Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Given that $\sqrt{64^x} = \dfrac{2^{1+9x}}{16}$, find the value of x and hence, $\sqrt{64^x}$ in the form $n\sqrt{2}$ where n is an integer. x=1/2, n=2 [4]

2 Solve the equation $\sqrt{7x+2} + 2x = 0$, explaining why you would or would not reject the solutions obtained. x=-1/4, x=2(rej) [4]

3 Given the area of a rectangle $ABCD$ is $(46-14\sqrt{7})$ cm and the length of the rectangle AB is $(10+\sqrt{7})$ cm . Find, without using a calculator,

(i) the width of the rectangle BC and show that $BC = (6-2\sqrt{7})$ cm , [3]

(ii) the exact value of AC^2 in the form $(c+d\sqrt{7})$, where c and d are integers.

c=171, d=-4 [3]

4 **(a)** Solve the simultaneous equations:

$$5^y = 25^{x+\frac{3}{2}},$$
$$y = 3x^2 - 5.$$

x=-4/3, y=1/3
or
x=2, y=7

[4]

(b) Find the greatest value of the integer a for which $ax^2 + 5x - 2$ is negative ~~positive~~ for all real x. a=-4 [3]

(c) Find the range of values of k for which the line $x + 3ky + 1 = 0$ meets the curve $y^2 = 2x - 7$. k<=-1 or k>=1 [3]

5 **(i)** Express $y = -2x^2 + 12x - 11$ in the form $y = a(x+h)^2 + k$, where a, h and k are constants. -2(x-3)^2+7 [3]

(ii) Sketch the graph of $y = -2x^2 + 12x - 11$, labelling the turning point, the line of symmetry and the y-intercept of the curve. [3]

End of Paper