



NATIONAL JUNIOR COLLEGE
SENIOR HIGH 2 PRELIMINARY EXAMINATION
Higher 2

NAME

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SUBJECT
CLASS

2ma2

REGISTRATION
NUMBER

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MATHEMATICS

9758/01

Paper 1

15 September 2025

3 hours

Candidates answer on the Printed Answer Booklet.

Additional Materials: Printed Answer Booklet
 List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Write your name, class and registration number on the work you hand in.
Write in dark blue or black pen.

Answer **all** the questions.

Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.

You must show all necessary working clearly.

You are reminded of the need for clear presentation in your answers. Up to 2 marks may be deducted for improper presentation.

The number of marks is given in brackets [] at the end of each question or part question.

- 1 Paul is helping his friends to convert their foreign currencies back to Singapore Dollars. The amounts of foreign currencies converted, and the total amount received in Singapore Dollars are shown in the following table.

	Alex	Nicholas	Palmer	Maybelline
US Dollar (USD)	150	250	425	a
Japanese Yen (JPY)	5500	9500	1000	0
Chinese Yuan (CNY)	1000	2200	2000	1200
Total amount: Singapore Dollars (SGD)	419.30	797.20	913.10	568.40

However, he has forgotten the amount of US Dollar that Maybelline has passed to him. Assuming that, for each foreign currency, the exchange rate quoted for each of the friends is the same, calculate the value of a . [4]

2 **Do not use a calculator in answering this question.**

- (i) Find the values of z and w that satisfy the equations $(1+i)z + 2w = -2 + 4i$ and $3z - w = 4 + 2i$, expressing your answers in the form $c + di$, where $c, d \in \mathbf{R}$. [4]

- (ii) Points W and Z represent w and z found in part (i). Find $\frac{w}{z}$ in the form $p + iq$, where $p, q \in \mathbf{R}$. Hence, state the transformation that maps line segment OZ onto line segment OW . [2]

- 3 (i) On the same axes, sketch the graphs of $y = -\frac{b}{x-a}$ and $y = |x-a|$, where a and b are positive constants and $a > b > 1$. State, in terms of a and b , the coordinates of the points where the curves cross the x - and y - axes. [3]

- (ii) Hence or otherwise, solve the inequality $-\frac{b}{x-a} < |x-a|$. [4]

- 4 A curve has equation $y = f(x)$, where $f(x) = 1 - \sqrt{q^2 - x^2}$ for $q > 1$. State the shape of $y = f(x)$. [1]

- (i) Sketch the curve $y = \frac{1}{f(x)}$, giving the equations of any asymptotes and the coordinates of the end-points. [3]

- (ii) Describe the transformations that map the graph of $y = f(x)$ to $y = -\sqrt{1-x^2}$. [3]

- 5 The curve C has equation

$$y = \frac{2x^2 + kx + 8}{x + p},$$

where k and p are constants.

It is given that C has a vertical asymptote $x = 2$ and a stationary point at $x = -4$.

- (i) Find the equation of the oblique asymptote of C . [5]
- (ii) Sketch C , clearly labelling the equations of asymptotes and the coordinates of stationary points. [3]

- 6 (a) An infinite geometric series S has first term 1 and non-zero common ratio r . It is given that the sum to infinity of S is equal to the square of the sum of the first three terms of S .

- (i) Show that r satisfies the equation $r^4 + ar^3 + br^2 + cr + d = 0$, where a, b, c , and d are constants to be determined. [3]
- (ii) Find the possible values of r . [1]

- (b) An arithmetic progression with $4n$ terms has first term 7 and common difference d . Every 4th term is removed. Find the sum of the remaining terms in terms of n and d . [4]

- 7 The curve C is defined parametrically by $x = e^{4t}$, $y = t^2$, where $t \geq 0$.

- (i) Find the Cartesian equation of C . [2]
- (ii) The tangent at point P has the steepest gradient. Find the exact coordinates of P .
[You do not need to show that the gradient at P is the steepest.] [3]
- (iii) Sketch C , indicating the coordinates of P and the point where C crosses the axes clearly. [2]

- 8 In this question, you may use expansions from the List of Formulae and Results (MF27).

It is given that $a > 0$.

- (i) Find, in terms of a , the series expansion of $\frac{a}{a-x} - 1$, in ascending powers of x , up to and including the term in x^2 . State, in terms of a , the range of x for which the expansion is valid. [4]
- (ii) Hence, find the Maclaurin expansion of $e^{\frac{a}{a-x}-1}$ in ascending powers of x , up to and including the term in x^2 . [2]
- (iii) Use the expansion in part (ii) to approximate $\int_0^{\frac{a}{2}} e^{\frac{a}{a-x}-1} dx$. Explain why this approximation is an under-estimation. [4]

- 9 (a) Find $\int \cos(3 \ln x) dx$. [4]
- (b) Let I be the indefinite integral $\int \frac{P(x)}{1-\sqrt{x}} dx$, $0 < x < 1$, where $P(x)$ is a polynomial in x .
- (i) Find I when $P(x) = 1 - x$. [2]
- (ii) By using the substitution $u = 1 - \sqrt{x}$, find I when $P(x) = 1$. [3]
- Hence find I when $P(x) = x$. [2]

- 10 A sequence of numbers $u_1, u_2, u_3, \dots$ has a sum S_n , where $S_n = \sum_{r=1}^n u_r$. It is given that $S_n = A - \frac{2}{(n+1)!}$, where A is a non-zero constant.
- (i) Find the value of A if $u_1 = 1$. [1]
- (ii) Show that $u_n = \frac{2}{(n+1)[(n-1)!]}$ for $n \geq 1$. [3]
- (iii) Find a recurrence relation in the form $u_{n+1} = [f(n)]u_n$. [2]
- (iv) Explain why S_n converges as $n \rightarrow \infty$. [1]
- (v) Hence, find the least value of m such that the sum of the infinite series
- $$u_m + u_{m+1} + u_{m+2} + \dots$$
- does not exceed 10^{-10} . [3]

- 11 The function f is defined by

$$f(x) = \begin{cases} \frac{a}{2} + \frac{4}{3} \left(x - \frac{1}{2}\right)^2 & \text{for } x \in \mathbf{R}, \frac{1}{2} < x < 2, \\ \frac{a}{x} & \text{for } x \in \mathbf{R}, x \geq 2, \end{cases}$$

where a is a positive constant.

- (i) Find the range of f . [3]
- (ii) Find $f^{-1}(x)$ and state its domain. [3]

The function g is defined by $g(x) = 3 + e^x$, for $x \in \mathbf{R}$.

- (iii) Show that fg exists. [1]
- (iv) Find the exact value of k for which $fg(k) = \frac{a}{7}$. [3]

- 12 A model of a triangular canopy that provides shade outdoors is shown in **Figure 1** below.

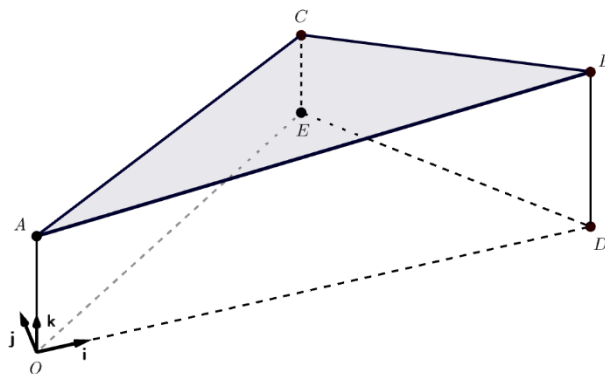


Figure 1

With point O taken as the origin, the canopy ABC is held taut using three vertical columns given by OA , DB and EC . The unit vectors $\mathbf{i}$ and $\mathbf{k}$ are defined with $\mathbf{i}$ along OD , $\mathbf{k}$ along OA , and unit vector $\mathbf{j}$ is perpendicular to both. The bases of the vertical columns are anchored to the horizontal ground ODE , which is perpendicular to OA .

- (i) State the cartesian equation of plane OAB . [1]

Points A and B have position vectors given by $\overrightarrow{OA} = 3\mathbf{k}$ and $\overrightarrow{OB} = 10\mathbf{i} + 4\mathbf{k}$. A marking, given by point M , is to be placed on the line segment AB .

- (ii) Find $\overrightarrow{OM}$ in terms of a parameter λ , stating the range of λ . [2]

Point C has position vector given by $\overrightarrow{OC} = 6\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$. The plan is to lay cables to connect M to C and then C to E . All cables are laid in straight lines and have negligible thickness.

- (iii) Explain why it is not possible for angle MCE to be 90° . [1]

With reference to **Figure 2** below, points F and G lie on lines AB and OD respectively. Party streamers, with negligible thickness, are laid in straight lines to connect E , C , F and G . The quadrilateral formed lies on a plane with cartesian equation $x - y = 2$.

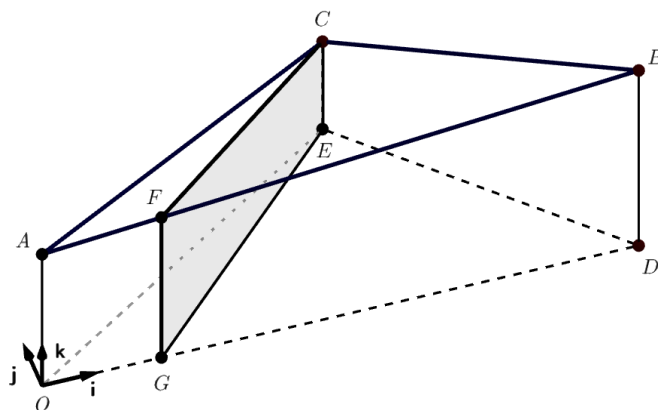


Figure 2

- (iv) Show that quadrilateral $ECFG$ is a trapezium. [3]
- (v) Find the shortest distance between F and the line CE . Hence or otherwise, find the area enclosed by the streamers. [5]