1 Differentiate $\dfrac{\left(e^{x}-e^{-x}\right)^{2}}{e^{x}}$ with respect to x, leaving your answer as a single fraction. [2]

$$\text{Let} \quad y=\frac{\left(e^{x}-e^{-x}\right)^{2}}{e^{x}}$$
$$=\frac{e^{2x}-2+e^{-2x}}{e^{x}}$$
$$=e^{x}-2e^{-x}+e^{-3x}$$
$$\frac{\mathrm{d}y}{\mathrm{d}x}=e^{x}+2e^{-x}-3e^{-3x}$$
$$=e^{x}+\frac{2}{e^{x}}-\frac{3}{e^{3x}}$$
$$=\frac{e^{4x}+2e^{2x}-3}{e^{3x}}$$

2 Differentiate each of the following with respect to x, simplifying your answers.

(i) $2e^{x^2}\left(\sqrt[3]{9x-5}\right)$, [3]

Let $y=2e^{x^2}\left(\sqrt[3]{9x-5}\right)$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=4xe^{x^2}\sqrt[3]{9x-5}+2e^{x^2}\left[\frac{1}{3}(9x-5)^{-\frac{2}{3}}(9)\right]$$

$$=2e^{x^2}(9x-5)^{-\frac{2}{3}}\left[2x(9x-5)+3\right]$$

$$=\frac{2e^{x^2}\left(18x^2-10x+3\right)}{\sqrt[3]{(9x-5)^2}}$$

2 **(ii)** $\dfrac{4-x^2}{\sqrt{3x-7}}$. [3]

$$\text{Let } y=\frac{4-x^2}{\sqrt{3x-7}}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-2x\sqrt{3x-7}-\frac{1}{2}(3x-7)^{-\frac{1}{2}}(3)\left(4-x^2\right)}{3x-7}$$

$$=\frac{\frac{1}{2}(3x-7)^{-\frac{1}{2}}\left[-4x(3x-7)-(3)\left(4-x^2\right)\right]}{3x-7}$$

$$=\frac{-9x^2+28x-12}{2\sqrt{(3x-7)^3}}$$

OR

$$\text{Let } y=\left(4-x^2\right)(3x-7)^{-\frac{1}{2}}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=-2x(3x-7)^{-\frac{1}{2}}-\frac{1}{2}(3x-7)^{-\frac{3}{2}}(3)\left(4-x^2\right)$$

$$=\frac{1}{2}(3x-7)^{-\frac{3}{2}}\left[-4x(3x-7)-(3)\left(4-x^2\right)\right]$$

$$=\frac{1}{2}(3x-7)^{-\frac{3}{2}}\left[-9x^2+28x-12\right]$$

$$=\frac{-9x^2+28x-12}{2\sqrt{(3x-7)^3}}$$

3 Find

(a) $\displaystyle\int \frac{8}{3\sqrt{(5-2x)^7}}\ \mathrm{d}x$, [2]

$$\int \frac{8}{3\sqrt{(5-2x)^7}}\ \mathrm{d}x$$
$$=\frac{8}{3}\int (5-2x)^{-\frac{7}{2}}\ \mathrm{d}x$$
$$=\frac{8}{3}\left[\frac{(5-2x)^{-\frac{5}{2}}}{\left(-\frac{5}{2}\right)(-2)}\right]+c, \text{ where } c \text{ is a constant}$$
$$=\frac{8}{15\sqrt{(5-2x)^5}}+c$$

3 **(b)** $\int_1^3 \frac{6x-1}{4-9x}\,dx$, leaving your answer to 3 significant figures. [3]

$$\int_1^3 \frac{6x-1}{4-9x}\,dx$$
$$=\int_1^3 \left(\frac{-\frac{2}{3}(4-9x)+\frac{5}{3}}{4-9x}\right)dx$$
$$=\int_1^3 \left(-\frac{2}{3}+\frac{5}{3(4-9x)}\right)dx$$
$$=\left[-\frac{2}{3}x-\frac{5}{27}\ln|4-9x|\right]_1^3$$
$$=\left[-2-\frac{5}{27}\ln 23\right]-\left[-\frac{2}{3}-\frac{5}{27}\ln 5\right]$$
$$=-1.62$$

by long division

$$\begin{array}{r|l} & -\frac{2}{3} \\ \hline 4-9x & 6x-1 \\ & 6x-\frac{8}{3} \\ \hline & \frac{5}{3} \\ \hline \end{array}$$

$$\frac{6x-1}{4-9x}=-\frac{2}{3}+\frac{5}{3(4-9x)}$$

4 The height of a ball above ground, h metres, released by a machine can be modelled by the equation $h = -0.4x^2 + 6x + 3$, where x is the horizontal distance travelled by the ball in metres.

(i) State the height above ground at which the ball is being released. [1]

Height = 3 m

(ii) Express $h = -0.4x^2 + 6x + 3$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [2]

$$\begin{aligned} h &= -0.4x^2 + 6x + 3 \\ &= -0.4(x^2 - 15x) + 3 \\ &= -0.4\left[(x-7.5)^2 - 7.5^2\right] + 3 \\ &= -0.4(x-7.5)^2 + 25.5 \end{aligned}$$

4 **(iii)** Using your answers from **(i)** and/or **(ii)**,

(a) explain if the machine is safe for use in an indoor sports arena with a ceiling height of 25 metres, [1]

Since maximum height ball can reach = $25.5 > 25$,

it is unsafe.

(b) find how far horizontally will the ball land from the point it is released. [2]

$$-0.4(x-7.5)^2 + 25.5 = 0$$
$$(x-7.5)^2 = 63.75$$
$$x - 7.5 = \pm\sqrt{63.75}$$
$$x = 15.5 \text{ or } -0.484 \text{(rej)}$$
$$\text{Distance} = 15.5 \text{ m}$$

5 The equation of a curve is $y=\ln\left[(x-2)\sqrt{2x+1}\right]$ for $x>2$.

(i) Find $\dfrac{dy}{dx}$, leaving your answer as a single fraction. [2]

$$y=\ln\left[(x-2)\sqrt{2x+1}\right]$$

$$y=\ln(x-2)+\frac{1}{2}\ln(2x+1)$$

$$\begin{aligned}\frac{dy}{dx}&=\frac{1}{x-2}+\frac{1}{2}\left(\frac{2}{2x+1}\right)\\&=\frac{2x+1+x-2}{(x-2)(2x+1)}\\&=\frac{3x-1}{(x-2)(2x+1)}\end{aligned}$$

5 **(ii)** Find the coordinates of the point on the curve where the tangent is perpendicular to the line $11y+18x=6$, leaving your answer in the form $(a,\ \ln b)$, where a and b are constants. [3]

$$11y+18x=6$$
$$y=-\frac{18}{11}x+\frac{6}{11}$$
$$\text{Gradient of tangent} = -\frac{1}{-\frac{18}{11}}$$
$$=\frac{11}{18}$$

$$\frac{3x-1}{(x-2)(2x+1)}=\frac{11}{18}$$
$$11(x-2)(2x+1)=54x-18$$
$$22x^2-33x-22=54x-18$$
$$22x^2-87x-4=0$$
$$(22x+1)(x-4)=0$$
$$x=-\frac{1}{22}\text{(reject)} \quad \text{or } x=4$$

when $x=4, y=\ln 6$

The coordinates is $(4,\ln 6)$

6 The equation $\log_3 x + 4\log_3 5 = \log_{27} x$ has the solution $x = 5^k$.

Find the value of k. [4]

$$\log_3 x + 4\log_3 5 = \log_{27} x$$
$$\log_3 x + \log_3 5^4 = \frac{\log_3 x}{\log_3 27}$$
$$\log_3\left(5^4 x\right) = \log_3 x^{\frac{1}{3}}$$
$$x^{\frac{1}{3}} = 5^4 x$$
$$x^{\frac{2}{3}} = 5^{-4}$$
$$x = 5^{-6}$$
$$k = -6$$

OR

$$\log_3 5^k + 4\log_3 5 = \log_{27} 5^k$$
$$\log_3\left(5^k 5^4\right) = \frac{\log_3 5^k}{\log_3 27}$$
$$\left(\log_3 5^{k+4}\right)^3 = \log_3 5^k$$
$$12 + 3k = k$$
$$2k = -12$$
$$k = -6$$

7 A is a point (1, 2), $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

(i) Find the exact value of $\left|\overrightarrow{BC}\right|$. [1]

$$\left|\overrightarrow{BC}\right| = \sqrt{2^2 + 3^2}$$
$$= \sqrt{13} \text{ units}$$

(ii) Express $\overrightarrow{AC}$ as a column vector. [1]

$$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$
$$= \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

7 **(iii)** Given that $\overrightarrow{CD} = \begin{pmatrix} -2 \\ p \end{pmatrix}$, find the two possible values of p such that $ABCD$ is a trapezium. [2]

When AB // to CD,

$p = 0$

$\overrightarrow{AD} = \begin{pmatrix} 6 \\ 3 \end{pmatrix} + \begin{pmatrix} -2 \\ p \end{pmatrix} = \begin{pmatrix} 4 \\ 3+p \end{pmatrix}$

When BC // to AD,

$p = 3$

OR

When AD // to BC,

$\overrightarrow{AD} = k\overrightarrow{BC}$

$\begin{pmatrix} 4 \\ 3+p \end{pmatrix} = k\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$k = 2$

$3 + p = 6$

$p = 3$

7 **(iv)** A student claims that $E(-8,-2)$ lies on CA produced.

Use **vectors** to determine whether his claim is true or not. [3]

$$\overrightarrow{EA}=\begin{pmatrix}1\\2\end{pmatrix}-\begin{pmatrix}-8\\-2\end{pmatrix}$$

$$=\begin{pmatrix}9\\4\end{pmatrix}$$

$$\overrightarrow{AC}=\begin{pmatrix}6\\3\end{pmatrix}$$

If E, A, C are collinear,

$$\begin{pmatrix}9\\4\end{pmatrix}=k\begin{pmatrix}6\\3\end{pmatrix}, k\in\mathbb{R}$$

However, k does not exist $\left(\frac{9}{6}\neq\frac{4}{3} \text{ or } \frac{9}{4}\neq\frac{6}{3}\right)$

His claim is false.

or

E does not lie on CA produced.

8 The gradient function of a curve is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k}{\sqrt{x^3}} - 2x + 1$ and the gradient to the curve at the point $(4, -6)$ is –9. Find

(i) the value of *k*, [1]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k}{\sqrt{x^3}} - 2x + 1$$

When $x = 4, \frac{\mathrm{d}y}{\mathrm{d}x} = -9$

$$k = -16$$

(ii) the equation of the curve, [2]

$$y = \int \left(-\frac{16}{\sqrt{x^3}} - 2x + 1 \right) \mathrm{d}x$$

$$= \frac{32}{\sqrt{x}} - x^2 + x + c, \text{ where } c \text{ is a constant}$$

When $x = 4, y = -6$

$$-6 = 16 - 16 + 4 + c$$

$$c = -10$$

Equation of curve is $y = \frac{32}{\sqrt{x}} - x^2 + x - 10$

8 **(iii)** the range of values of x for which $\frac{\mathrm{d}y}{\mathrm{d}x} < -2x - 15$. [2]

$$\frac{\mathrm{d}y}{\mathrm{d}x} < -2x - 15$$

$$-\frac{16}{\sqrt{x^3}} - 2x + 1 < -2x - 15$$

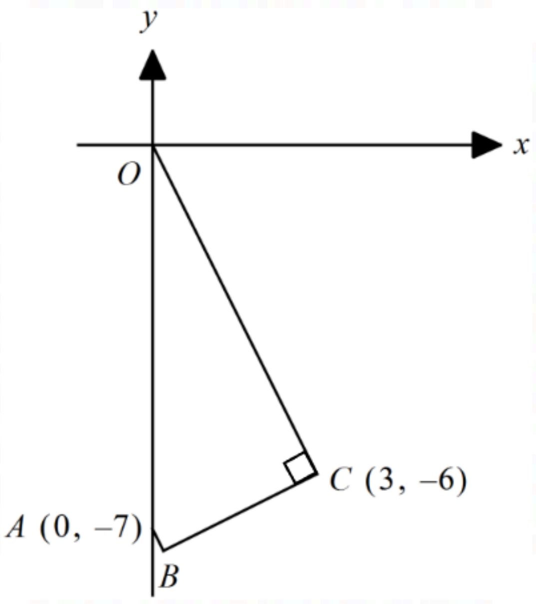
$$-\frac{16}{\sqrt{x^3}} < -16$$

$$\sqrt{x^3} < 1$$

$$x < 1$$

The range of values of x is $0 < x < 1$.

9 **Solution to this question by accurate drawing will not be accepted.**



In trapezium $OABC$ (not drawn to scale), point A has coordinates $(0,-7)$ and point C has coordinates $(3,-6)$.

(i) Find the coordinates of B. [4]

Let B be $(x. y)$

Equation AB:

$$\frac{y+7}{x}=-2$$

$$y=-2x-7$$

Equation BC:

$$y+6=\frac{1}{2}(x-3)$$

$$y=\frac{1}{2}x-\frac{15}{2}$$

$$-2x-7=\frac{1}{2}x-\frac{15}{2}$$

$$-\frac{5}{2}x=-\frac{1}{2}$$

$$x=\frac{1}{5},\ y=-7\frac{2}{5}$$

$$B\left(\frac{1}{5},-7\frac{2}{5}\right)$$

9 **(ii)** E is a point which lies on the perpendicular bisector of OC such that the area of $OACE$ is 25.5 units^2. Find the coordinates of E. [4]

Mid-pt of $OC = \left(\frac{3}{2},\ -3\right)$

Equation of perp. bisector:

$y+3=\frac{1}{2}\left(x-\frac{3}{2}\right)$

$y=\frac{1}{2}x-\frac{15}{4}$

Area of $OACE = \frac{1}{2}\begin{vmatrix} 0 & 0 & 3 & x & 0 \\ 0 & -7 & -6 & \frac{1}{2}x-\frac{15}{4} & 0 \end{vmatrix} = 25.5$

$\frac{1}{2}\left[3\left(\frac{1}{2}x-\frac{15}{4}\right)-(-6x-21)\right]=25.5$

$\frac{15}{4}x+\frac{39}{8}=25.5$

$x=5\frac{1}{2},\ y=-1$

$E\left(5\frac{1}{2},-1\right)$

10 **(i)** Given that $4x^3-19x^2+6x+45 \equiv (x-A)^2(Bx+5)$, find the values of A and B, where A and B are positive integers.

[2]

$$4x^3-19x^2+6x+45 \equiv (x-A)^2(Bx+5)$$
$$=\left(x^2-2Ax+A^2\right)(Bx+5)$$

Compare coeff. of x^3:

$$B=4$$

Compare coeff. of x^2:

$$5-8A=-19$$
$$A=3$$

OR

Compare constant or sub $x=0$:

$$5A^2=45$$
$$A=3$$

OR

Compare coeff. of x:

$$4A^2-10A=6$$
$$2A^2-5A-3=0$$
$$(2A+1)(A-3)=0$$
$$A=-\frac{1}{2} \text{ (rej) or } A=3$$

10 **(ii)** Hence, express $\dfrac{3x^2-27x+88}{4x^3-19x^2+6x+45}$ as partial fractions. [5]

$$\frac{3x^2-27x+88}{4x^3-19x^2+6x+45} \equiv \frac{3x^2-27x+88}{(4x+5)(x-3)^2} \equiv \frac{C}{4x+5}+\frac{D}{x-3}+\frac{E}{(x-3)^2}$$

By cover-up method,

Put $x=-\dfrac{5}{4}$: $\quad C=\dfrac{3\left(-\frac{5}{4}\right)^2-27\left(-\frac{5}{4}\right)+88}{\left(-\frac{5}{4}-3\right)^2}$

$=7$

Put $x=3$: $\quad E=\dfrac{27-81+88}{17}=2$

$$\frac{3x^2-27x+88}{(4x+5)(x-3)^2}=\frac{7}{4x+5}+\frac{D}{x-3}+\frac{2}{(x-3)^2}$$

sub $x=0$: $\quad \dfrac{88}{45}=\dfrac{7}{5}-\dfrac{D}{3}+\dfrac{2}{9}$

$D=-1$

OR

$3x^2-27x+88=7(x-3)^2+D(4x+5)(x-3)+2(4x+5)$

comparing coeff. of x^2: $\quad 7+4D=3$

$D=-1$

OR

comparing coeff. of x: $\quad -42-7D+8=-27$

$D=-1$

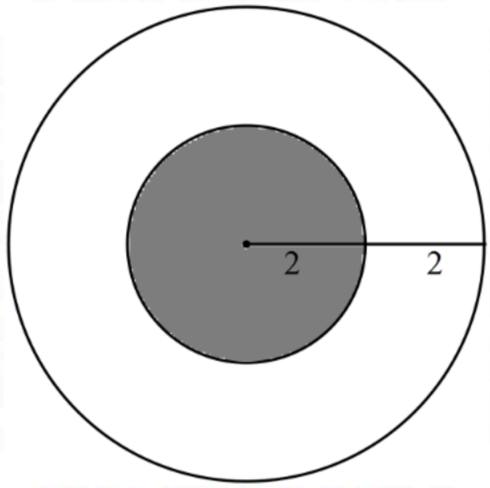
OR

comparing constant: $\quad 63-15D+10=88$

$D=-1$

$$\therefore \frac{3x^2-27x+88}{4x^3-19x^2+6x+45} \equiv \frac{7}{4x+5}-\frac{1}{x-3}+\frac{2}{(x-3)^2}$$

11



The figure above shows a circular target board formed by 2 concentric circles.

The radius of the entire target board is 4 cm.

The radius of the shaded circular region is 2 cm.

(i) A dart is thrown randomly inside the target board. Find the probability that the dart hits the shaded region. [1]

$$\text{P (Hits shaded region)} = \frac{\pi\left(2^2\right)}{\pi\left(4^2\right)}$$
$$= \frac{1}{4}$$

Darts are thrown randomly inside the target board until a dart hits the shaded region.

(ii) Find the probability that exactly 3 throws are required. [2]

$$\text{P (exactly 3 throws)} = \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4}$$
$$= \frac{9}{64}$$

11 **(iii)** Find the probability that no more than 3 throws are required. [2]

$P\ (\leq\ 3 \text{ throws})$

$= P(1) + P(2) + P(3)$ or $1 - P(\text{more than 3 throws})$

$= \frac{1}{4} + \frac{3}{4} \times \frac{1}{4} + \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4}$ $\quad = 1 - \left(\frac{3}{4}\right)^3$

$= \frac{37}{64}$ $\quad = \frac{37}{64}$

(iv) How many darts would need to be thrown so that the probability that at least one dart hits the shaded region exceeds 0.99? [3]

Let the no. of throws needed be n:

$$1 - \left(\frac{3}{4}\right)^n > 0.99$$

$$\left(\frac{3}{4}\right)^n < 0.01$$

$$n \ln 0.75 < \ln 0.01$$

$$n > \frac{\ln 0.01}{\ln 0.75} = 16.008$$

No. of throws needed = 17

12 Food companies Raffles and Gryphon sell the same type of chicken satays, fish ball sticks and sausages. The following table shows the selling price, in dollars, of a box of chicken satays, a box of fish ball sticks and a box of sausages in Raffles and Gryphon.

	Selling price in Raffles ($)	Selling price in Gryphon ($)
A box of chicken satays	a	7.00
A box of fish ball sticks	5.80	6.50
A box of sausages	7.50	b

Kayden and Leo intend to organise a class party. Kayden decides to buy 12 boxes of chicken satays, 15 boxes of fish ball sticks and 8 boxes of sausages. Leo decides to buy 14 boxes of chicken satays, 10 boxes of fish ball sticks and 12 boxes of sausages.

(i) Using relevant information from above, write down two matrices such that the product would give the total amount spent by Kayden and by Leo at each company. Find this product. [3]

$$\begin{pmatrix} a & 5.8 & 7.5 \\ 7 & 6.5 & b \end{pmatrix}\begin{pmatrix} 12 & 14 \\ 15 & 10 \\ 8 & 12 \end{pmatrix}$$

$$= \begin{pmatrix} 12a+147 & 14a+148 \\ 8b+181.5 & 12b+163 \end{pmatrix}$$

OR

$$\begin{pmatrix} 12 & 15 & 8 \\ 14 & 10 & 12 \end{pmatrix}\begin{pmatrix} a & 7 \\ 5.8 & 6.5 \\ 7.5 & b \end{pmatrix}$$

$$= \begin{pmatrix} 12a+147 & 8b+181.5 \\ 14a+148 & 12b+163 \end{pmatrix}$$

12 The next table shows the total amount, in dollars, that Kayden and Leo will spend if they buy the above quantities from Raffles and Gryphon respectively.

	Total amount spent in Raffles ($)	Total amount spent in Gryphon ($)
Kayden	255	249.50
Leo	274	265

(ii) Using your matrix found in **(i)**, find the value of a and of b. [2]

$12a+147=255$

$a=9$

$8b+181.5=249.5$

$b=8.5$

OR

$14a+148=274$

$a=9$

$12b+163=265$

$b=8.5$

(iii) The total amount spent by Kayden and Leo buying the quantities from Raffles and Gryphon respectively can be represented by matrix, $\mathbf{D}=\begin{pmatrix}255 & 249.50\\274 & 265\end{pmatrix}$.

To entice customers, Raffles and Gryphon each runs a promotion offering a 15% discount and a 12% discount on the orders they receive respectively.

(a) Using a 2×2 matrix **E**, find the product **DE** which will give the discounted prices that Kayden and Leo will pay to Raffles and to Gryphon. [2]

$$\mathbf{DE}=\begin{pmatrix}255 & 249.50\\274 & 265\end{pmatrix}\begin{pmatrix}0.85 & 0\\0 & 0.88\end{pmatrix}$$
$$=\begin{pmatrix}216.75 & 219.56\\232.90 & 233.20\end{pmatrix}$$

(b) Hence, explain which company should Leo order from, if he wants to keep his costs as low as possible. [1]

Leo should order from Raffles because cost from Raffles is less than cost from Gryphon or 232.9 is less than 233.2 .

13 It is given that $f(x)=3ax^3+(a-2b)x^2-x+2$ is exactly divisible by $x-1$ but leaves a remainder of -10 when it is divided by $x+1$.

(i) Calculate the values of a and b. [3]

$$f(x)=3ax^3+(a-2b)x^2-x+2$$
$$f(1)=3a+a-2b-1+2=0$$
$$4a-2b=-1 \qquad (1)$$
$$f(-1)=-3a+a-2b+1+2=-10$$
$$-2a-2b=-13 \qquad (2)$$
$$(1)-(2)$$
$$6a=12$$
$$a=2,\ b=4.5$$

(ii) Factorise $f(x)$ completely. [2]

$$f(x)=6x^3-7x^2-x+2$$
$$=(x-1)(6x^2-x-2)$$
$$=(x-1)(3x-2)(2x+1)$$

OR

$$f(x)=6x^3-7x^2-x+2$$
$$=(x-1)(6x^2+kx-2)$$

comparing coeff of x:

$$-2-k=-1$$
$$k=-1$$
$$f(x)=(x-1)(6x^2-x-2)$$
$$=(x-1)(3x-2)(2x+1)$$

13 **(iii)** Hence, solve the equation $3a\left(4^{y}\right)+2\left(2^{-y}\right)=1-(a-2b)\left(2^{y}\right)$. [2]

$$3a\left(4^{y}\right)+2\left(2^{-y}\right)=1-(a-2b)\left(2^{y}\right)$$
$$6\left(4^{y}\right)+2\left(2^{-y}\right)-1-7\left(2^{y}\right)=0$$
$$6\left(2^{y}\right)^{2}-7\left(2^{y}\right)-1+\frac{2}{2^{y}}=0$$
$$6\left(2^{y}\right)^{3}-7\left(2^{y}\right)^{2}-2^{y}+2=0$$

Let $x=2^{y}$

$2^{y}=1$ or $2^{y}=\frac{2}{3}$ or $2^{y}=-\frac{1}{2}$ (NA)

$y=0$ or $y=\frac{\ln\frac{2}{3}}{\ln 2}=-0.585$ (to 3 sf)

(iv) Show that one of the solutions from part **(iii)** may be written in the form $1-\log_{p} q$, where p, q are constants and $p \neq q$. [2]

$$\begin{aligned} y &= \frac{\ln\frac{2}{3}}{\ln 2} \\ &= \frac{\ln 2-\ln 3}{\ln 2} \\ &= 1-\frac{\ln 3}{\ln 2} \\ &= 1-\log_{2} 3 \end{aligned}$$

OR

$$2^{y}=\frac{2}{3}$$
$$\begin{aligned} y &= \log_{2}\frac{2}{3} \\ &= 1-\log_{2} 3 \end{aligned}$$

END OF PAPER