Name	Index Number	Class

WOODGROVE SECONDARY SCHOOL

A COMMUNITY OF FUTURE-READY LEARNERS AND THOUGHTFUL LEADERS

N LEVEL PRELIMINARY EXAMINATION 2025

LEVEL & STREAM	:	SECONDARY 4 NORMAL (ACADEMIC)
SUBJECT (CODE)	:	ADDITIONAL MATHEMATICS (4051)
PAPER NO.	:	01
DATE (DAY)	:	29 JULY 2025 (TUESDAY)
DURATION	:	1 HOUR 45 MINUTES

MARKING SCHEME

READ THESE INSTRUCTIONS FIRST

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Student's Signature		**Parent's Signature**	
Date		**Date**	

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This document consists of **13** printed pages including this cover page.
Setters: Mdm Zuraidah

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** questions.

1 Circle C has equation $(x+1)^2+(y-3)^2=36$.

(a) Write down the radius and the coordinates of the centre of the circle. [2]

$(x+1)^2+(y-3)^2=36$

$(x-(-1))^2+(y-3)^2=6^2$

centre of circle $=(-1,\ 3)$ and radius $=6$ units [B2]

(b) Explain why any line that passes through the point $(-1,\ 1)$ will intersect circle C at two points. [1]

distance of $(-1, 1)$ from centre $=\sqrt{(-1-(-1))^2+(1-3)^2}$ or distance $=3-1$

$=2<6$ units

$=\sqrt{4}$

$=2\ <6$ units (radius of circle)

As the point $(-1, 1)$ lies inside the circle,

$\therefore$ any line that passes through it will intersect circle C at two points. [B1]

2 Given that $\cos A=\dfrac{3}{7}$ where $0\le A\le\dfrac{\pi}{2}$, find the exact value of

(a) $\sin A$, [2]

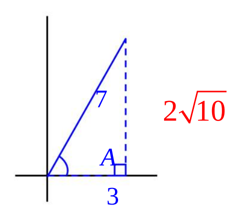
$y^2+3^3=7^2$

$y=\sqrt{7^2-3^2}$

$y=\sqrt{40}$ [M1]

$y=2\sqrt{10}$

$\sin A=\dfrac{2\sqrt{10}}{7}$ [A1]



(b) $\sin 2A$. [2]

$\sin 2A=2\sin A\cos A$

$=2\left(\dfrac{2\sqrt{10}}{7}\right)\left(\dfrac{3}{7}\right)$ [M1] or $=2\left(\dfrac{\sqrt{40}}{7}\right)\left(\dfrac{3}{7}\right)$

$=\dfrac{12\sqrt{10}}{49}$ [A1]

3 Do not use a calculator in answering this question.

Express $\dfrac{1+\sqrt{7}}{5-\sqrt{7}}$ in the form $\dfrac{a+b\sqrt{7}}{c}$, stating the values of the integers a, b and c. [4]

$$\frac{1+\sqrt{7}}{5-\sqrt{7}}=\frac{1+\sqrt{7}}{5-\sqrt{7}}\times\frac{5+\sqrt{7}}{5+\sqrt{7}}$$ [M1]

$$=\frac{5+\sqrt{7}+5\sqrt{7}+\left(\sqrt{7}\right)^2}{(5)^2-\left(\sqrt{7}\right)^2}$$ [M1]

$$=\frac{12+6\sqrt{7}}{18}$$ [A1]

$$=\frac{2+\sqrt{7}}{3}\to\frac{a+b\sqrt{7}}{c}$$

$\therefore\ a=2,\ b=1,\ c=3$ [B1]

4 (a) It is given that $y=\dfrac{4x-1}{2x+5}$ for $x\neq-\dfrac{5}{2}$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [3]

Given: $y=\dfrac{4x-1}{2x+5}$

$$\frac{dy}{dx}=\frac{(2x+5)(4)-(4x-1)(2)}{(2x+5)^2}$$ [M2]

M1: correct numerator
M1: correct denominator

$$=\frac{8x+20-8x+2}{(2x+5)^2}$$

$$=\frac{22}{(2x+5)^2}$$ [A1]

(b) Explain why y is an increasing function. [1]

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{22}{(2x+5)^2}$$

For $x\neq-\dfrac{5}{2}$, $\quad(2x+5)^2>0$

Hence $\dfrac{22}{(2x+5)^2}>0$ and y is an increasing function. [B1]

5 Express $\dfrac{6x^2-21x+25}{2x^2-5x}$ in the form $A+\dfrac{B}{x}+\dfrac{C}{2x-5}$. [5]

$$\begin{array}{r} 3 \\ 2x^2-5x\,\overline{)\,6x^2-21x+25} \\ \underline{-\left(6x^2-15x\right)} \\ -6x+25 \end{array}$$

Given: $\dfrac{6x^2-21x+25}{2x^2-5x}=A+\dfrac{B}{x}+\dfrac{C}{2x-5}$

$\dfrac{6x^2-21x+25}{2x^2-5x}=3+\dfrac{25-6x}{2x^2-5x}$ [M1]

$\dfrac{25-6x}{2x^2-5x}=\dfrac{25-6x}{x(2x-5)}=\dfrac{B}{x}+\dfrac{C}{2x-5}$ [M1]

$\therefore\quad 25-6x=B(2x-5)+Cx$

when $x=0$,

$25=B(-5)$

$B=-5$ [M1]

when $x=\dfrac{5}{2}$,

$25-6\left(\dfrac{5}{2}\right)=B\left(2\left(\dfrac{5}{2}\right)-5\right)+C\left(\dfrac{5}{2}\right)$

$10=\dfrac{5}{2}C$

$\therefore\quad C=4$ [M1]

$\therefore\quad \dfrac{6x^2-21x+25}{2x^2-5x}=3+\dfrac{-5}{x}+\dfrac{4}{2x-5}$

$=3-\dfrac{5}{x}+\dfrac{4}{2x-5}$ [A1]

Alternative method:

$6x^2-21x+25=Ax(2x-5)+B(2x-5)+Cx$ [M1]

when $x=0$, $\quad -5B=25$

$B=-5$ [M1]

when $x=\dfrac{5}{2}$, $\quad 10=\dfrac{5}{2}C$

$C=4$ [M1]

when $x=1$, $\quad 10=-3A+15+4$

$A=3$ [M1]

$\therefore\quad \dfrac{6x^2-21x+25}{2x^2-5x}=3-\dfrac{5}{x}+\dfrac{4}{2x-5}$ [A1]

6 **(a)** Express $2x^2-6x+7$ in the form $p(x-q)^2+r$, where p, q and r are constants. [3]

$$2x^2-6x+7=2\left(x^2-3x\right)+7 \quad \text{[M1]}$$

$$=2\left(x^2-3x+\left(-\frac{3}{2}\right)^2-\left(-\frac{3}{2}\right)^2\right)+7$$

$$=2\left(x-\frac{3}{2}\right)^2-2\left(-\frac{3}{2}\right)^2+7 \quad \text{[M1]} \quad \text{or} \quad 2\left[\left(x-\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2\right]+7$$

$$=2\left(x-\frac{3}{2}\right)^2+\frac{5}{2} \quad \text{[A1]}$$

(b) Hence explain why the minimum point of the curve $y=2x^2-6x+7$ is $\left(\frac{3}{2},\ \frac{5}{2}\right)$. [1]

$$2x^2-6x+7=2\left(x-\frac{3}{2}\right)^2+\frac{5}{2}$$

At the minimum point, y must be the least.

$\because \left(x-\frac{3}{2}\right)^2 \geq 0, \qquad 2x^2-6x+7 \geq \frac{5}{2}$

$\therefore \quad y \geq \frac{5}{2}$

when $y=\frac{5}{2}$, $\qquad x-\frac{3}{2}=0$

$x=\frac{3}{2}$

Hence minimum point is $\left(\frac{3}{2},\ \frac{5}{2}\right)$. [B1]

(c) State the range of values of k for which $y=k$ has no solutions. [1]

$k<2\frac{1}{2}$ [B1]

7 The volume, V cm^3, of a spherical balloon is given by $V = \frac{4}{3}\pi r^3$, where r cm is the radius of the balloon. The radius is increasing at a rate of 2 cm/s.

Calculate the rate at which the volume of the balloon is increasing when the radius is 10 cm. [4]

Given: $V = \frac{4}{3}\pi r^3$, $\frac{dr}{dt} = 2$ cm/s, $r = 10$ cm

$$\frac{dV}{dr} = 4\pi r^2 \quad \text{[M1]}$$

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$= 4\pi r^2 \times 2 \quad \text{[M1]}$$

$$= 8\pi r^2$$

When $r = 10$ cm, $\frac{dV}{dt} = 8\pi(10)^2$ [M1]

$= 800\pi$ cm^3/s [A1] or 2510 cm^3/s (3 sig fig)

8 Show that $\frac{\cos\theta}{\sec\theta + \tan\theta} = 1 - \sin\theta$. [5]

Show: $\frac{\cos\theta}{\sec\theta + \tan\theta} = 1 - \sin\theta$

$$\text{LHS} = \frac{\cos\theta}{\sec\theta + \tan\theta} = \frac{\cos\theta}{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}} \quad \text{[M1]}$$

$$= \frac{\cos\theta}{\left(\frac{1+\sin\theta}{\cos\theta}\right)}$$

$$= \frac{\cos^2\theta}{1+\sin\theta} \quad \text{[M1]}$$

$$= \frac{1-\sin^2\theta}{1+\sin\theta} \quad \text{[M1]}$$

$$= \frac{(1+\sin\theta)(1-\sin\theta)}{1+\sin\theta} \quad \text{[M1]}$$

$$= 1 - \sin\theta = \text{RHS} \quad \text{[A1]}$$

9 **(a)** Find $\int \frac{7}{\sqrt{11x+5}}\,dx.$ [3]

$$\int \frac{7}{\sqrt{11x+5}}\,dx = 7\int (11x+5)^{-\frac{1}{2}}\,dx$$

$$= 7\frac{(11x+5)^{\frac{1}{2}}}{\frac{1}{2}.11} + c \quad \text{[M2]}$$

M1: for power -1

M1: correct denominator, $\frac{1}{2} \times 11$

$$= \frac{14}{11}\sqrt{11x+5} + c \quad \text{[A1]}$$

(b) Hence evaluate $\int_1^4 \frac{14}{\sqrt{11x+5}}\,dx.$ [3]

$$\int_1^4 \frac{14}{\sqrt{11x+5}}\,dx = 2\int_1^4 \frac{7}{\sqrt{11x+5}}\,dx$$

$$= 2\left[\frac{14}{11}\sqrt{11x+5}\right]_1^4 \quad \text{[M1]}$$

$$= \frac{28}{11}\left(\sqrt{11(4)+5} - \sqrt{11(1)+5}\right) \quad \text{[M1]}$$

$$= \frac{28}{11}(7-4)$$

$$= \frac{84}{11}$$

$$= 7\frac{7}{11} \quad \text{[A1]}$$

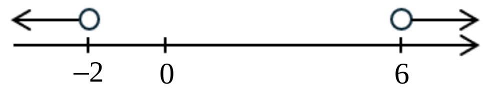
10 **(a)** Solve $x(x-4)>12$ and represent the solution set on a number line. [4]

$x(x-4)>12$

$x^2-4x-12>0$

$(x+2)(x-6)>0$ [M1]

$x<-2$ or $x>6$ [A2]



[A1]

(b) Without using a diagram, determine whether the line $y=2x+3$ and the curve $y=2x^2-5x+4$ have 0, 1 or 2 points of intersection. [4]

$y=2x+3 \quad \rightarrow (1)$

$y=2x^2-5x+4 \quad \rightarrow (2)$

Sub (1) into (2),

$2x^2-5x+4=2x+3$ [M1]

$2x^2-7x+1=0$

$D=(-7)^2-4(2)(1)$ [M1]

$=41>0$ [M1]

$\therefore$ There are 2 points of intersections. [A1]

11 **(a)** Write $2\sin\theta + 3\cos\theta$ in the form $R\sin(\theta+\alpha)$, where $R > 0$ and $0° \le \alpha \le 90°$. [3]

$2\sin\theta + 3\cos\theta = R\sin(\theta+\alpha)$

$R = \sqrt{(2)^2 + (3)^2}$ [M1] and $\alpha = \tan^{-1}\left(\frac{3}{2}\right)$ [M1]

$= \sqrt{13}$ $\qquad = 56.3099°$

$= 56.3°$ (1 dec pl)

$\therefore \quad 2\sin\theta + 3\cos\theta = \sqrt{13}\sin(\theta + 56.3°)$ [A1]

(b) Hence find the greatest value of $2\sin\theta + 3\cos\theta - 1$, giving your answer in exact value. [1]

$2\sin\theta + 3\cos\theta - 1 = \sqrt{13}\sin(\theta + 56.3°) - 1$

$\therefore$ max value of $2\sin\theta + 3\cos\theta = \sqrt{13}$

greatest value of $2\sin\theta + 3\cos\theta - 1 = \sqrt{13} - 1$ [B1]

(c) Determine the value of θ for $0° \le \theta \le 180°$ at which the greatest value occurs. [2]

Greatest value occurs

when $\sin(\theta + 56.3099°) = 1$ [M1]

$\theta + 56.3099° = 90°$

$\theta = 90° - 56.3099°$

$\theta = 33.6901°$

$\therefore \quad \theta = 33.7°$ (1 dec pl) [A1]

12 **(a)** Show that the equation $3\tan^2\theta = 10 - \sec\theta$ can be written in the form $3\sec^2\theta + \sec\theta - 13 = 0$. [2]

$$3\tan^2\theta = 10 - \sec\theta$$

$$3\left(\sec^2\theta - 1\right) + \sec\theta - 10 = 0 \quad \text{[M1]}$$

$$3\sec^2\theta - 3 + \sec\theta - 10 = 0$$

$$3\sec^2\theta + \sec\theta - 13 = 0 \quad (\text{shown}) \quad \text{[A1]}$$

(b) Hence solve $3\sec^2\theta + \sec\theta - 13 = 0$ for $0° < \theta < 360°$. [5]

$$3\sec^2\theta + \sec\theta - 13 = 0$$

$$\sec\theta = \frac{-(1) \pm \sqrt{(1)^2 - 4(3)(-13)}}{2(3)} \quad \text{[M1]}$$

$$= \frac{-1 \pm \sqrt{157}}{6}$$

$$\therefore \cos\theta = \frac{6}{-1 \pm \sqrt{157}} \quad \text{[M1]}$$

$$\cos\theta = \frac{6}{-1+\sqrt{157}} \qquad \text{or} \qquad \cos\theta = \frac{6}{-1-\sqrt{157}}$$

1st and 4th quadrants,

basic $\angle = 58.642°$

$$\theta = 58.642°, 360° - 58.642°$$

$$= 58.642°, 301.358°$$

$$= 58.6°,\ 301.4° \ (1 \text{ dec pl}) \quad \text{[A1]}$$

2nd and 3rd quadrants,

basic $\angle = 63.675°$

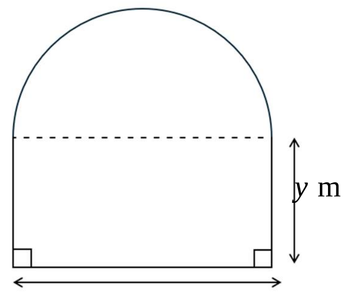
$$\theta = 180° - 63.675°, 180° + 63.675°$$

$$= 116.325°,\ 243.675°$$

$$= 116.3°,\ 243.7° \ (1 \text{ dec pl}) \quad \text{[A1]}$$

M1: correct quadrants

13 The diagram shows a windowpane which is formed by a rectangle with sides $2x$ m by y m, and a semicircle of radius x m.
The perimeter of the window is 12 m.



(a) Show that $y = \frac{1}{2}(12 - 2x - \pi x)$. [2]

Given: perimeter of windowpane $= 12$ m

$$2x + 2y + \frac{1}{2}(2\pi x) = 12 \quad \text{[M1]}$$
$$2x + 2y + \pi x = 12$$
$$2y = 12 - 2x - \pi x$$
$$\therefore \quad y = \frac{1}{2}(12 - 2x - \pi x) \quad (\text{shown}) \quad \text{[A1]}$$

(b) Hence show that the area, $A\,\text{m}^2$, of the windowpane is $A = 12x - 2x^2 - \frac{\pi x^2}{2}$. [2]

area of windowpane, $A = (2x)(y) + \frac{1}{2}(\pi x^2)$ [M1]

$$= (2x)\left(\frac{1}{2}(12 - 2x - \pi x)\right) + \frac{1}{2}\pi x^2$$
$$= 12x - 2x^2 - \pi x^2 + \frac{1}{2}\pi x^2$$
$$\therefore \quad A = 12x - 2x^2 - \frac{\pi x^2}{2} \quad (\text{shown}) \quad \text{[A1]}$$

(c) Find the value of x, correct to 2 significant figures, for which A has a stationary value. [3]

$A = 12x - 2x^2 - \dfrac{\pi x^2}{2}$

$\dfrac{dA}{dx} = 12 - 4x - \pi x$ [M1]

When $\dfrac{dA}{dx} = 0$, $12 - 4x - \pi x = 0$ [M1]

$$4x + \pi x = 12$$
$$(4 + \pi)x = 12$$
$$\therefore \quad x = \frac{12}{4+\pi}$$
$$= 1.6803$$
$$= 1.7 \ (2 \text{ sig fig})$$ [A1]

(d) Determine whether the stationary value of A is a maximum or a minimum. [2]

$\dfrac{dA}{dx} = 12 - 4x - \pi x$

$\dfrac{d^2A}{dx^2} = -4 - \pi$ [M1]

$\because \dfrac{d^2A}{dx^2} < 0$, A is maximum. [A1]

END OF PAPER