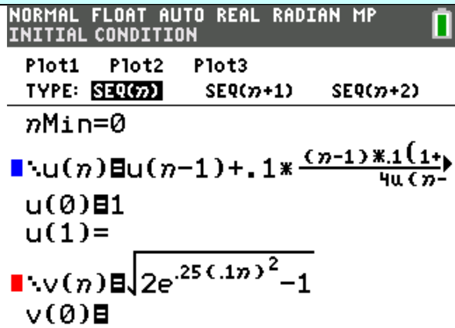
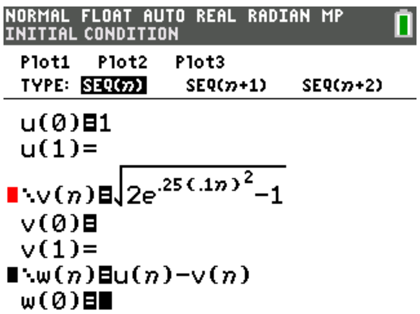
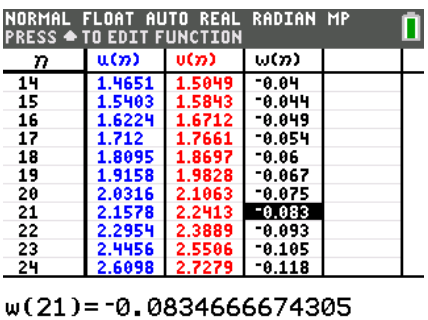
SOLUTIONS:

Qn	Suggested Solution	
1	$4y\frac{\mathrm{d}y}{\mathrm{d}x}-xy^2=x$ $\int\frac{2y}{\left(1+y^2\right)}\mathrm{d}y=\int\frac{x}{2}\mathrm{d}x$ $\ln\left(1+y^2\right)=\frac{x^2}{4}+c$ $1+y^2=A\mathrm{e}^{\frac{x^2}{4}}$, $A=\mathrm{e}^c$ When $x=0, y=1$, $A=2$ $y=\sqrt{2\mathrm{e}^{\frac{x^2}{4}}-1}$ Using Euler Method, $x_0=0, y_0=1, h=0.1$, $x_n=x_0+nh=0+n(0.1)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x\left(1+y^2\right)}{4y}=\mathrm{f}\left(x,y\right)$ $y_n=y_{n-1}+\mathrm{h}\times\mathrm{f}\left(x_{n-1},y_{n-1}\right)$ $y_n=y_{n-1}+0.1\frac{(n-1)(0.1)(1+y_{n-1}{}^2)}{4y_{n-1}}, n\ge1$, $y_0=1$	

Qn	Suggested Solution	







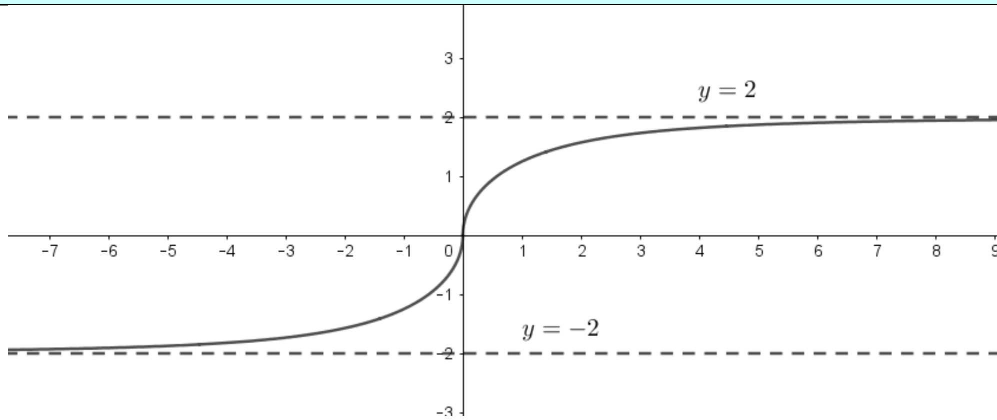
n	u(n)	v(n)	w(n)
14	1.4651	1.5049	-0.04
15	1.5403	1.5843	-0.044
16	1.6224	1.6712	-0.049
17	1.712	1.7661	-0.054
18	1.8095	1.8697	-0.06
19	1.9158	1.9828	-0.067
20	2.0316	2.1063	-0.075
21	2.1578	2.2413	-0.083
22	2.2954	2.3889	-0.093
23	2.4456	2.5506	-0.105
24	2.6098	2.7279	-0.118

n	y_n	y_{actual}	Difference
22	2.2954	2.3889	-0.0935
23	2.4456	2.5506	-0.1050

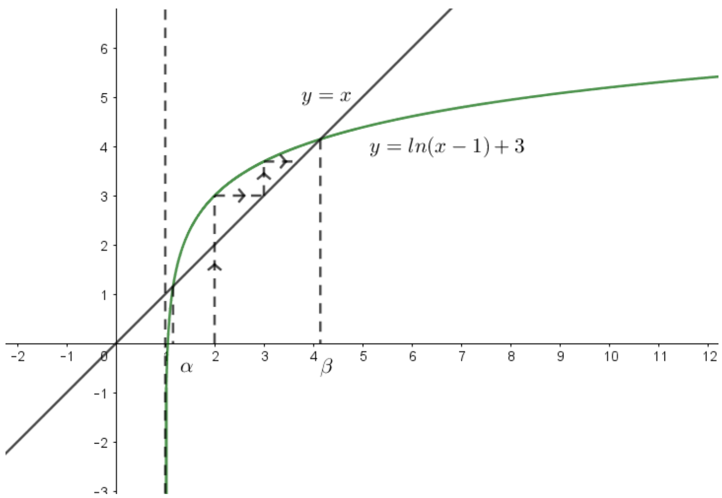
From GC, max no of iterations = 22

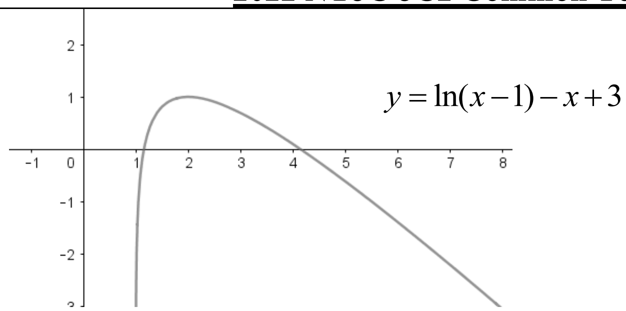
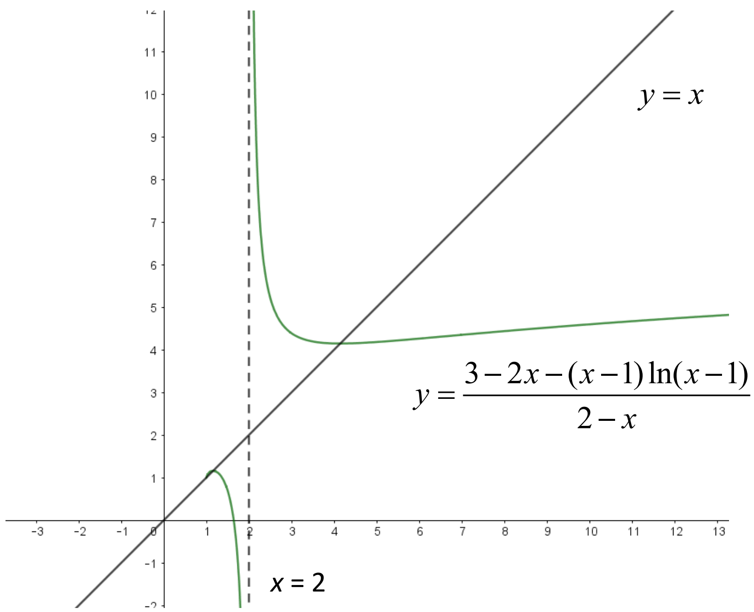
Qn	Suggested Solution	
2	**(i)** $\mathbf{0} \in V$ as $\mathbf{0} = \mathbf{0}^t$ and $\text{trace}(\mathbf{0}) = 0$ Let $\mathbf{A}_1, \mathbf{A}_2 \in V$ and $k \in \mathbb{R}$ $\left(k\mathbf{A}_1 + \mathbf{A}_2\right)^t = k\mathbf{A}_1^{\,t} + \mathbf{A}_2^{\,t} = k\mathbf{A}_1 + \mathbf{A}_2$ and $\text{trace}(k\mathbf{A}_1 + \mathbf{A}_2) = k\text{trace}(\mathbf{A}_1) + \text{trace}(\mathbf{A}_2) = 0 + 0 = 0$ Hence $k\mathbf{A}_1 + \mathbf{A}_2 \in V$. Since V is closed under vector addition and scalar multiplication, V is a subspace.	
	(ii) Note that matrices in V are of the form $\begin{pmatrix} x & y \\ y & -x \end{pmatrix} = x\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + y\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Hence a basis for V is $\left\{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right\}$. Note that matrices in W are of the form $\begin{pmatrix} 0 & a \\ 0 & b \end{pmatrix}$. Hence a basis for W is $\left\{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\right\}$. Since dim($\mathbf{M}_{2\times 2}(\mathbb{R})$) = 4 and the 4 basis matrices are independent of each other, $V + W = \mathbf{M}_{2\times 2}(\mathbb{R})$.	

Qn	Suggested Solution	
3(i)	$r = \dfrac{2\cos\theta}{\sin\theta}$ As $\theta \to 0, \pi$, $r \to \infty$, thus $y = r\sin\theta = 2\cos\theta \to 2$, if $\theta \to 0$ $y = r\sin\theta = 2\cos\theta \to -2$, if $\theta \to \pi$	

Qn	Suggested Solution	
(ii)	 $x=0$	
(iii)	When $y=\sqrt{3}$, $r\sin\theta=2\cos\theta=\sqrt{3}$, $\theta=\frac{\pi}{6}$ x-coordinates of the point of intersection $r\cos\theta=2\cot\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}\right)=3$ Area bounded by the curve from $\theta=\frac{\pi}{6}$ to $\theta=\frac{\pi}{2}$ $=\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}4\cot^2\theta\,\mathrm{d}\theta$ $=2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\operatorname{cosec}^2\theta-1\,\mathrm{d}\theta$ $=2\left[-\cot\theta-\theta\right]_{\pi/6}^{\pi/2}$ $=-2\left[\cot\theta+\theta\right]_{\pi/6}^{\pi/2}=-2\left[-\sqrt{3}+\frac{\pi}{3}\right]=2\sqrt{3}-\frac{2}{3}\pi$ Required Area $=\frac{1}{2}(3)\sqrt{3}-\left[2\sqrt{3}-\frac{2}{3}\pi\right]$ $=\frac{2}{3}\pi-\frac{\sqrt{3}}{2}$	

Qn	Suggested Solution (2025 VJC Promo Q9)	
4	(a) $F_n = a - bP_n$ and $P_n = cF_{n-1}$, Sub $F_{n-1} = a - bP_{n-1}$ into $P_n = cF_{n-1}$ $P_n = c\left(a - bP_{n-1}\right)$ $P_n = ac - bcP_{n-1}$ General Solution: $\therefore p_n = A\left(-bc\right)^n + B$ Particular solution: Consider $p_n = p_{n-1} = p$ $P = ac - bcP$ $P = \dfrac{ac}{1+bc}$ General solution: $p_n = A\left(-bc\right)^n + \dfrac{ac}{1+bc}$ $n = 0$: $A + \dfrac{ac}{1+bc} = p_0$ Solving, $A = p_0 - \dfrac{ac}{1+bc}$ $\therefore p_n = \left(-bc\right)^n\left(p_0 - \dfrac{ac}{1+bc}\right) + \dfrac{ac}{1+bc}$	
	(b) For p_n to converge, $\lvert -bc \rvert < 1$, so that $\left(-bc\right)^n \to 0$, i.e. $0 < bc < 1$ as b and c are positive constants. $P_e = \dfrac{ac}{1+bc}$	
	(c) Since P_n contains the term $\left(-bc\right)^n$ and $bc < 1$, P_n oscillates in sign and decays in magnitude. Thus, P_n converges to $\dfrac{ac}{1+bc}$ in an oscillatory manner.	

Qn	Suggested Solution	
5 (i)(a)	$x_1 = 4.15$ $x_2 = 4.14740$ $x_3 = 4.14658$ $x_4 = 4.14631$ $x_5 = 4.14623$ $\beta = 4.146$	
(i)(b)	 If the initial value is less than α, the iteration will give $x_{n+1} < 1$, then value becomes undefined as $\ln(x_{n+1} - 1)$ is undefined. If the initial value is more than α, the iteration will converge to β.	
(ii)	$\mathrm{f}(x) = \ln\left(x-1\right) - x + 3$ $\mathrm{f}'(x) = \dfrac{1}{x-1} - 1$ $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$ $y = x$ $= x_n - \dfrac{\ln(x_n - 1) - x_n + 3}{\dfrac{1}{x_n - 1} - 1}$ $\quad y = \ln(x-1)+3$ $= x_n - \dfrac{\left(x_n - 1\right)\left[\ln(x_n - 1) - x_n + 3\right]}{1 - x_n + 1}$ $= x_n - \dfrac{\left(x_n - 1\right)\left[\ln(x_n - 1) - x_n + 3\right]}{2 - x_n}$ $= \dfrac{2x_n - x_n^{\ 2} - \left(x_n - 1\right)\ln(x_n - 1) + \left(x_n - 1\right)\left(x_n - 3\right)}{2 - x_n}$ $= \dfrac{3 - 2x_n - \left(x_n - 1\right)\ln(x_n - 1)}{2 - x_n}$	

(ii)(a)	 $y=\mathrm{f}(x)$ has a turning point at $x=2$. If we take $x_1=2.2$, drawing a tangent at $x_1=2.2$, it will cuts the x-axis at value bigger than β, and subsequent iterations will converge to β. **Or**  If we start at x_1 = 2.2, then the iteration will converge to β.	
(ii)(b)	$x_1=1.2$ $x_2=1.152359$ $x_3=1.158448$ $x_4=1.158594$ $x_5=1.158594$ $\alpha=1.159$	

Qn	Suggested Solution	
6(a)	$I_n - I_{n-2} = \int \frac{\sin nx}{\sin x}\ \mathrm{d}x - \int \frac{\sin(n-2)x}{\sin x}\ \mathrm{d}x$ $= \int \frac{\sin nx - \sin(n-2)x}{\sin x}\ \mathrm{d}x$ $= \int \frac{2\cos(n-1)x\sin x}{\sin x}\ \mathrm{d}x$ $= 2\int \cos(n-1)x\ \mathrm{d}x$ $= \frac{2\sin(n-1)x}{n-1} + C \quad (*)$ Integrating factor = $\mathrm{e}^{\int -\cot x \mathrm{d}x} = \mathrm{e}^{-\int \frac{\cos x}{\sin x}\mathrm{d}x} = \mathrm{e}^{-\ln(\sin x)} = \frac{1}{\sin x}$. Multiplying the DE by IF gives $\frac{\mathrm{d}}{\mathrm{d}x}\left\{\frac{y}{\sin x}\right\} = \frac{\sin 5x}{\sin x}$. Integrating w.r.t x: $\frac{y}{\sin x} = \int \frac{\sin 5x}{\sin x}\ \mathrm{d}x = I_5$ By (*), $I_5 - I_3 = \frac{2\sin 4x}{4} + C_1 = \frac{\sin 4x}{2} + C_1$ $I_3 - I_1 = \frac{2\sin 2x}{2} + C_2 = \sin 2x + C_2$ Adding gives $I_5 - I_1 = \frac{\sin 4x}{2} + \sin 2x + C_1 + C_2$ $= \frac{\sin 4x}{2} + \sin 2x + C$ where $C = C_1 + C_2$ $I_5 = \frac{\sin 4x}{2} + \sin 2x + I_1 + C$ $= \frac{1}{2}\sin 4x + \sin 2x + \int \frac{\sin x}{\sin x}\ \mathrm{d}x + C$ $= \frac{1}{2}\sin 4x + \sin 2x + x + C$ Thus $\frac{y}{\sin x} = \frac{1}{2}\sin 4x + \sin 2x + x + C \Rightarrow y = \left(\frac{1}{2}\sin 4x + \sin 2x + x + C\right)\sin x$	

Qn	Suggested Solution	
(b)	$y=\dfrac{u}{x}\Rightarrow xy=u\overset{\text{diff w.r.t }x}{\Rightarrow} x\dfrac{\mathrm{d}y}{\mathrm{d}x}+y=\dfrac{\mathrm{d}u}{\mathrm{d}x}$ $\overset{\text{diff w.r.t }x}{\Rightarrow} x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{d}^2u}{\mathrm{d}x^2}$ Dividing the last equation by x and transferring terms, $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\dfrac{1}{x}\dfrac{\mathrm{d}^2u}{\mathrm{d}x^2}-\dfrac{2}{x}\dfrac{\mathrm{d}y}{\mathrm{d}x}$ $=\dfrac{1}{x}\dfrac{\mathrm{d}^2u}{\mathrm{d}x^2}-\dfrac{2}{x}\left[\dfrac{1}{x}\left(\dfrac{\mathrm{d}u}{\mathrm{d}x}-y\right)\right]$ $=\dfrac{1}{x}\dfrac{\mathrm{d}^2u}{\mathrm{d}x^2}-\dfrac{2}{x}\left(\dfrac{1}{x}\dfrac{\mathrm{d}u}{\mathrm{d}x}-\dfrac{u}{x^2}\right)$ $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\dfrac{1}{x}\dfrac{\mathrm{d}^2u}{\mathrm{d}x^2}-\dfrac{2}{x^2}\dfrac{\mathrm{d}u}{\mathrm{d}x}+\dfrac{2u}{x^3}$ which is the required equation. Substitute all the above into given DE gives $\dfrac{1}{x}\dfrac{\mathrm{d}^2u}{\mathrm{d}x^2}-\dfrac{2}{x}\dfrac{\mathrm{d}y}{\mathrm{d}x}+\dfrac{2}{x}\dfrac{\mathrm{d}y}{\mathrm{d}x}+25\left(\dfrac{u}{x}\right)=0$ $\dfrac{\mathrm{d}^2u}{\mathrm{d}x^2}+25u=0$ AE: $k^2+25=0\Rightarrow k=\pm 5\mathrm{i}$ Therefore GS is $u=A\cos 5x+B\sin 5x$ $\Rightarrow y=\dfrac{1}{x}\left(A\cos 5x+B\sin 5x\right)$	

Qn	Suggested Solution	

7

(i) $\begin{vmatrix} 2 & 1 & 2 \\ 0 & 2u_{n-1} & 1 \\ 2 & u_n & 2 \end{vmatrix} = u_{n-2} + 2$

$2\begin{vmatrix} 2u_{n-1} & 1 \\ u_n & 2 \end{vmatrix} + 2\begin{vmatrix} 1 & 2 \\ 2u_{n-1} & 1 \end{vmatrix} = u_{n-2} + 2$

$2(4u_{n-1} - u_n) + 2(1 - 4u_{n-1}) = u_{n-2} + 2$

$2u_n + u_{n-2} = 0$

Auxilliary equation: $2m^2 + 1 = 0 \Rightarrow m = \frac{1}{\sqrt{2}}\left(\cos\left(\frac{\pi}{2}\right) \pm i\sin\left(\frac{\pi}{2}\right)\right)$

General solution: $u_n = \left(\frac{1}{\sqrt{2}}\right)^n \left[A\cos\left(\frac{n\pi}{2}\right) + B\sin\left(\frac{n\pi}{2}\right)\right]$

When $n = 0,\ u_0 = A \Rightarrow A = 1$

When $n = 1,\ u_1 = \frac{1}{\sqrt{2}}B = \sqrt{2} \Rightarrow B = 2$

$\therefore u_n = \left(\frac{1}{\sqrt{2}}\right)^n \left(\cos\left(\frac{n\pi}{2}\right) + 2\sin\left(\frac{n\pi}{2}\right)\right).$

(ii) $u_n = \left(\frac{1}{\sqrt{2}}\right)^n \left(\cos\left(\frac{n\pi}{2}\right) + 2\sin\left(\frac{n\pi}{2}\right)\right)$

$u_n = \left(\frac{1}{\sqrt{2}}\right)^n \sqrt{5}\cos\left(\frac{n\pi}{2} - \alpha\right)$ where $\alpha = \tan^{-1} 2$

$x_n = 20\left(\sqrt{2}\right)^n u_n + \sum_{r=1}^{n}(u_r - u_{r-1}) = 20\left(\sqrt{2}\right)^n u_n + u_n - u_0$

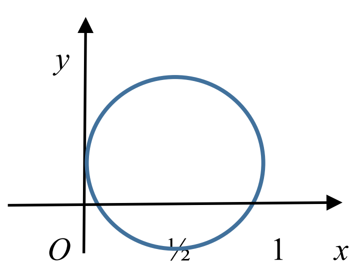
When $n \to \infty, u_n \to 0$ since $-1 \le \cos\left(\frac{n\pi}{2} - \alpha\right) \le 1$ and $\left(\frac{1}{\sqrt{2}}\right)^n \to 0$

$\therefore x_n \to 20\left(\sqrt{2}\right)^n u_n - u_0$ as $n \to \infty$

Since $20\left(\sqrt{2}\right)^n u_n = 20\sqrt{5}\cos\left(\frac{n\pi}{2} - \alpha\right)$, thus the long run, amount of

bacteria cannot exceed $20\sqrt{5} - u_0 = 20\sqrt{5} - 1$ billion since $u_0 = 1$ **A1**

Qn	Suggested Solution	
	(iii) The new recurrence relation: $x_n = 0.8x_{n-1} + 10$ Iteratively, $x_n = 0.8^{n-1}x_1 + \dfrac{10\left(1-0.8^{n-1}\right)}{1-0.8}$ $= 0.8^{n-1} + 50(1-0.8^{n-1})$ As $n \to \infty, x_n \to 50$ Amount of bacteria would approach 50 billion in the long run.	

Qn	Suggested Solution	
8	<u>Sketch locus of *P*:</u> $z = 1 - e^{i\theta}\cos\theta = 1 - (\cos\theta + i\sin\theta)\cos\theta$ $= \sin^2\theta - i\sin\theta\cos\theta$ $= \sin\theta(\sin\theta - i\cos\theta)$ Set $x = \sin^2\theta,\ \ y = -\sin\theta\cos\theta$ $y^2 = \sin^2\theta\cos^2\theta = \sin^2\theta\left(1-\sin^2\theta\right) = x(1-x) \Rightarrow \left(x-\frac{1}{2}\right)^2 + y^2 = \frac{1}{4}$. Since $0 \le \theta < \pi,\ \ 0 \le x \le 1$. $y = -\sin\theta\cos\theta = -\frac{1}{2}\sin 2\theta$. Thus $-\frac{1}{2} \le y \le \frac{1}{2}$.  Locus of *P* is a circle, centre $\left(\frac{1}{2}, 0\right)$ and radius $\frac{1}{2}$. Alternatively,	

Qn	Suggested Solution	
	$z=1-e^{i\theta}\cos\theta \Rightarrow z-\frac{1}{2}=\frac{1}{2}-e^{i\theta}\cos\theta$ $=\frac{1}{2}\left[1-e^{i\theta}\left(2\cos\theta\right)\right]$ $=\frac{1}{2}\left[1-e^{i\theta}\left(e^{i\theta}+e^{-i\theta}\right)\right]$ $=\frac{1}{2}\left(1-e^{i2\theta}-1\right)$ $=-\frac{1}{2}e^{i2\theta}$ Taking modulus, $\left\|z-\frac{1}{2}\right\|=\left\|-\frac{1}{2}e^{i2\theta}\right\|=\frac{1}{2}$ which is a circle, centre $\left(\frac{1}{2},0\right)$ and radius $\frac{1}{2}$. $z=\frac{1}{2}-\frac{1}{2}e^{i2\theta}$ $=\frac{1}{2}e^{i\theta}\left(e^{-i\theta}-e^{i\theta}\right)$ $=-2i\sin\theta\left(\frac{1}{2}e^{i\theta}\right)$ $=\sin\theta e^{\left(\theta-\frac{\pi}{2}\right)i}$ Since $0\le\theta<\pi,\quad \sin\theta>0$, thus $\|z\|=\sin\theta$, $\arg(z)=\theta-\frac{\pi}{2}$ Find modulus and argument of z: $z=\sin\theta\left(\sin\theta-i\cos\theta\right)=\sin\theta\left[\cos\left(\frac{\pi}{2}-\theta\right)-i\sin\left(\frac{\pi}{2}-\theta\right)\right]$ $=\left(\sin\theta\right)e^{-i\left(\frac{\pi}{2}-\theta\right)}$ $=\left(\sin\theta\right)e^{i\left(\theta-\frac{\pi}{2}\right)}$	

Qn	Suggested Solution	

Since $0 \le \theta < \pi$, $\sin\theta \ge 0$ and $-\frac{\pi}{2} \le \theta - \frac{\pi}{2} < \frac{\pi}{2}$.

So $z = (\sin\theta)\mathrm{e}^{\mathrm{i}\left(\theta-\frac{\pi}{2}\right)}$ is in exponential form.

$|z| = \sin\theta$ and $\arg z = \theta - \frac{\pi}{2}$.

<u>To prove summation formula:</u>

$$z^n = \left(1-\mathrm{e}^{\mathrm{i}\theta}\cos\theta\right)^n = \sum_{r=0}^{n}\left[\binom{n}{r}\left(-\mathrm{e}^{\mathrm{i}\theta}\cos\theta\right)^r\right] \text{ (Binomial Expansion)}$$

$$= \sum_{r=0}^{n}\left[(-1)^r\binom{n}{r}\mathrm{e}^{\mathrm{i}r\theta}\cos^r\theta\right]$$

$$= \sum_{r=0}^{n}\left[(-1)^r\binom{n}{r}(\cos r\theta + \mathrm{i}\sin r\theta)\cos^r\theta\right]$$

But on the other hand, $z = (\sin\theta)\mathrm{e}^{\mathrm{i}\left(\theta-\frac{\pi}{2}\right)} \Rightarrow z^n = (\sin^n\theta)\mathrm{e}^{\mathrm{i}n\left(\theta-\frac{\pi}{2}\right)}$.

Hence

$$\sum_{r=0}^{n}\left[(-1)^r\binom{n}{r}(\cos r\theta + \mathrm{i}\sin r\theta)\cos^r\theta\right] = (\sin^n\theta)\,\mathrm{e}^{\mathrm{i}n\left(\theta-\frac{\pi}{2}\right)}$$

$$\sum_{r=0}^{n}\left[(-1)^r\binom{n}{r}\cos^r\theta\cos r\theta\right] + \mathrm{i}\sum_{r=0}^{n}\left[(-1)^r\binom{n}{r}\cos^r\theta\sin r\theta\right]$$

$$= \sin^n\theta\left[\cos n\left(\theta-\frac{\pi}{2}\right) + \mathrm{i}\sin n\left(\theta-\frac{\pi}{2}\right)\right]$$

Equating real parts,

$$\sum_{r=0}^{n}\left[(-1)^r\binom{n}{r}\cos^r\theta\cos r\theta\right] = \sin^n\theta\cos n\left(\theta-\frac{1}{2}\pi\right).$$

Qn	Suggested Solution	Marking
9 (i)	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ When $x=6,\ \ y=0$ $\frac{6^2}{a^2}=1, \therefore a=6$ When $x=10, y=4$ $\frac{10^2}{6^2}-\frac{4^2}{b^2}=1$ $b^2=9$ $\therefore\ \frac{x^2}{6^2}-\frac{y^2}{3^2}=1$	
(ii)	Volume $=\pi\int_{-14}^{4} x^2 \text{ d}y$ $=36\pi\int_{-14}^{4}\left(1+\frac{y^2}{3^2}\right)\text{d}y$ $=36\pi\left[y+\frac{y^3}{3^3}\right]_{-14}^{4}$ $=36\pi\left[4-(-14)+\frac{4^3}{3^3}-\frac{(-14)^3}{3^3}\right]$ $=4392\pi$	
(iii)	$\frac{x^2}{36}-\frac{y^2}{9}=1$ Differentiate implicitly w.r.t. y, $\frac{2x}{36}\frac{\text{d}x}{\text{d}y}-\frac{2y}{9}=0$ $\frac{\text{d}x}{\text{d}y}=\frac{4y}{x}=\frac{4y}{6\left(\sqrt{1+\frac{y^2}{9}}\right)}=\frac{2y}{\sqrt{9+y^2}}$ Therefore, by GC, the length of one arc:	

Qn	Suggested Solution	Marking
	$\int_{-14}^{4}\sqrt{1+\left(\frac{\mathrm{dx}}{\mathrm{dy}}\right)^2}\,\mathrm{d}y$ $=\int_{-14}^{4}\sqrt{1+\left(\frac{2y}{\sqrt{9+y^2}}\right)^2}\,\mathrm{d}y$ $=32.838$ total perimeter of the outline is $2(32.838)+2(10)+2\left(2\sqrt{205}\right)$ $=142.95$ ≈ 143 m (to the nearest integer)	
(iv)	$\frac{x^2}{6^2}-\frac{y^2}{3^2}=1$ $\frac{2}{36}x-\frac{2}{9}y\frac{\mathrm{d}y}{\mathrm{d}x}=0$ $x=4y\frac{\mathrm{d}y}{\mathrm{d}x}$ $x\left(\frac{\mathrm{d}x}{\mathrm{d}y}\right)=4y$ Surface Area $=\int_{y=-14}^{y=4}2\pi x\sqrt{1+\left(\frac{\mathrm{d}x}{\mathrm{d}y}\right)^2}\,\mathrm{d}y$ $=\int_{y=-14}^{y=4}2\pi\sqrt{x^2+x^2\left(\frac{\mathrm{d}x}{\mathrm{d}y}\right)^2}\,\mathrm{d}y$ $=\int_{y=-14}^{y=4}2\pi\sqrt{\left(1+\frac{y^2}{9}\right)36+16y^2}\,\mathrm{d}y$ $=12\pi\int_{y=-14}^{y=4}\sqrt{\left(1+\frac{5y^2}{9}\right)}\,\mathrm{d}y$ Let $\mathrm{f}(y)=\sqrt{1+\frac{5y^2}{9}}$	(

Qn	Suggested Solution	Marking

$$h=\frac{18}{4}=\frac{9}{2}$$

y	-14	-9.5	-5	-0.5	4
f(y)	10.483	7.151	3.859	1.067	3.145

Curved Surface Area

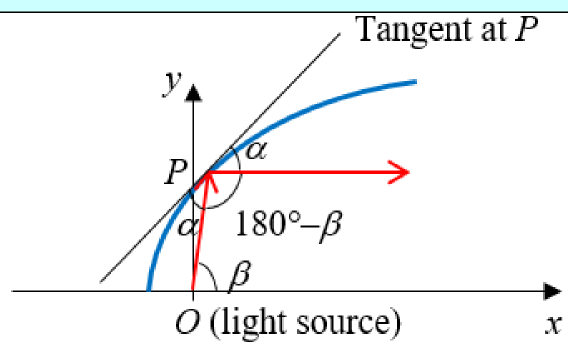
$$\approx 12\pi\left(\frac{h}{3}\right)\left(\mathrm{f}(-14)+4\mathrm{f}(-9.5)+2\mathrm{f}(-5)+4\mathrm{f}(-0.5)+\mathrm{f}(4)\right)$$

$$\approx 3065.956$$

$$=3066$$

Qn		Suggested Solution	Marking

10 (i)



$$2\alpha+180^{\circ}-\beta=180^{\circ}$$

$$\Rightarrow \beta=2\alpha$$

(ii)

$$\tan\alpha=\frac{\mathrm{d}y}{\mathrm{d}x} \text{ and } \tan\beta=\frac{y}{x}.$$

(iii)

$$\beta=2\alpha \Rightarrow \tan\beta=\tan 2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$\Rightarrow \frac{y}{x}=\frac{2\frac{\mathrm{d}y}{\mathrm{d}x}}{1-\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}$$

$$\Rightarrow \left[1-\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right]\left(\frac{y}{x}\right)=2\frac{\mathrm{d}y}{\mathrm{d}x} \quad (1)$$

	(iv)	Letting $p=\dfrac{\mathrm{d}y}{\mathrm{d}x}$, (1) becomes $yp^2+2xp-y=0$ (Quadratic eqn in p) Solving, $p=\dfrac{-2x\pm\sqrt{4x^2+4y^2}}{2y}=\dfrac{-x\pm\sqrt{x^2+y^2}}{y}$. That is, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{-x\pm\sqrt{x^2+y^2}}{y}$ $\Rightarrow y\dfrac{\mathrm{d}y}{\mathrm{d}x}+x=\pm\sqrt{x^2+y^2}$ (2)	
	(v)	Let $z=x^2+y^2 \Rightarrow \dfrac{\mathrm{d}z}{\mathrm{d}x}=2x+2y\dfrac{\mathrm{d}y}{\mathrm{d}x}$. Substituting the above into (2) gives $\dfrac{\mathrm{d}z}{\mathrm{d}x}=\pm2\sqrt{z}$ (Separable DE) $\displaystyle\int z^{-\frac{1}{2}}\mathrm{d}z=\pm2\int \mathrm{d}x$ $2\sqrt{z}=\pm2x+2C$ $z=(\pm x+C)^2=x^2\pm2Cx+C^2$ $x^2+y^2=x^2+Bx+\dfrac{1}{4}B^2$ $(B=\pm2C)$	
	(vi)	Given $y^2=Bx+\dfrac{1}{4}B^2$ When $y=0, x=-a$ $0=B(-a)+\dfrac{1}{4}B^2$ $0=-a+\dfrac{1}{4}B\Rightarrow B=4a$ $y^2=4ax+4a^2$ Using $y=r\sin\theta$, $y=r\cos\theta$ $r^2\sin^2\theta=4ar\cos\theta+4a^2$	

		$r^2\sin^2\theta - 4a\cos\theta r - 4a^2 = 0$ $r = \dfrac{4a\cos\theta \pm \sqrt{16a^2\cos^2\theta + 16a^2(\sin\theta)^2}}{2(\sin^2\theta)}$ $= \dfrac{4a\cos\theta \pm 4a}{2\sin^2\theta} = 2a\left(\dfrac{\cos\theta \pm 1}{1-\cos^2\theta}\right)$ Thus $r = 2a\left(\dfrac{\cos\theta+1}{1-\cos^2\theta}\right) \Rightarrow r = 2a\left(\dfrac{1}{1-\cos\theta}\right)$ or $r = 2a\left(\dfrac{\cos\theta-1}{1-\cos^2\theta}\right) \Rightarrow r = -2a\left(\dfrac{1}{1+\cos\theta}\right)$ (Reject as it does not satisfy when $\theta = \pi, r = a$)	