Student's Copy

Unit 15: Currents

Learning Outcomes

Candidates should be able to:

(a) show an understanding that electric current is the rate of flow of charge and solve problems using the equation $I = Q/t$

(b) derive and use the equation $I = nAvq$ for a current-carrying conductor, where n is the number density of charge carriers and v is the drift velocity

(c) recall and solve problems using the equation for potential difference in terms if electrical work done per unit charge, $V = \frac{W}{Q}$

(d) recall and solve problems using the equations for electrical power $P = VI$, $P = I^2R$ and $P = \frac{V^2}{R}$

(e) distinguish between electromotive force (e.m.f.) and potential difference (p.d.) using energy considerations

(f) show an understanding of and use the terms period, frequency, peak value and root-mean-square (r.m.s.) value as applied to an alternating current or voltage

(g) represent a sinusoidal alternating current or voltage by an equation of the form $x = x_o \sin \omega t$

(h) deduce that the mean power in a resistive load is half the maximum (peak) power for a sinusoidal alternating current

(i) distinguish between r.m.s. and peak values, and recall and use $I_{rms} = \frac{I_0}{\sqrt{2}}$ and $V_{rms} = \frac{V_o}{\sqrt{2}}$ for the sinusoidal case

(j) explain the use of a single diode for the half-wave rectification of an alternating current

1 Charge and electric current

LO (a)(b)

1.1 Charge

The property of matter that causes the electrical force is **charge**. All bodies are either positively charged, negatively charged, or neutral (zero charge). Bodies with the same charge repel each other and bodies with opposite charge attract. Charge cannot be created or destroyed so therefore charge is conserved. The law of conservation of charge states that in a closed system the amount of charge is constant.

Charge is a scalar quantity. The SI unit of charge is the coulomb (C).

The charge of electron is $e = -1.60 \times 10^{-19}$ C. Any charge can only exist as an integral multiple of this value.

i.e. $$\boxed{Q = Ne}$$

Examples of charged particles or charged carriers in conductors are

- negative electrons in copper wire
- positive and negative ions in an electrolyte
- positive ions and negative electrons in a discharge tube
- negative electrons and positive holes in a semiconductor

1.2 Electric current

Electric current I is the rate of flow of charge.

i.e. $$\boxed{I = \frac{Q}{t}}$$ The SI unit of current is ampere (A).

Hence, in a circuit carrying a **constant current** I, the **charge** Q which flows in time t, is given by

charge = current x time

i.e. $$\boxed{Q = It}$$

The **coulomb** is defined as the charge which passes a point in a circuit when a constant current of one ampere flows for one second.

If N electrons, each of charge e, pass through a point in a circuit in time t, the total charge is given by $Q = Ne$ and the current

$$I = \frac{Q}{t} = \frac{Ne}{t}.$$

Note: If the current is not constant, but is changing with time, then the instantaneous current is

$$I = \frac{dQ}{dt}.$$

And the charge which flows can be found from the area under the current-time graph according to the equation

$$Q = \int I dt.$$

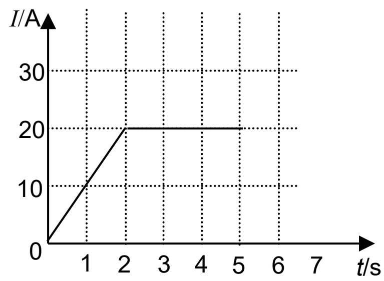
Example 1:
The current in a wire is 200 mA. Calculate
(a) the charge which passes a point in the wire in 5 minutes,
(b) the number of electrons needed to carry this charge

Solution:

(a) $Q = It$
$= 200 \times 10^{-3} (5 \times 60)$
$= 60$ C

(b) $Q = Ne$
$60 = N (1.60 \times 10^{-19})$
$N = 3.75 \times 10^{20}$

Example 2:
A car battery is used to supply varying current for 5 s. Calculate the total charge delivered from the battery if the variation in current supplied is given by the graph as shown on the right.



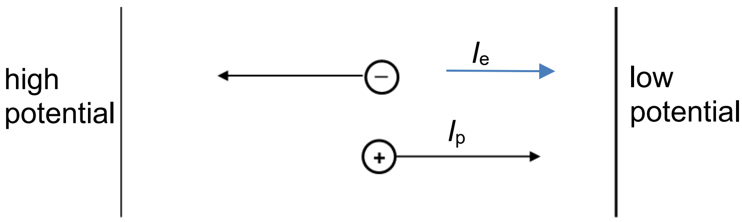
Solution:

Total charge, Q = area under I-t graph

$Q = \frac{1}{2}(3 + 5) \times 20 = 80$ C

Example 3:
A high p.d is applied between the electrodes of a hydrogen discharge tube so that the gas is ionised. Electrons then move towards the positive electrode and the protons towards the negative electrode. In one second, 5.0×10^{18} electrons and 5.0×10^{18} protons pass a cross-section of the tube. Calculate the current in the discharge tube.

Solution:
Oppositely-charged particles moving in opposite directions under the influence of an electric field. By convention, current has the same direction as the flow of positive charge.



The positive protons moving to the right results in a current I_p to the right.
The negative electrons moving to the left results in a current I_e to the right.
So the total current is

$$I = \frac{Q}{t} = \frac{(N_e + N_p)e}{t}$$

$$= \frac{(5.0 \times 10^{18} + 5.0 \times 10^{18})1.6 \times 10^{-19}}{1} = 1.6 \text{ A}$$

1.3 Drift velocity and the derivation of $I = nAvq$ *LO (b)*

A conducting wire is made of metal, such as copper or silver, as shown in Fig. 1.1.

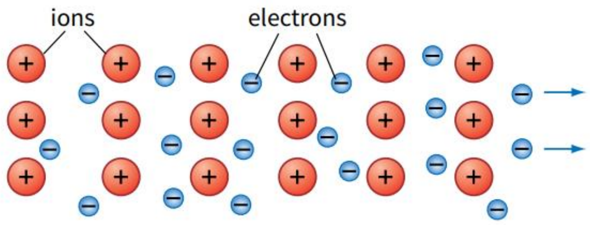


Fig. 1.1

In a typical metal, one electron from each atom breaks free to become a free or conduction electron. The atom remains as a positively charged ion. Since there are equal numbers of free electrons (negative) and ions (positive), the metal has no overall charge , i.e. it is neutral.

When a battery is connected across the ends of a metal wire in Fig. 1.2, an electric field is set up across the wire.

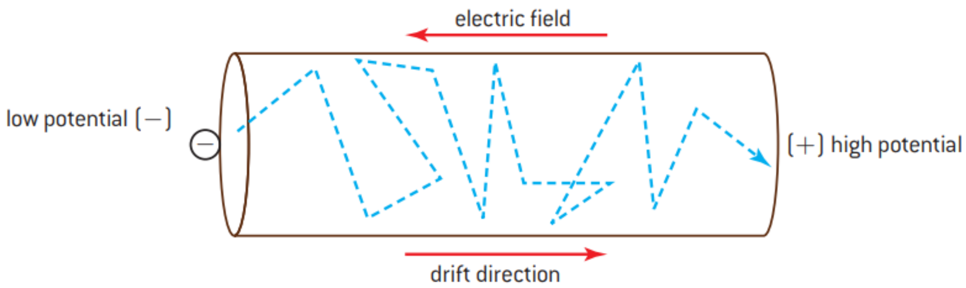


Fig. 1.2

Under the influence of this external electric field, the 'free' electrons in the wire experience a force, and accelerate from a region of high potential (i.e. +ve terminal) to a region of low potential (-ve terminal). As the electrons move, it collide with the lattice ions and are scattered repeatedly. The resultant motion is irregular, but it has a slow drift towards the positive terminal with an average velocity called the **drift velocity**, *v*.

Consider a current I flowing through a section of cylindrical wire of length l and cross-sectional area *A* as shown in Fig. 1.3. This current is caused by free electrons drifting in the opposite direction to the current. (Note that conventionally current is the flow of positive charges)

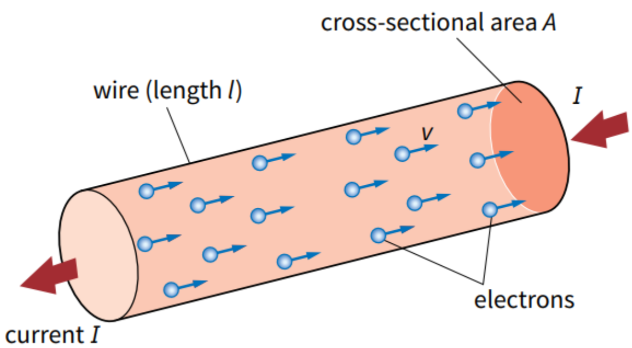


Fig. 1.3

If there are *n* electrons per unit volume in the wire with a drift velocity *v*, then total charge passing through the wire is given by

$$Q = Nq$$

where N is total number of electrons and *q* is the charge of each electron.

Substitute N = $n \times (Al)$ where Al is the volume of the wire,

Charge $\quad Q = n \times (Al)\, q$

Current $\quad I = Q/t$

$I = n \times (Al)\, q/t$

Sub drift speed $v = l/t$

$$\boxed{I = nAvq}$$

Note:

- Number of charge carriers per unit volume is called number density n.
- For metals, the number density, n of charge carriers is approximately 10^{29} electrons per m^3 and drift velocity, v is typically of the order of around 10^{-4} m s^{-1}.
- When a domestic lighting circuit is switched on, the light comes on almost instantly. This is because the electric field is established almost immediately. So all electrons in the wire and filament bulb **move simultaneously** under the electric field.

Example 4:
The number of free electrons per cubic metre in a copper wire is approximately 8×10^{28}. A typical diameter of wire is 1.0 mm. If the current is 1.0 A, calculate the drift speed of the electrons along the wire.

Solution:

$$I = nAvq = n\pi\left(\frac{d}{2}\right)^2 vq$$

$$\Rightarrow 1 = 8 \times 10^{28} \times \pi\left(\frac{1.0 \times 10^{-3}}{2}\right)^2 v\left(1.60 \times 10^{-19}\right)$$

$$\Rightarrow v = 9.9 \times 10^{-5} \text{ m s}^{-1}$$

2 Electromotive force and potential difference

LO (c)(e)

2.1 Electromotive force

Fig. 2.1 below shows a filament bulb connected to a cell and a current flows as a result of electrons moving in the circuit. Inside the cell, chemical energy stored is transferred to electrical energy carried by electrons in the wires. Electrical energy carried by electrons in the wires is transferred to light energy by the bulb, as well as energy wasted in heating.

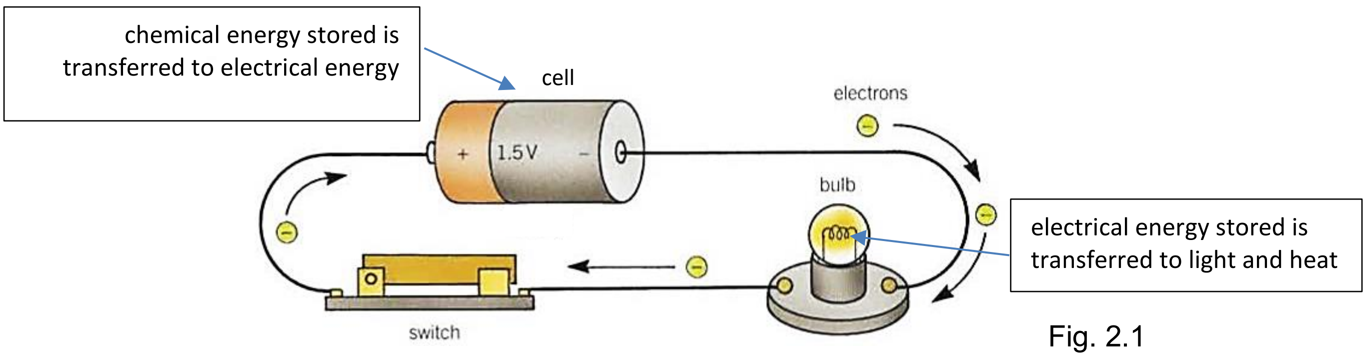


Fig. 2.1

In order for an electric current to flow in a circuit, there must be an energy source – this is often a chemical cell or battery. When no current is flowing through the source, we have a situation in which there is the maximum possible potential difference across the source. This is called the electromotive force or e.m.f. of the supply.

The electromotive force of a source can be defined as the energy converted from other forms into electrical energy per unit charge when charge flows round a complete circuit.

The above definition for e.m.f. E can be expressed by the following equation:

$$\boxed{E = \frac{W}{Q}}$$

where
W denotes the energy converted into electrical energy from other forms of energy, and
Q denotes the amount of charge flowing round a complete circuit.
The SI unit of electromotive force is the volt (abbreviated V). Another unit is J C^{-1}.

Electromotive force of a source is also **defined as the electrical power supplied by the source per unit current passing through it**.

i.e. $$\boxed{E = \frac{P}{I}}$$

Note that electromotive force (e.m.f.) is not a force, it is associated with energy transferred per unit charge!

2.2 Potential difference

The potential difference across the bulb in Fig. 2.1 is a measure of the electrical energy transferred, or the work done, by each coulomb of charge as it moves across the bulb.

Potential difference (p.d.) between two points in a circuit is defined as the amount of electrical energy converted to other forms of energy per unit charge passing across the two points.

i.e. $$\boxed{V = \frac{W}{Q}}$$

SI unit is also J C^{-1} or volt (V). Since p.d. is measured in volts, sometimes it is called the voltage.

One volt is defined as the p.d. across a conductor or between two points in a circuit if one joule of electrical energy is converted to other forms of energy per coulomb of charge passing through it or across the two points (i.e. 1 V = 1 J C^{-1}).

We can turn this equation round to get an expression for the electrical energy transferred when a charge Q is moved through a potential difference V.

Or

Energy transferred (work done) = potential difference x charge
$$W = VQ$$

Potential difference across an electrical device in a circuit is also the **amount of electrical power supplied to the device per unit current** passing through it.

i.e. $$\boxed{V = \frac{P}{I}}$$

<u>Question to ponder</u>: What is the difference between e.m.f. and p.d. ?

Example 5:
A heater is rated at 3.0 x 10^3 W and is switched on for 2.0 x 10^3 s. During this time a charge of 2.5 x 10^4 C is supplied to the heater. Calculate
(i) the heat energy supplied by the heater, and
(ii) the p.d. across the heater.

Solution:
(i) *Heat supplied = Pt*
= 3.0 x 10^3 (2.0 x 10^3)
= 6.0 x 10^6 J

(ii) $V = \frac{W}{Q} = \frac{3.0\times10^3\left(2.0\times10^3\right)}{2.5\times10^4} = 240\text{ V}$

3 Electrical energy and power

LO (d)

On a microscopic level, as the electrons flow through the metal they collide with the vibrating positive ion cores of the metal structure. The collisions between the electrons and the positive ion cores transfer electrical energy from the electrons to the structure of the metal, causing the metal ion cores to vibrate more, thus heating up the wire. The heating effect is called **joule heating**. The opposition of a conductor to the flow of electric current through it is called **resistance**.

The resistance of a conductor is defined as the ratio of the potential difference to the current.

$$\text{resistance} = \frac{\text{potential difference}}{\text{current}} \quad \text{or} \quad R = \frac{V}{I}$$

The unit of resistance is the ohm (symbol Ω); named after Georg Simon Ohm, a German physicist.

Alternative forms of the equation are: $V = IR$ and $I = V/R$

If a p.d. V is applied to a conductor of resistance R and a current I flows through it, the power dissipated in the conductor is

$$\boxed{P = VI}$$

SI unit of power is watt (W).

Using $V = IR$, $P = VI = (IR)I = I^2R$ or $P = V\left(\frac{V}{R}\right) = \frac{V^2}{R}$

i.e. $\boxed{P = I^2R}$ or $\boxed{P = \frac{V^2}{R}}$

Energy dissipated as heat is $E = Pt = (VI)t = (I^2R)t = \left(\frac{V^2}{R}\right)t$.

Energy values are sometimes expressed in kilowatt-hour (kWh), the energy supplied by 1 kilowatt of power for an hour. 1 kWh = 1000 W (3600 s) = 3.6 x 10^6 J = 3.6 MJ.

Example 6:
An iron is marked '240 V, 800 W'.
(i) Calculate resistance of the iron.
(ii) When the supply voltage is only 220 V, what power will be supplied to it?

Solution:
(i) $P = \frac{V^2}{R} \Rightarrow 800 = \frac{240^2}{R} \Rightarrow R = 72\ \Omega$

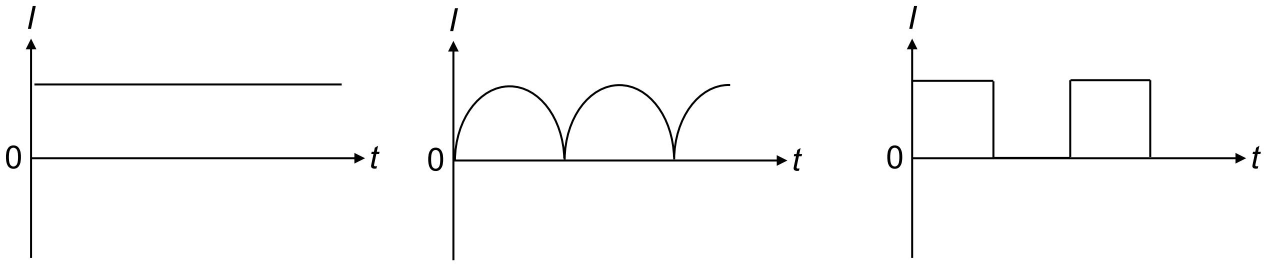
(ii) $P = \frac{V^2}{R} = \frac{220^2}{72} = 672\text{ W}$

4 Alternating Current (A.C.)

LO (f)

In previous section, we have dealt with circuits in which there is a current in one direction only. This sort of current is a direct current (d.c.).

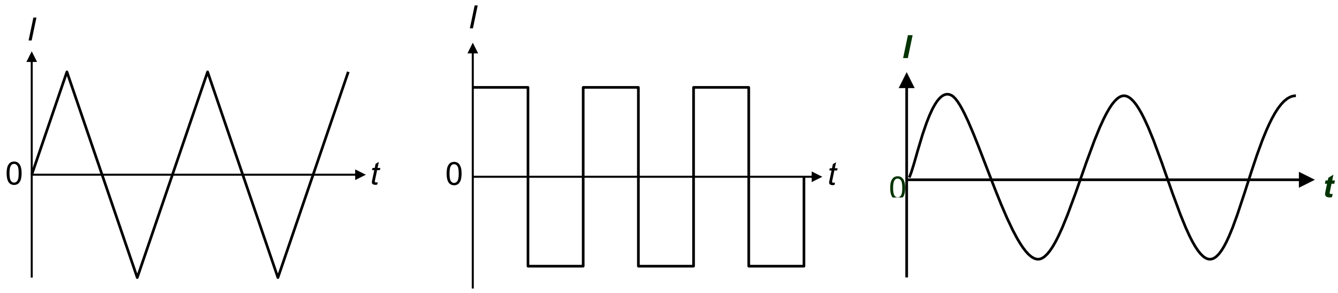
Typical d.c. waveforms look like those in Fig. 4.1.



(a) Steady (b) Pulsating D.C. (c) Pulsating D.C.

Fig. 4.1

However, the domestic electricity supply produced by generators at a power station, is one which uses alternating current (a.c.). An alternative current or voltage reverses its direction regularly, as shown in Fig. 4.2.



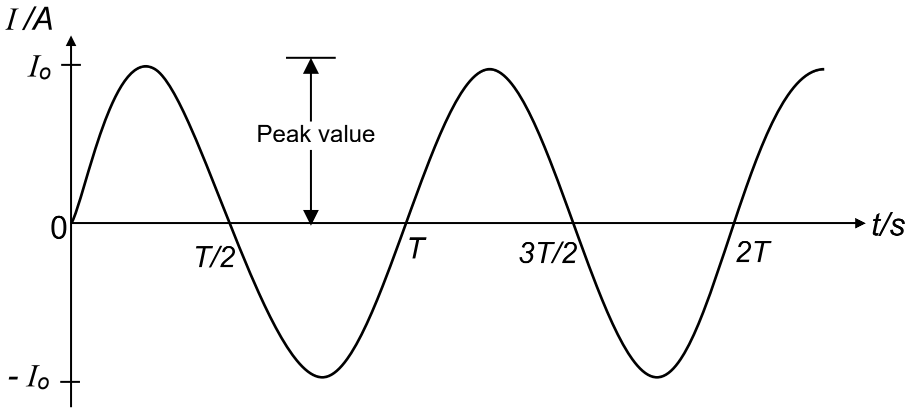
(a) triangular or saw-tooth (b) square (c) sinusoidal

Fig. 4.2

- Saw-tooth waveforms are often used in TV sets and monitors.
- Square waves are often used in digital computer circuits.

4.1 TERMINOLOGY *LO (f) (g)*

A typical alternating current or voltage has a sinusoidal waveform as seen in the diagram below. It varies periodically with time in magnitude and direction.



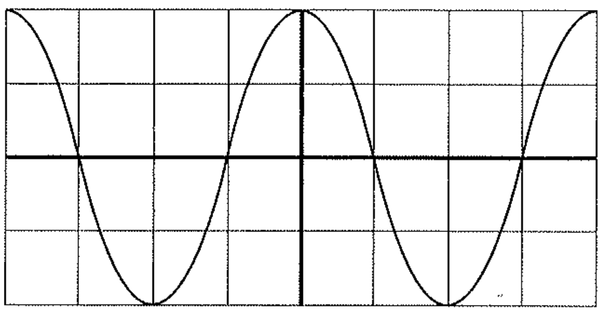
A sinusoidal alternating current I and voltage V with respect to time t are often expressed mathematically as

$$I = I_o \sin \omega t \quad \text{and} \quad V = V_o \sin \omega t \qquad \text{where } \omega = 2\pi f$$

The characteristics of an a.c. are described in terms of:

period *T*	Time taken for the a.c. to complete one cycle.
frequency *f*	Number of complete cycles of the a.c. per unit time.
peak value or amplitude I_0	Maximum value of the a.c. in either direction.
angular frequency ω	2π x frequency

Example 7:
A CRO screen with a grid of 1 cm x 1 cm square displays an a.c. waveform. The settings of the oscilloscope are: Y sensitivity = 0.50 V cm^{-1}, time-base = 5.0 ms cm^{-1}.



What is the expression of the voltage V of this waveform with respect to time t?

Solution:

Period $T = 4.0 \text{ cm} \times 5.0 \text{ ms cm}^{-1} = 20 \text{ ms}$,
Frequency $f = 1/T$ = 1/20 ms = 50 Hz
Peak voltage V_o = 2.0 cm x 0.50 V cm^{-1} = 1.0 V
Equation of waveform: $V = V_o \cos \omega t = 1.0 \cos 2\pi(50)\, t = 1.0 \cos 100\pi t$

4.2 MEASUREMENT OF A.C. *LO (i)*

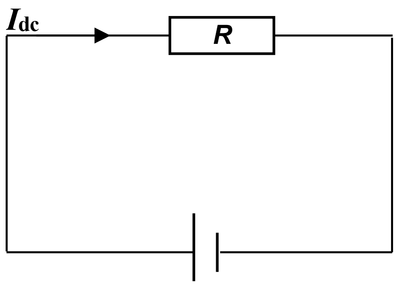
If an alternating current of frequency 50 Hz passes through a moving-coil ammeter, the pointer does not move because it cannot respond to the rapid changes in the direction of the current. In a more sensitive ammeter, the pointer may vibrate with some amplitude but centred on zero.

How then, can we measure an alternating current?

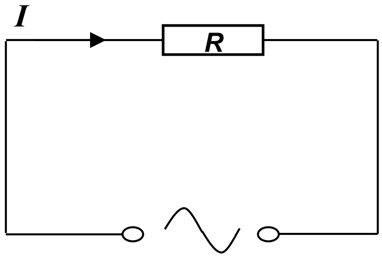
Well, we can make use of the *joule heating effect* of a current! When an alternating current flows through a resistor, there is a joule heating effect just as there would be when a direct current flows. The rate at which energy is delivered to a resistor R is the power $P = I^2R$, where I is the instantaneous current in the resistor. Because power is proportional to (current)^2, there is always power developed in a resistor whether the current is moving in the positive or in the negative direction.

4.3 Root-mean-square current and voltage

Consider the two circuits below. In one circuit, a resistor R is connected across a d.c. source whereas in the other circuit, an identical resistor is connected across an a.c. source. The current in the d.c. circuit is I_{dc} and the current in the a.c. circuit is the alternating current $I = I_o \sin \omega t$.



d.c.



a.c.

Left circuit	Right circuit
Power dissipated in R, $P_{dc} = I_{dc}^2 R$ Average power = $\langle P \rangle_{dc} = I_{dc}^2 R$	Instantaneous power dissipated in R, $P_{ac} = I^2 R$ Average power = $\langle P \rangle_{ac} = \langle I^2 R \rangle = \langle I^2 \rangle R$

Suppose the average power dissipated across the resistors in both circuits are equal,

That is $\quad \langle P \rangle_{dc} = \langle P \rangle_{ac}$

then $\quad I_{dc}^2 R = \langle I^2 \rangle R$

Since R is the same, $\quad I_{dc}^2 = \langle I^2 \rangle$

$$\Rightarrow \quad I_{dc} = \sqrt{\langle I^2 \rangle} = I_{rms}$$

Thus, the steady current I_{dc} is equal to the square root of the mean of the square of the instantaneous a.c. current, or simply the **root-mean-square current** I_{rms} of the a.c..

The root mean square value of an a.c. is defined as the value of the steady direct current that would dissipate heat at the same rate as the a.c. in a given resistor.

4.4 Steps to determine root-mean-square value of an a.c.

Step 1	Square the a.c., i.e. I^2, and plot the I^2-t graph.
Step 2	Determine the mean of I^2, i.e. $< I^2>$, using (a) a graphical method or (b) an integration method (a) Graphical method: Determine the **area** under the I^2- t graph for the duration of **one period** ***T*** and then **divide by the period**, i.e. $< I^2 > = \dfrac{Area\ under\ \ I^2 - t\ \ graph\ \ within\ \ one\ \ period}{T}$ (b) Integration method: $< I^2 > = \dfrac{\int_0^T I^2 dt}{T}$ This second method is not used as calculus is not required in the current syllabus.
Step 3	Determine the square root of $<I^2>$, i.e. I_{rms}. $I_{rms} = \sqrt{< I^2 >} = \sqrt{\dfrac{Area\ under\ \ I^2 - t\ \ graph\ \ within\ \ one\ \ period}{T}}$

<u>Note:</u> The above steps can be applied to determine r.m.s current or voltage of any a.c..

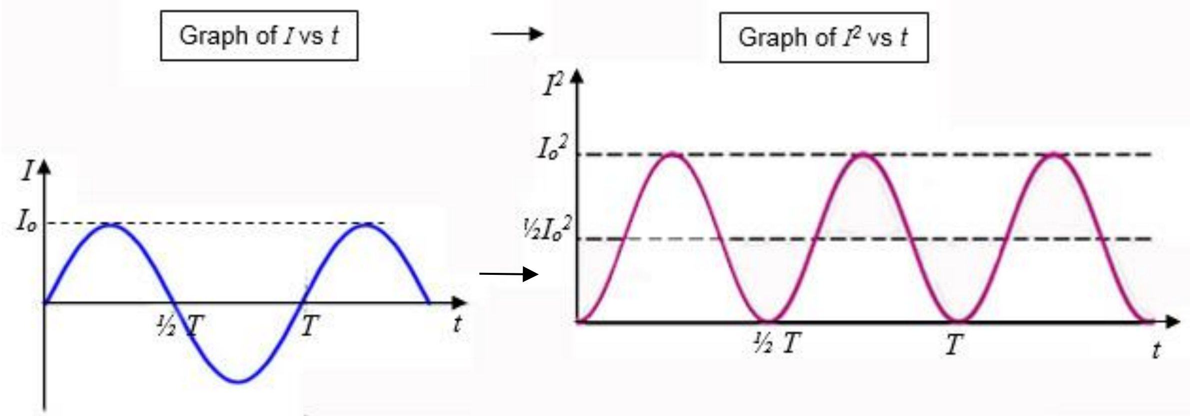
<u>Example 8:</u>

Show that the r.m.s. value of a sinusoidal a.c. is given by $I_{rms} = \dfrac{I_o}{\sqrt{2}}$ where I_o is the peak value.

Solution:

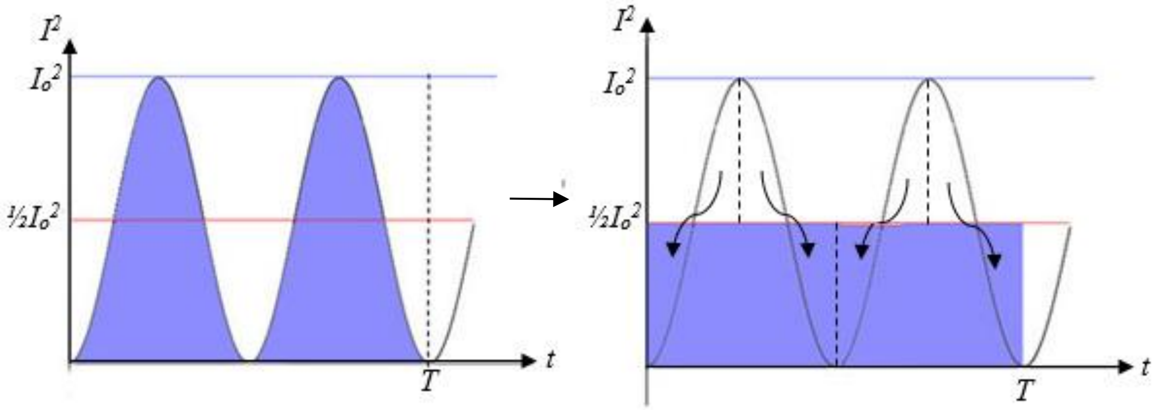
For a sinusoidal a.c., $I = I_o \sin \omega t$

Step 1 **Square the current, i.e. $I^2 = I_o^2 \sin^2 \omega t$ and plot the corresponding $I^2 - t$ graph.**



Step 2 **Determine the mean square current $<I^2>$.**

The shaded area under the $I^2 - t$ graph over one period can be found by flattening the peaks and filling in the 'valleys' and is equal to the area of the shaded rectangle.



Area under $I^2 - t$ graph for one period = Area of shaded rectangle = $\frac{1}{2} I_o^2 \times T$

Mean square current $\langle I^2 \rangle = \dfrac{\text{Area under } I^2 - t \text{ graph within one period}}{T} = \dfrac{\frac{1}{2} I_o^2 \times T}{T}$

Step 3 **Determine the r.m.s. current I_{rms}.**

$$I_{rms} = \sqrt{\langle I^2 \rangle} = \sqrt{\frac{\text{Area under } I^2 - t \text{ graph within one period}}{T}}$$

$$= \sqrt{\frac{\frac{1}{2} I_o^2 x T}{T}} = \frac{I_o}{\sqrt{2}}$$

Thus, the root mean-square values for both alternating sinusoidal current and voltage signals can be expressed as: $I_{rms} = \dfrac{I_o}{\sqrt{2}}$ and $V_{rms} = \dfrac{V_o}{\sqrt{2}}$

where I_o and V_o are the peak values of the current and voltage.
Note: Most current and voltage specifications for power outlets are usually quoted in terms of r.m.s. values. Therefore, a 120 V a.c. supply has an r.m.s. value of 120 V.

Questions to ponder:

1. Do AC ammeters and voltmeters read maximum, r.m.s., or average values of an a.c?

2. Fluorescent lights should flicker on and off 100 times every second if a.c. of 50 Hz is delivered to the lights. Explain what causes this. Why can't you see it happening?

Example 9:

Complete the table and determine the rms current I_{rms} of each of the a.c.waveform.

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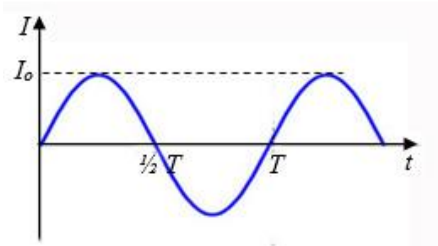
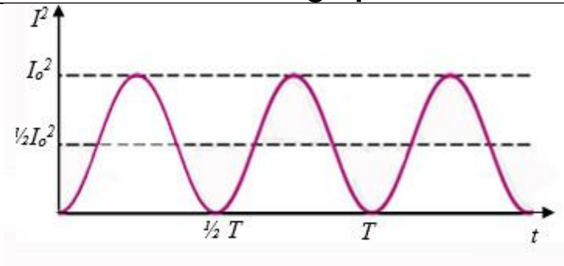
<tr><th>$I - t$ graph</th><th>Plot $I^2 - t$ graph</th><th>I_{rms}</th></tr>
<tr><td>
</td><td>
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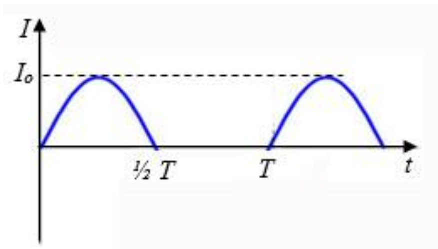
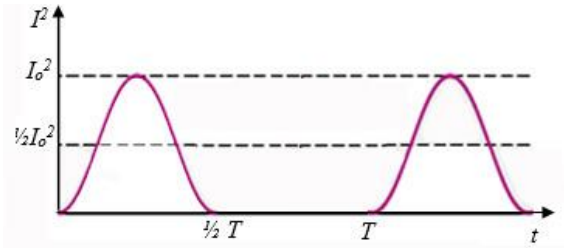
<tr><td>
</td><td>
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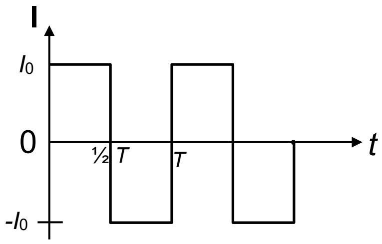
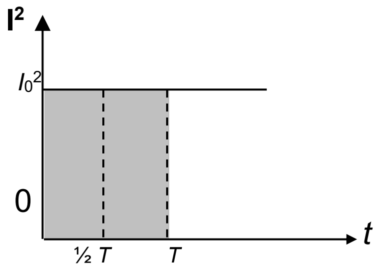
<tr><td>
</td><td>
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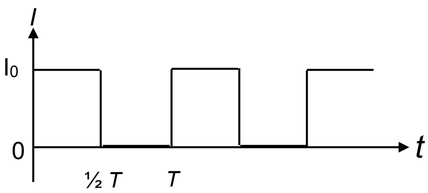
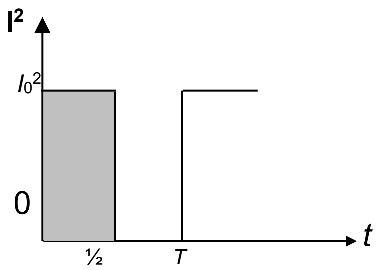
$= I_o$</td></tr>
<tr><td>
</td><td>
</td><td>$I_{rms} = \sqrt{\frac{Io^2 x \frac{1}{2}T}{T}}$
$= \frac{Io}{\sqrt{2}}$</td></tr>
</table>

4.5 POWER DISSIPATION IN A RESISTIVE LOAD

Consider the circuit shown.

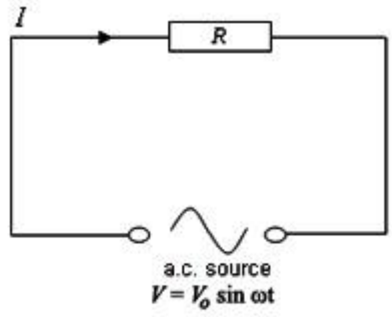
E.m.f. of source is $V = V_o \sin \omega t$.

At any instant, current $I = \dfrac{V}{R}$

$= \dfrac{V_o \sin \omega t}{R}$

$= I_o \sin \omega t$

where I_o is the peak current given by $\dfrac{V_o}{R}$.



Note that both the e.m.f. V and current I given by $V = V_o \sin \omega t$ and $I = I_o \sin \omega t$ are in phase.

The instantaneous power delivered to the load is $P = I^2R$

$= I_o^2 R \sin^2 \omega t$

The maximum power delivered to the load is $P_o = I_o^2 R$

The mean or average power absorbed by the load is $\langle P \rangle = \langle I^2 \rangle R = I_{rms}^2 R$

$= \left(\dfrac{I_o}{\sqrt{2}}\right)^2 R$

$= \frac{1}{2} I_o^2 R = \frac{1}{2} P_o$

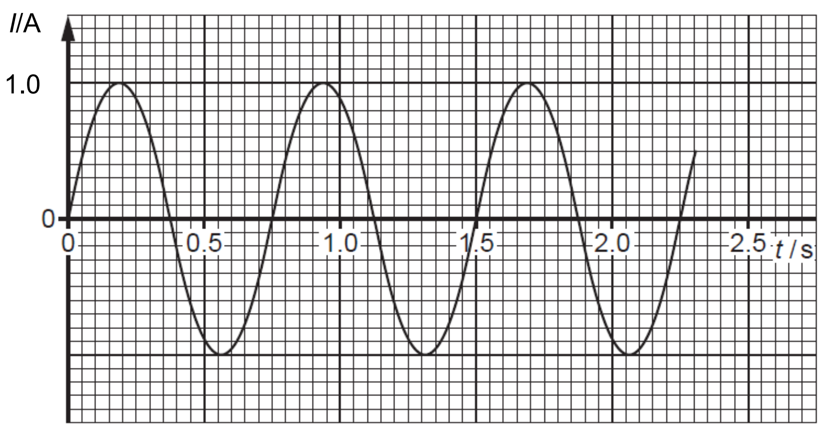
Hence, the mean power $\langle P \rangle$ is equal to half the maximum power P_o for a sinusoidal a.c..

The useful power equations of an a.c. are

$$\boxed{\langle P \rangle = I_{rms}^2 R = V_{rms} I_{rms} = \frac{V_{rms}^2}{R} = \tfrac{1}{2} P_o}$$

Example 10:

The graph below shows how the current through a 50 Ω resistor varies with time.



Calculate

(a) the r.m.s current,

(b) the average power dissipated in the resistor,

(c) the maximum power dissipated in the resistor.

Solution:

(a) $I_{rms} = \dfrac{I_o}{\sqrt{2}}$ = 1.0/√2 = 0.71 A

(b) $\langle P \rangle = I_{rms}^{\ 2} R$

$= (\dfrac{1.0}{\sqrt{2}})^2 \times 50$

= 25 W

(c) $P_o = I_o^2 R$ or $P_o = 2\langle P \rangle$

$= 1.0^2 \times 50$ $= 2 \times 25$

= 50 W = 50 W

5 RECTIFICATION *LO (j)*

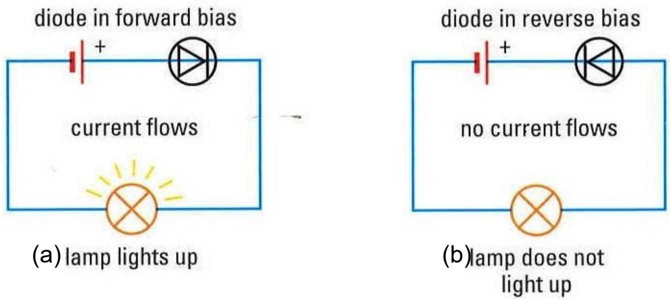
Although electrical power is transmitted as a.c., many electrical and electronic devices require d.c. Hence, a means of converting a.c. to d.c. is required and this is achieved using a diode. Such a process is termed **rectification**.

A diode is a semi-conductor device that allows a current to flow easily in one direction only.

It prevents a current from flowing in the reverse direction. Circuit Symbol: 

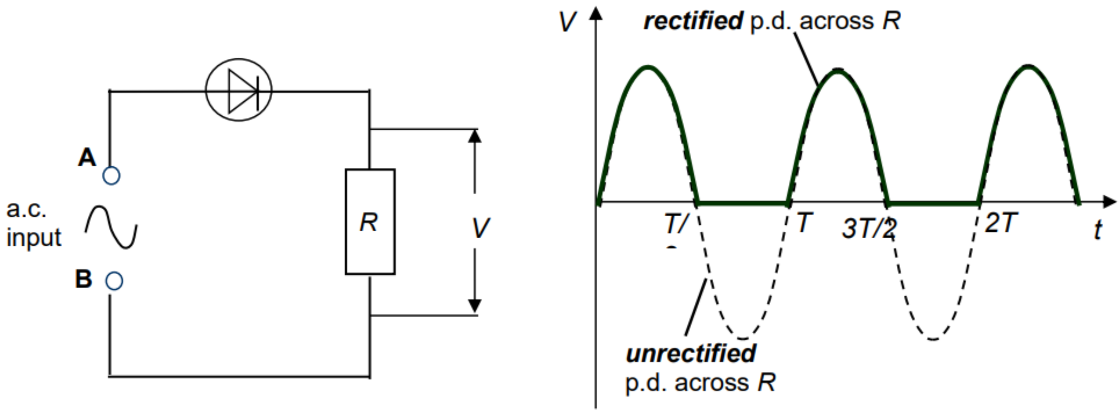
The circuits shown below illustrate the workings of a diode. When the diode is connected in circuit (a), the lamp lights up. This means that the diode allows current to pass through easily.

When the diode is reversed and is connected in circuit (b), the diode prevents the current from flowing through it, resulting in little or no current flow thus the lamp fails to light up. The connection of the diode in (a) is known as forward bias and that in (b) is known as reverse bias.



5.1 Half-Wave Rectification

A single diode connected to an a.c. supply can act as a rectifier to perform half-wave rectification.



Explanation:

(a) The diode conducts during the first half-cycle when it is forward-biased (terminal **A** is positive). A pulse of current flows round the circuit, creating a p.d. across the electrical load R which is equal to the applied alternating p.d..

(b) During the second half-cycle, the diode is reversed-biased (now, terminal **B** is positive instead). *Little or no current flows* and the p.d. across R is zero.

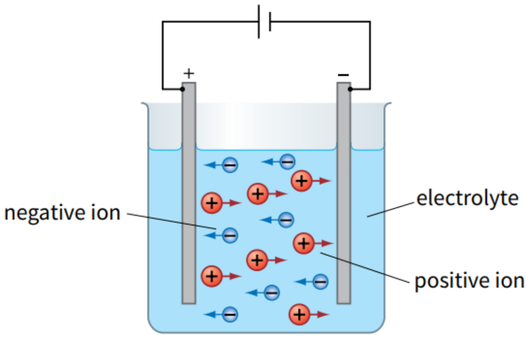
(c) The V-t graph shows the rectified output p.d. across the load R.

<u>Question to ponder</u>:
What modifications will you make to smoothen the rectified p.d. across R?

Appendix:

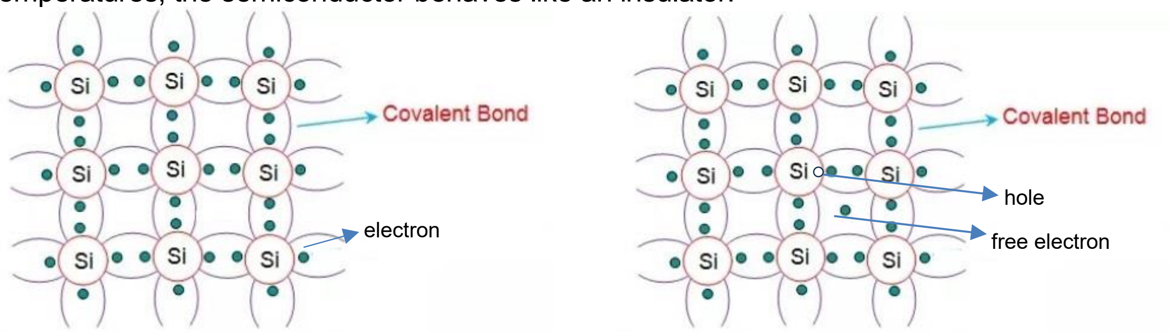
1 Charge Carriers in an electrolyte

Any charged particles which contribute to an electric current are known as charge carriers; these can be electrons, protons or ions. Charged ions in a solution (called an electrolyte) can also flow and create a current. Car batteries work due to the flow of hydrogen (H^+) ions and sulphate (SO_4^{2-}) ions and can deliver currents of up to 450 A for 2.5 s. Figure shows an electrolyte which contains positive and negative ions which flow in opposite directions when the solution are connected to a cell.



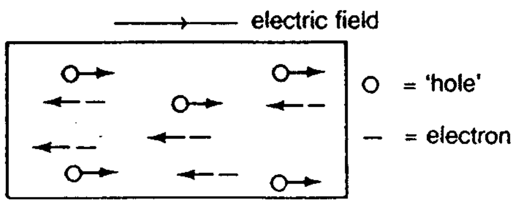
2 Charge carriers in a semiconductor

Semiconductors are mainly Group IV elements in the Periodic Table. In pure semiconductors such as silicon and germanium, the four valence electrons of each atom form covalent bonds with those of neighbouring atoms, with no free electrons available for conduction. Thus, at low temperatures, the semiconductor behaves like an insulator.



At low T, Crystal Lattice of Silicon **At high T**, Crystal Lattice of Silicon

When the temperature is raised, thermal agitation can free the electrons from the covalent bonds leaving behind vacancies called **holes** in the covalent bonds. Holes are positively charged and have the same magnitude of charge as the electrons. Thus, thermal agitation in pure semiconductors produces equal number of free electrons and holes which render the semiconductor conducting.



The current in a pure semiconductor

In the presence of an electric field, the free electrons drift opposite to the electric field and the holes drift in the direction of the electric field, resulting in a current. At room temperature, the typical drift speed of charge carriers in semi-conductors is of the order **10^3 m s^{-1}** for a current of 1 A. This is 10^7 times more than that of metals with the same physical dimensions and same current. This is due to the relatively low concentration of the charge carriers in semiconductors compared to metals (since $v \propto 1/n$ for same current I in the equation $I = nAvq$).

3 Sources of Electromotive Force (emf)

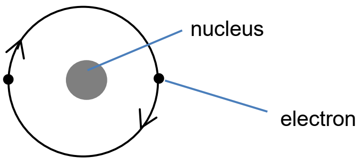
Primary source	Principle	Examples	Applications
Light	Photoelectric effect	Solar cell	Satellite power, calculator, light sensing device
Chemical energy	Chemical reaction between electrodes and an electrolyte	Dry cell, rechargeable battery	Car battery, all portable electronic equipment
Thermoelectric effect	e.m.f. created between two junctions of different metals at different temperatures	Thermocouple, thermopile (which are thermocouples in series)	Heat sensing device, thermocouple thermometer
Electromagnetic induction	Induced e.m.f. – by changing the magnetic flux of a conductor	a.c. generator, dynamo	Power station, dynamo inside a car for recharging battery

Unit 15: Currents Tutorial

Solutions to Q4, 9, 14 are provided in student's copy.

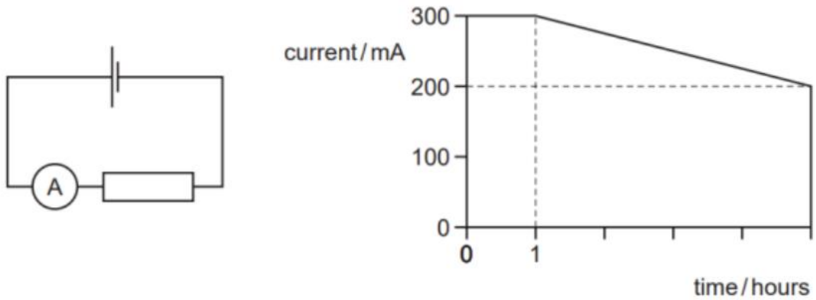
Charge and Direct Current

1. The diagram shows a model of an atom in which two electrons move round a nucleus in a circular orbit. The electrons complete one full orbit in 1.0×10^{-15} s



What is the current caused by the motion of the electrons in the orbit? [3.2×10^{-4} A]

2. A cell is connected to a resistor and the current is measured. The graph shows how the current varies with time

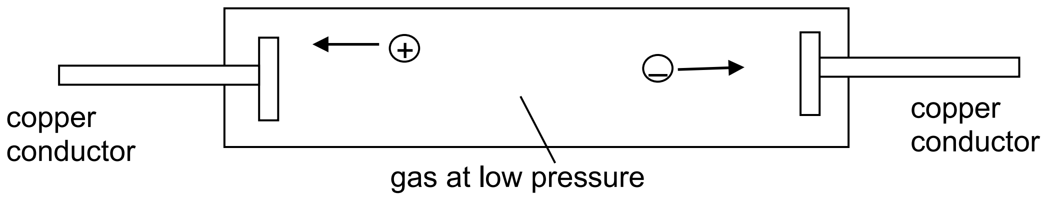


Calculate the total charge that flows through the cell in 5 hours. [4680 C]

3. In a particular wire, electrons move with an average drift speed of 0.50 mm s^{-1} through a cross-section of area 4.0 mm^2. There are 1.0×10^{29} electrons in each m^3 of the wire. By considering a cylinder of wire of length 12.0 mm, calculate

(a) the number of electrons within the cylinder,
(b) the total charge carried by the electrons in the cylinder,
(c) the current in the conductor. [4.8×10^{21}, 7.68×10^{2} C, 32 A]

4. In a gas, conduction occurs as a result of negative particles flowing one way and positive particles flowing in the opposite direction.



In this case, the copper conductors to the gas carry a current of 0.28 mA. The number of negative particles passing any point in the gas per unit time is 1.56×10^{15} s^{-1} and the charge on each negative particle is -1.60×10^{-19} C. Calculate

(a) the negative charge flowing past any point in the gas per second,

(b) the positive charge flowing past any point in the gas per second,

(c) the number of positively charged particles passing any point in the gas per second, given that the charge on each positive particle is +3.20 x 10^{-19} C.

By considering the significant figures available, explain why your answers to (b) and (c) are unreliable. [0.25 mA, 0.03 mA, 9.4 x 10^{13} s^{-1}]

Solution:

(a) $I_- = \frac{Q}{t} = \frac{Ne}{t} = 1.56\times10^{15}\left(1.60\times10^{-19}\right) = 0.25\ mA$

(b) $I_{total} = I_+ + I_- \Rightarrow I_+ = I_{total} - I_- = 0.28 - 0.25 = 0.03\ mA$

(c) $0.03\times10^{-3} = \frac{N_+}{t}\left(3.20\times10^{-19}\right) \Rightarrow \frac{N_+}{t} = 9.4\times10^{13}\ s^{-1}$

Subtraction of two nearly equal numbers increases the percentage uncertainty considerably.

$\Delta I_+ = \Delta I_{total} + \Delta I_- = 0.01 + 0.01 = 0.02\ mA$

$\Rightarrow \frac{\Delta(N_+/t)}{N_+/t} = \frac{\Delta I_+}{I_+} = \frac{0.02}{0.03} = 0.67$

$\Rightarrow$ % uncertainty = 67%!!

Hence answers to (b) and (c) are unreliable.

5. 2018 P2 Q5

   (a) Copper has one conduction electron per atom. The density of copper is 8960 kg m^{-3}. The mass of 1.00 mole of copper is 63.5 g. Show that the number density of charge carriers in copper is 8.49 x 10^{28} m^{-3}.

   (b) A tungsten wire of diameter 0.36 mm has resistance 30 Ω. The power dissipated in the wire is 5.0 W. The number density of charge carriers in tungsten is 3.4 x 10^{28} m^{-3}. Calculate the average drift velocity *v* of the conduction electrons in tungsten.
   [7.37 x 10^{-4} m s^{-1}]

6. (a) Define charge.

   (b) A heater is made from a wire of resistance 18.0 Ω and is connected to a power supply of 240 V. The heater is switched on for 2.60 Ms. Calculate
   (i) the power transformed in the heater,
   (ii) the current in the heater,
   (iii) the charge passing through the heater in this time.
   (iv) the number of electrons per second passing a given point in the heater.
   [3200 W, 13.3 A, 3.47 × 10^7 C , 8.35 × 10^{19}]

7. A filament lamp is rated as 30W, 120V. A potential difference of 120V is applied across the lamp. For the filament wire of the lamp, calculate

   (a) the current,
   (b) the number of electrons passing a point in 3.0 hours
   (c) the resistance of the filament wire.

   [0.25 A, 1.7 × 10^{22}, 480 Ω]

Alternating Currents

8. Explain, in terms of heating effect, what is meant by the *root-mean-square (r.m.s.) value* of an alternating current.

9. Calculate the r.m.s. value of the a.c. in the figures.

Fig. (a)

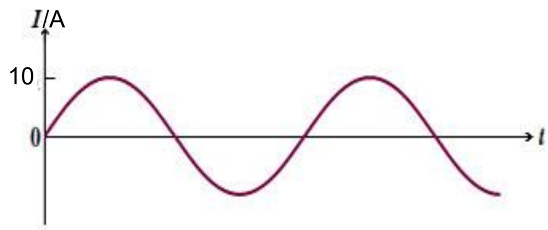
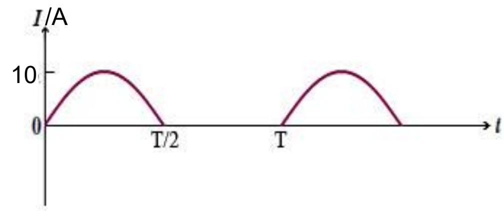


Fig. (b)

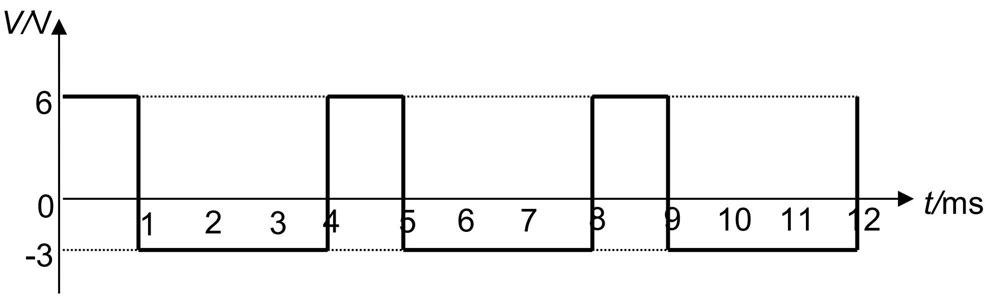


[7.1 A, 5.0 A]

Solution:
From the figures, the peak current $I_o = 10$ A.
Fig. (a): For sinusoidal a.c., $I_{rms} = I_o/\sqrt{2} = 10/\sqrt{2} = 7.1$ A
Fig. (b): For rectified a.c., $I_{rms} = I_o/2 = 10/2 = 5.0$ A

10. An AC source has an output voltage given by the expression $V = 200 \sin(100\pi t)$. This source is connected to a 100 Ω resistor. Calculate the
(a) frequency of the source,
(b) r.m.s. voltage and
(c) r.m.s. current in the resistor. [50 Hz, 141 V, 1.41 A]

11. The following alternating voltage is applied across a resistive load of 100 Ω.



Determine the
(a) mean value of V.
(b) r.m.s. value of *V*.
(c) average power dissipated in the load.

[-0.75 V, 4.0 V, 0.16 W]

12. An a.c. power supply is connected to a resistor *R*, as shown in Fig. A below.

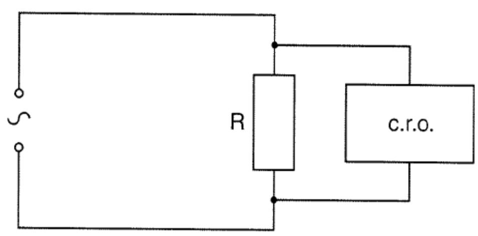


Fig. A

A cathode ray oscilloscope (c.r.o.) is used to show the potential difference (p.d.) across R. The screen of the c.r.o. displays the variation with time of the p.d. across R, as shown in Fig. B.

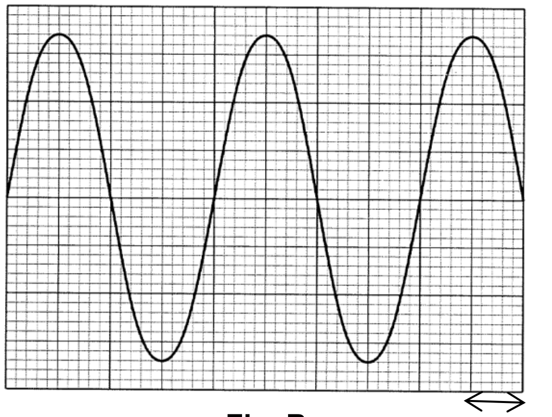


Fig. B

On the vertical scale, 1.0 cm represents 5.0 V. On the horizontal scale, 1.0 cm represents 10 ms.

(a) Use Fig. B to determine

i. the frequency of the a.c. supply,

ii. the peak p.d. across resistor R.

(a) R has a resistance of 500 Ω. Calculate

i. the r.m.s. current in R,

ii. the mean power transformed in R.

[25 Hz, 17 V, 0.024 A, 0.29 W]

13. An alternating current of r.m.s value $2\sqrt{2}$ A and frequency 50 Hz passes through a resistor of resistance 5.0 Ω. Sketch a labelled graph showing the variation with time t of the power P dissipated in the resistor for one cycle of the alternating current.

14. An electric shaver designed to operate at 110 V, 20 W is plugged into a 240 V a.c. supply. Calculate,

(a) the average and peak power of the shaver.

(b) the resistance of this shaver.

(c) the r.m.s. current that flows when operating on the 240 V supply. [95 W, 190 W, 605 Ω, 0.40 A]

Solution:

(a)

$$\langle P \rangle = \frac{{V_{rms}}^2}{R} \propto {V_{rms}}^2 \qquad \therefore \frac{P'}{P} = \left(\frac{V'}{V}\right)^2$$

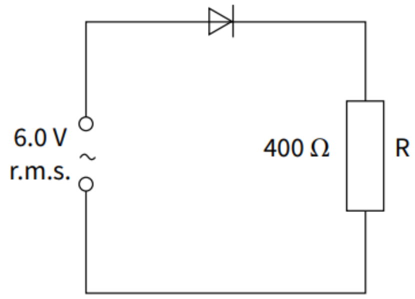
$$\frac{P'}{20} = \left(\frac{240}{110}\right)^2 \Rightarrow P' = 95\ \mathrm{W}$$

Peak Power = 2 $\langle P \rangle$ = 190 W

(b) Resistance of shaver $R = {V_{rms}}^2/P = 110^2/20 = 605\ \Omega$

(c) $I_{rms} = V_{rms}/R = 240/605 = 0.40$ A

15. A sinusoidal voltage of 6.0 V r.m.s. and frequency 50 Hz is connected to a diode and a resistor R of resistance 400 Ω as shown in the figure.



Draw a sketch graph showing the variation with time of both the supply waveform and the voltage across R. Put numerical scales on both the voltage and time axes.