Bukit Batok Secondary School

GCE ‘O’ Level Preliminary Examination 2021

Secondary 4 Express

ADDITIONAL MATHEMATICS **(Marking Scheme)**

4049/02

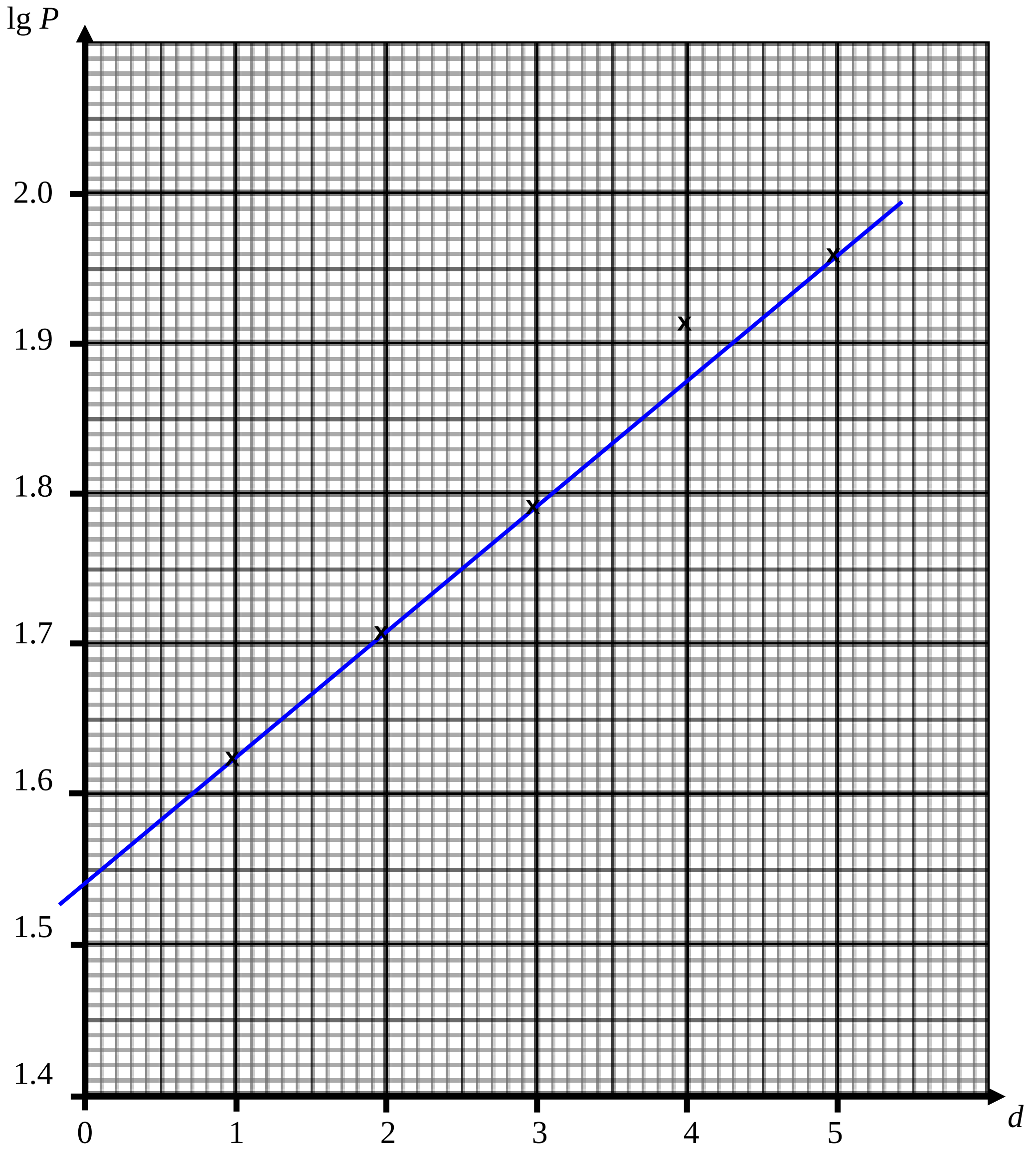
1	**(i)**	LHS = $\frac{1-coscos\,\theta\;+sinsin\,\theta}{1+coscos\,\theta\;+sinsin\,\theta}$ = $\frac{1-\left(1-2\frac{\theta}{2}\right)+\frac{\theta\theta}{22}}{1+\left(2\frac{\theta}{2}-1\right)+\frac{\theta\theta}{22}}$ = $\frac{2\frac{\theta}{2}+\frac{\theta\theta}{22}}{2\frac{\theta}{2}+\frac{\theta\theta}{22}}$ = $\frac{\frac{\theta}{2}\left(\frac{\theta}{2}+\frac{\theta}{2}\right)}{\frac{\theta}{2}\left(\frac{\theta}{2}+\frac{\theta}{2}\right)}$ = $tan\,tan\,\frac{\theta}{2}$ = RHS	M1 for converting at least one cos θ correctly M1 for converting sin θ correctly M1 for factorizing A1
	(ii)	1 + sin θ = 2 cos θ ------ (1) $\frac{1-coscos\,\theta\;+sinsin\,\theta}{1+coscos\,\theta\;+sinsin\,\theta}=tan\,tan\,\frac{\theta}{2}$ --------- (2) Sub (1) into (2): $\frac{2coscos\,\theta\;-coscos\,\theta}{\theta\;+coscos\,\theta}=tan\,tan\,\frac{\theta}{2}$ $\frac{1}{3}=tan\,tan\,\frac{\theta}{2}$ (since cos $\theta \neq$ 0) Let β be a basic angle. tan $\beta = \frac{1}{3}$ $\beta \approx 18.435^{o}$ $\frac{\theta}{2} \approx 18.435^{o}$ $\theta \approx 36.9^{o}$	M1 for substitution M1 for simplifying A1 [7]
2	**(i)**	$x^2 + y^2 - 6x + 8y = 0$ Centre of circle = (3, – 4) Radius of circle = $\sqrt{3^2+(-4)^2}$ = 5 units Diameter of circle = 10 units	B1 for centre M1 for radius A1 for diameter
	(ii)	tangent at R: $3x + 4y = 18$ $y = -\frac{3}{4}x + \frac{9}{2}$ Gradient of normal = $\frac{4}{3}$ Normal will pass through centre of circle (3, – 4).	M1 for making y the subject M1 for gradient of normal

		Equation of normal: $-4 = \frac{4}{3}(3) + c$ $c = -8$ Equation of normal: $y = \frac{4}{3}x - 8$	A1
	(iii)	Since the origin (0, 0) satisfies the equation, $x^2 + y^2 - 6x + 8y = 0$, Thus the origin lies on the circle.	B1 [7]
3	**(i)**	$2x^3 + cx^2 + dx - 6 = (x^2 - 2x - 3)\ \mathrm{Q}(x) + 23x + 21$ $2x^3 + cx^2 + dx - 6 = (x - 3)(x + 1)\ \mathrm{Q}(x) + 23x + 21$ When $x = 3$, $2(3)^3 + c(3)^2 + d(3) - 6 = 23(3) + 21$ $9c + 3d = 42$ $3c + d = 14$ ----- (1) When $x = -1$, $2(-1)^3 + c(-1)^2 + d(-1) - 6 = 23(-1) + 21$ $c - d = 6$ ----- (2) (1)+ (2): $4c = 20$ $c = 5$ Sub $c = 5$ into (1): $d = 14 - 15$ $= -1$	M1 for factorising $(x^2 - 2x - 3)$ correctly M1 for sub $x = 3$ or sub $x = -1$ correctly A1 for correct c A1 for correct d
	(ii)	$\mathrm{f}(x) = 2x^3 + 5x^2 - x - 6$ $\mathrm{f}(1) = 2(1)^3 + 5(1)^2 - (1) - 6$ $= 0$ $(x - 1)$ is a factor of $\mathrm{f}(x)$. $\mathrm{f}(x) = (x - 1)(2x^2 + 7x + 6)$ $= (x - 1)(2x + 3)(x + 2)$	M1 for finding one factor M1 for finding another factor by long division or comparing coefficients. A1 [7]
4	**(a)**	$(3n) = \frac{(3n)}{\sqrt{27}}$ $= \frac{3 + n}{\frac{3}{2}3}$ $= \frac{(1+m)}{3}$	M1 for change of base M1 for either numerator or denominator correct A1
	(b)	$\sqrt{x+3} = 1 - \frac{1}{2}(1 - 2x)$ $\frac{1}{2}(x + 3) + \frac{1}{2}(1 - 2x) = 1$ $(x + 3)(1 - 2x) = 2$ $(x + 3)(1 - 2x) = 2^2$ $-2x^2 - 5x - 1 = 0$ $2x^2 + 5x + 1 = 0$ $x = \frac{-5 \pm \sqrt{(5)^2 - 4(2)(1)}}{2(2)}$ $= \frac{-5 \pm \sqrt{17}}{4}$ ≈ -0.219 or -2.28	M1 combining log M1 for removing log M1 for quadratic eq. M1 for formula A1 [8]
5	**(i)**	Height of triangle = $\sqrt{(13x)^2 - (\frac{10x}{2})^2}$	M1

		$= 12x$ cm Area of triangle $= \frac{1}{2}(10x)(12x)$ $= 60x^2$ cm^2 Volume of box, $3840 = 60x^2 h$ $h = \frac{3840}{60x^2}$ $= \frac{64}{x^2}$ Surface area, $A = (13x + 13x + 10x)h + 60x^2$ $= (36x)\left(\frac{64}{x^2}\right) + 60x^2$ $= \frac{2304}{x} + 60x^2$	M1 M1 A1
	(ii)	$A = \frac{2304}{x} + 60x^2$ $\frac{dA}{dx} = -\frac{2304}{x^2} + 120x$ $-\frac{2304}{x^2} + 120x = 0$ $x^3 = \frac{2304}{120}$ $x = \sqrt[3]{\frac{96}{5}}$ $\frac{d^2A}{dx^2} = \frac{4608}{x^3} + 120$ When $x^3 = \frac{2304}{120}$, $\frac{d^2A}{dx^2} = \frac{4608}{\frac{2304}{120}} + 120$ $= 360 > 0$ Minimum internal surface area, $A = \frac{2304}{\sqrt[3]{\frac{96}{5}}} + 60\left(\sqrt[3]{\frac{96}{5}}\right)^2$ ≈ 1290 cm^2	M1 A1 M1 M1 A1 [9]

6 (i)

d	1	2	3	4	5
lg P	1.623	1.708	1.792	1.914	1.959



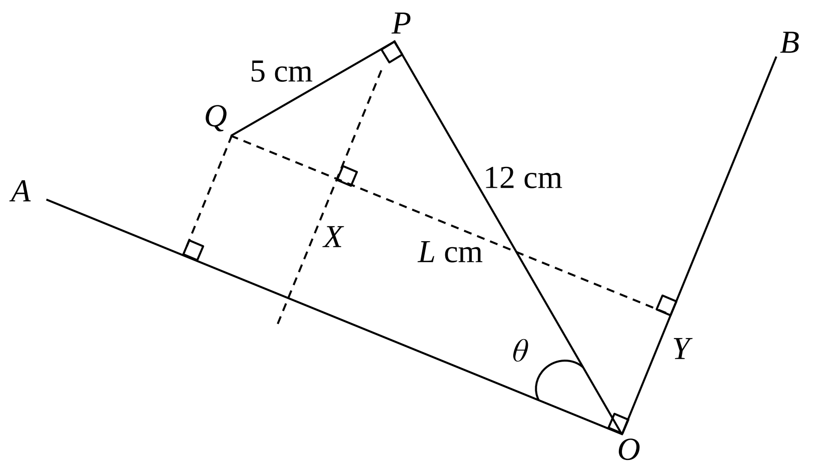
C1 for all points plotted correctly, C1 for straight line.

(ii)

lg P = $2d$ lg b + lg a	
Gradient = $\frac{1.959-1.623}{5-1}$	
2lg b = 0.084 (acceptable range: 0.07 to 0.1)	M1
$b = 10^{0.042}$	
≈ 1.10	A1
Vertical intercept = 1.54	M1 for vertical intercept (1.53 to 1.55)
lg a = 1.54	
$a = 10^{1.54}$	A1

		$\approx$ 34.7 (acceptable range: 33.9 to 35.5)	

	(iii)	Incorrect P = 82 When d = 4, lg P = 1.875 $P \approx 75.0$ (acceptable range: 73.2 to 76.7)	B1 B1 [8]
7	**(i)**	$\frac{x^3+4x^2+3}{(x^2+3)(x-1)} = \frac{x^3+4x^2+3}{x^3-x^2+3x-3}$ $= 1 + \frac{5x^2-3x+6}{(x^2+3)(x-1)}$ Let $\frac{5x^2-3x+6}{(x^2+3)(x-1)} = \frac{Ax+B}{(x^2+3)} + \frac{C}{(x-1)}$ $5x^2 - 3x + 6 = (Ax+B)(x-1) + C(x^2+3)$ When $x = 1$, $5 - 3 + 6 = 0 + 4C$ $C = 2$ When $x = 0$, $6 = -B + 3(2)$ $B = 0$ When $x = 2$, $5(2)^2 - 3(2) + 6 = 2A + 2(2^2+3)$ $A = 3$ $\frac{x^3+4x^2+3}{(x^2+3)(x-1)} = 1 + \frac{3x}{(x^2+3)} + \frac{2}{(x-1)}$	M1 for expansion M1 for division M1 M1 for one correct A, B or C Another M1 for one correct A, B or C A1
	(ii)	$\frac{d}{dx}[ln\ ln\ (x^2+3)] = \frac{2x}{x^2+3}$	B1
	(iii)	$\int \frac{x^3+4x^2+3}{(x^2+3)(x-1)}\,dx = \int \left[1 + \frac{3x}{(x^2+3)} + \frac{2}{(x-1)}\right] dx$ $= x + \frac{3}{2} ln\ ln\ (x^2+3) + 2\ ln\ ln\ (x-1) + c$, where c is an arbitrary constant.	M1 A1 for $\frac{3}{2} ln\ ln\ (x^2+3)$ A1 for $2\ ln\ ln\ (x-1)$ A1 for all final correct answer [11]
8	**(i)**	$\frac{d^2y}{dx^2} = cos\ cos\ x\ \ cos\ cos\ 2x - sin\ sin\ x\ \ sin\ sin\ 2x - 4\ sin\ sin\ x\ \ cos\ cos\ x$ $= \cos(2x + x) - 2\sin 2x$ $= \cos 3x - 2\sin 2x$	M1 for sin $2x$ or cos $3x$ A1
	(ii)	$\frac{dy}{dx} = \int (cos\ 3x - 2\ sin\ 2x)\ dx$ $= \frac{1}{3}\sin 3x + \cos 2x + c$, where c is an arbitrary constant $y = \int \left(\frac{1}{3} sin\ 3x + cos\ 2x + c\right) dx$ $= -\frac{1}{9}\cos 3x + \frac{1}{2}\sin 2x + cx + d$, where c and d are arbitrary constants	M1 for integration M1 for integration of c M1 for all integration correct

		At origin, $0 = -\frac{1}{9}\cos 0 + \frac{1}{2}\sin 0 + c(0) + d$, $d = \frac{1}{9}$ At $\left(-\frac{\pi}{2}, -2\right)$, $-2 = -\frac{1}{9}\cos\left(-\frac{3\pi}{2}\right) + \frac{1}{2}\sin\left(-\frac{2\pi}{2}\right) + c\left(-\frac{\pi}{2}\right) + \frac{1}{9}$ $-2 = 0 + 0 + c\left(-\frac{\pi}{2}\right) + \frac{1}{9}$ $c = \frac{38}{9\pi}$ Equation of curve is $y = -\frac{1}{9}\cos 3x + \frac{1}{2}\sin 2x + \frac{38}{9\pi}x + \frac{1}{9}$	M1 A1 M1 A1 A1 o.e. but must be exact [10]
9			
	(i)	Points X and Y are added to the diagram as shown above. Angle $OPX = 90^o - \theta$ (angle sum of triangle) Angle $QPX = \theta$ $\sin\theta = \frac{QX}{5}$ $QX = 5\sin\theta$ $\cos\theta = \frac{XY}{12}$ $XY = 12\cos\theta$ $L = QX + XY$ $= 5\sin\theta + 12\cos\theta$	M1 for either QX or XY o.e. A1
	(ii)	$L = 5\sin\theta + 12\cos\theta$ $= 12\cos\theta + 5\sin\theta$ $R = \sqrt{5^2 + 12^2}$ $= 13$ $\tan\alpha = \frac{5}{12}$ □ $\alpha = 22.620^o$ $L = 13\cos(\theta - 22.6^o)$	M1 M1 for α A1
	(iii)	Max $L = 13$ $\cos(\theta - 22.6^o) = 1$	B1 M1

		$\theta - 22.6^\circ = 0$ Corresponding $\theta = 22.6^\circ$	A1
	(iv)	$13\cos(\theta - 22.620^\circ) = 8$ $\cos(\theta - 22.620^\circ) = \frac{8}{13}$ Let β be a basic angle. $\cos\beta = \frac{8}{13}$ $\beta \approx 52.020^\circ$ $\theta - 22.620^\circ = 52.020^\circ$ $\theta \approx 74.6^\circ$	M1 M1 A1 [11]
10	**(i)**	At x- axis, $0 = 8 - \frac{32}{x^2}$ $x^2 = 4$ $x = \pm 2$ $U = (-2, 0)$ Gradient of $UV = \frac{-2-0}{0-(-2)}$ $= -1$	M1 A1
	(ii)	$y = 8 - \frac{32}{x^2}$ $\frac{dy}{dx} = \frac{64}{x^3}$ At T, Gradient $= -1$ $\frac{64}{x^3} = -1$ $x^3 = -64$ $x = -4$ $y = 8 - \frac{32}{(-4)^2}$ $= 6$ $T = (-4, 6)$	M1 for differentiation M1 M1 for correct x A1
	(iii)	Equation of TS: $6 = -(-4) + c$ $c = 2$ $S = (0, 2)$ Area of shaded region $= \frac{1}{2}(6+2)(4) + \frac{1}{2}(2)(2) - \int_{-4}^{-2}\left(8 - \frac{32}{x^2}\right)dx$ $= 18 - \left[8x + \frac{32}{x}\right]_{-4}^{-2}$ $= 18 - \left[8(-2) + \frac{32}{(-2)} - 8(-4) - \frac{32}{(-4)}\right]$ $= 18 - 8$ $= 10$ units^2	M1 for y-coordinate of S M1 for area of trapezium, M1 for area of triangle, M1 for area under graph M1 for integration A1 [12]

– End of Marking Scheme –