α^*, using sum and product of roots. [3]

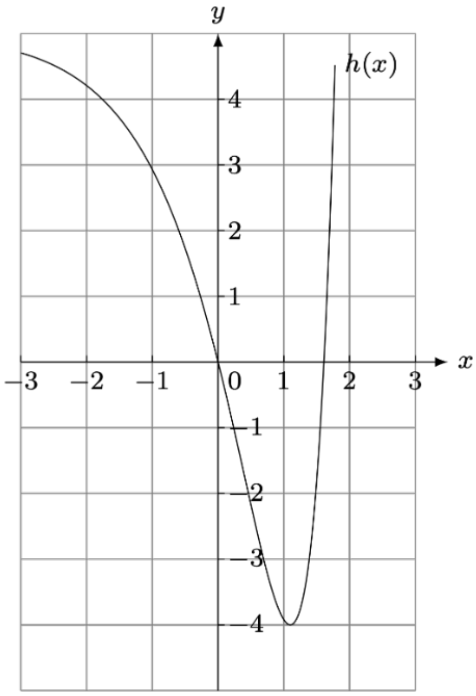
(e) Given that $\alpha=\omega+\omega^2+\omega^4$, show that $\alpha^*=\omega^3+\omega^5+\omega^6.$ [2]

(f) Hence or otherwise, find the value of b and of $c.$ [6]

Do **not** write solutions on this page.

12. [Maximum Mark : 17]

(a) The graph of a function $h(x)=e^{2x}-6e^{x}+5, x\in\mathbb{R},$ is shown in the following diagram.



(i) Find the value of $x-$ intercepts and the coordinates of the minimum point of the graph of $y=h(x)$. [4]

Another function $g(x)$ is defined by restricting the domain of $h(x)$ and $g(x)=e^{2x}-6e^{x}+5,\ \ x\le a.$

(ii) State the maximum value of a such that the inverse function $g^{-1}(x)$ exists. [2]

(iii) For the value of a found in part (ii), find an expression for $g^{-1}(x)$, stating its domain. [5]

(iv) Describe the graphical relationship between the graph of $y=g(x)$ and $y=g^{-1}(x)$. [1]

(b) A hypothesis states that a polynomial of even degree with odd coefficients has no rational root.

Prove by contradiction, that the polynomial equation $x^{2}+bx+c=0$, where b and c are odd, has no rational root. [5]

END OF PAPER