

Topic: Algebraic manipulation, surds and indices (including exponential equations)

Question 1 [2008 EOY, Q1]	marks
Express $\frac{1}{3-2\sqrt{2}}$ in the form $a+b\sqrt{2}$, where a and b are real numbers.	[2]
Question 2 [2008 EOY, Q2]	marks
Evaluate $25^{-1.5} \div 32^{0.4}$ without a calculator, leaving your answer as a fraction in its simplest form.	[2]
Question 3 [2009 EOY, Q2]	marks
Express $\frac{2\sqrt{5}-3}{2\sqrt{5}+4} + \frac{2}{\sqrt{3}}$ in the form $a+b\sqrt{3}+c\sqrt{5}$ where a, b and c are rational numbers.	[3]

Question 4 [2010 EOY, Q1]	marks
(i) Simplify $(\sqrt[3]{3} + \sqrt[3]{2})(\sqrt[3]{9} - \sqrt[3]{6} + \sqrt[3]{4})$.	[2]
(ii) A triangle has an area of $(5\sqrt{6} + 18)$ units² and the length of its base is $(2\sqrt{3} - 3)$ unit. Find its height in the form $(a\sqrt{2} + b\sqrt{3} + c\sqrt{6} + d)$ units, where a, b, c and d are integers.	[3]
Question 5 [2010 EOY, Q2]	marks
Solve for x when $\frac{1}{2}\sqrt{2x-3} = \sqrt{\frac{3x+5}{2}}$.	[3]

Question 6 [2010 EOY, Q3]	marks
(i) Make h the subject of the formula $\frac{g}{2r} = \frac{1}{y} \sqrt{\frac{h^2 - 1}{2 - h^2}}$.	[2]
(ii) Simplify $\frac{x^2 - 3x + 2}{2x^2 - x - 3} \div \frac{2x^2 + 7x + 6}{8x^2 - 18} \times \frac{x^2 + x - 2}{x^2 - 2x + 1}$.	[3]

Question 7 [2011 EOY, Q1]	marks
Without using a calculator, simplify $\frac{6}{1-\sqrt{3}} + \frac{9}{\sqrt{3}}$.	[3]
Question 8 [2011 EOY, Q3]	marks
Show that $(2x + y)(4x^2 - 2xy + y^2) = 8x^3 + y^3$.	[1]

(ii)

Hence simplify $\frac{8x^3 + y^3}{x^2 - xy - 6y^2} \times \left(\frac{x^2 - 3xy}{2x + y} \right) \div \sqrt{\frac{16x^4}{x^2 + 4xy + 4y^2}}$.

[4]

Question 9 [2012 EOY, Q2]

marks

Without using a calculator, simplify $\frac{3}{(2 - \sqrt{5})^2} - \frac{2}{(2 + \sqrt{5})^2}$.

[4]

Question 10 [2012 EOY, Q10 a, b]	marks
(a) Factorise $81a^4 - 27a^2b^2 - 4b^4$ completely	[2]
(b) Given that $2x = 3\sqrt{px^2 + q}$, express x in terms of p and q.	[2]
Question 11 [2013 EOY, Q1 a, b]	marks
(i) Find the product of $(x^{\frac{1}{3}} - y^{\frac{1}{3}})(x^{\frac{1}{3}} + y^{\frac{1}{3}}) \text{ and } (x^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}} + y^{\frac{2}{3}})(x^{\frac{2}{3}} + x^{\frac{1}{3}}y^{\frac{1}{3}} + y^{\frac{2}{3}})$ and simplify the answer.	[2]

(ii) If $y = \frac{\sqrt{x+a} - \sqrt{x-a}}{\sqrt{x+a} + \sqrt{x-a}}$, simplify $y + \frac{1}{y}$.	[3]
Question 12 [2013 EOY, Q6]	marks
(i) Simplify $\frac{a-b}{(b+c)(a+c)} + \frac{b-c}{(a+c)(a+b)} + \frac{c-a}{(a+b)(b+c)}$.	[3]
(ii) Simplify $\frac{2x^2 - 6x + 4}{x^2 - 6x + 5} \times \frac{x^2 - x - 20}{3x^2 + 6x - 24}$.	[3]

Question 13 [2014 EOY, Q1]	marks
(i) Factorise completely $(x-3)^3 + (x-1)^3$.	[3]
(ii) Simplify $\frac{x - \frac{x-1}{2}}{2 - \frac{x-1}{x}}$.	[3]

Question 14 [2014 EOY, Q2]	marks
The area of a rectangle $ABCD$ is given by $(1 + 3\sqrt{2}) \text{ m}^2$, and the length of side AB is $(2 + \sqrt{8}) \text{ m}$. Find, without using a calculator, the length of side BC in the form of $(a + b\sqrt{2}) \text{ m}$, where a and b are real numbers.	[4]
Question 15 [2015 EOY, Q3]	marks
(i) Make p the subject of the formula if $s = \frac{4(2p^2 + 1)}{u^2}$.	[3]

(ii)

Simplify into a single fraction $\frac{x^3 + 8y^3}{x^2 - 4y^2} - \frac{16xy^2 - 4x^3}{(4y - 2x)^2}$.

[4]

Question 16 [2015 EOY, Q4]

marks

(i)

Given that $x^2 - 4x = y^2 - 4y$ and that $x \neq y$, find the value of $3\sqrt{x+y}$.

[3]

(ii)

Using a suitable substitution, or otherwise, solve for x if

$$2(27^x) = 45(3^x) + \frac{243}{3^x}.$$

[4]