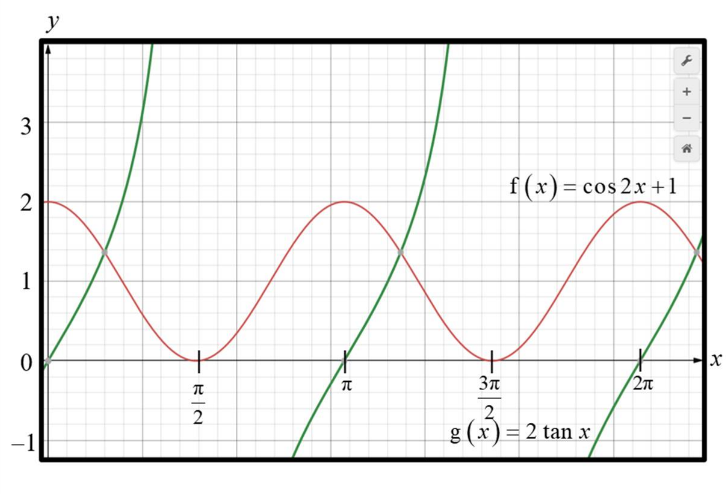
AHS 2022 S4 Preliminary Examinations AM P1 Answer Key

1(a)	$\mathrm{P}(x) = 2x+3$	**2**	$\frac{29}{121}$.	**3**	$x=9$

1(b)	$2x^3-5x^2+6x+27=0$ $\left(x^2-4x+9\right)(2x+3)=0$ For $x^2-4x+9=0$, Discriminant $=-20<0$. Hence there is no solution for $x^2-4x+9=0$. For $2x+3=0,\quad x=-\frac{3}{2}$. Hence the equation $\mathrm{P}(x)=0$ has only one real root

4(i)	0.000121	**4(ii)**	10.3%	**4(iii)**	$-0.0121e^{-0.000121t}$ percent/year

5	$\frac{x^2-10}{\left(x^2+3\right)(2x-1)}=\frac{2x+1}{x^2+3}-\frac{3}{2x-1}$	**6(i)**	Period of f $=\pi$	**6(iii)**	Number of solutions is 2

6(i)	

7(i)	$A>B$	**7(ii) (a)**	$\tan(A+B)=\frac{4}{3}$	**7(ii) (b)**	$\tan(A-B)=\frac{5}{12}$	**7b (iii)**	$\tan 2A=\tan\left[(A+B)+(A-B)\right]$

8(a)(i)	$k=-2$	**8(a) (ii)**	Coordinates of S are $(-5,-3)$	**8(b)**	Gradient of $QW=-\tan\alpha$	**8(c)**	Coordinates of W are $\left(\frac{11}{2},-\frac{5}{2}\right)$

8(d)	Show that *MQ*, *MP*, *MS* and *MR* are equal, these are radii, so a circle can be drawn to pass through the vertices of the quadrilateral *PQRS*. Centre of the circle is *M*.	**8(d)**	Show that $QR \perp SR$, angle $QRS = 90°$ (Right angle in a semicircle). Show that $QP \perp PS$, angle $SPQ = 90°$ (Right angle in a semicircle) *QS* is the diameter of a circle, with *P* and *R* lying on its circumference. Hence a circle can be drawn to pass through the vertices of the quadrilateral *PQRS* .

9(a)	$p = 7.1202$ $x = 10.2$	**9(b)**	$k = \frac{59}{3}$	**10(i)**	$2x + 2x\ln\left(x^2+1\right)$	**10(ii)**	$\frac{1}{2}\left(x^2+1\right)\ln\left(x^2+1\right) - \frac{1}{2}x^2 + c$

11(a)	$B\ (-2, 4)$	**11(b)**	$5\frac{1}{3}$ units^2	**12(a)**	$M = 30.4\sin 2\theta + 45.6\cos\theta$	**12(b)**	66.0 g, maximum.

13(a)	$\angle GBD = 90°$ (Right angle in a semi-circle) $\angle ECD = 90°$ (Right angle in a semi-circle) $\angle GBD = \angle ECD$ $\angle BDG = \angle CDE$ (Common angle) Therefore triangle *DBG* is similar to triangle *DCE* (AA)
13(b)	$\angle FGE = 90°$ (Tangent perpendicular to radius) $\angle FGE = \angle ECD$ $\angle GEF = \angle DEC$ (Vertically opposite angles) $GE = EC$ (Given) Therefore triangle *GEF* is congruent to triangle *CED*. (ASA) Hence $GF = CD$
13(c)	$\angle BGA = \angle BDG$ (Alternate Segment Theorem) $= \angle CDE$ $\angle GBD = 90°$ (Right angle in a semi-circle) $\angle ABG = 180° - 90°$ (sum of angles on a straight line) $= 90°$ $\angle ECD = 90°$ (Tangent perpendicular to radius) $\angle ABG = \angle ECD$ So triangle *ABG* is similar to triangle *ECD*. (AA) From (b), triangle *EGF* is congruent to triangle *ECD*. Therefore triangle *ABG* is similar to triangle *EGF*.