RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 5

Chapter 3: Equations and Inequalities

SYLLABUS INCLUDES

- Formulating an equation, a system of linear equations, or inequalities from a problem situation
- Solving an equation exactly or approximately using a graphing calculator
- Solving a system of linear equations using a graphing calculator
- Solving inequalities of the form $\frac{\mathrm{f}(x)}{\mathrm{g}(x)} > 0$ where $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are linear expressions or quadratic expressions
- Concept of $|x|$, and use of relations $|x-a| < b \Leftrightarrow a-b < x < a+b$ and $|x-a| > b \Leftrightarrow x < a-b$ or $x > a+b$
- Solving inequalities by graphical methods

PRE-REQUISITES

1. Sketching of simple graphs (e.g.: straight lines, quadratic, cubic, quartic, exponential and logarithmic)
2. Factorisation of quadratic and cubic expressions
3. Simultaneous equations
4. Completing the square for quadratic expressions
5. Applying the quadratic formula

CONTENT

1 Inequalities

1.1 Properties of inequalities

1.2 Solving inequalities by factorisation, completing the square, applying the quadratic formula and sketching simple graphs

1.3 Solving inequalities involving rational functions by reducing the inequalities to one involving polynomials

1.4 Solving inequalities involving modulus

2 System of Linear Equations

INTRODUCTION

Many problems in real life can be formulated by means of inequalities. They are inevitably an important part of mathematics – you will find yourself solving inequalities in many topics in mathematics. In this chapter, we shall look at some basic concepts of inequalities and the various techniques of solving them.

1 Inequalities

Inequalities are mathematical expressions involving the symbols >, <, ≥, ≤ and ≠.
In solving an inequality like f(x) > 0, where f(x) is an expression in x, we have to find all values for x such that when substituted into f(x), satisfies the given inequality.

For example, to solve $x^2 \geq 1$, we may rewrite the inequality as $x^2 - 1 \geq 0$ and proceed to factorise the LHS to get $\left(x-1\right)\left(x+1\right) \geq 0$. Thereafter, we have $x \leq -1$ or $x \geq 1$ as a solution to $x^2 - 1 \geq 0$.

We write $x^2 - 1 \geq 0 \quad \Leftrightarrow \quad x \leq -1$ or $x \geq 1$, where the symbol $\Leftrightarrow$ is used to indicate that the statements are equivalent.

Note:
"$x^2 \geq 1 \quad \Rightarrow \quad x \geq \pm 1$" is incorrect!
We **<u>CANNOT</u>** solve inequalities in the same way as we solve equations.

1.1 Properties of inequalities

The following basic concepts are useful in solving inequalities.

1. $a > b$ and $c > 0 \Rightarrow ac > bc$.	2. $a > b$ and $c < 0 \Rightarrow ac < bc$.
3. $\dfrac{a}{b} > 0 \Leftrightarrow ab > 0$.	4. $\dfrac{a}{b} < 0 \Leftrightarrow ab < 0$.

Note:

- *The inequality sign* ***remains unchanged*** *if a* ***positive*** *constant is multiplied or divided to both sides of an inequality.*
- *The inequality sign is* ***reversed*** *if a* ***negative*** *constant is multiplied or divided to both sides of an inequality.*

Example 1

Test yourself: Given a and b are non-zero real constants, state, with a reason, whether the following statements are true or false.

(a) If $a > 0,\ 5a > 4a$

True. Since $5 > 4$, multiplying by a which is positive does not change the sign.

(b) If $b < 0,\ 8b < 10b$

False. Since $8 < 10$, multiplying by b which is negative changes the sign.

(c) $\dfrac{3}{a} > 0 \quad \Rightarrow \quad 3a > 0$

True. Multiplying by a^2 which is positive does not change the sign.

(d) $\dfrac{9a}{b} < 0 \quad \Rightarrow 9ab > 0$

False. Multiplying by b^2 which is positive does not change the sign.

(e) $\dfrac{2}{b} < 1 \Rightarrow \ 2 < b$

False. Let $b = -1$.

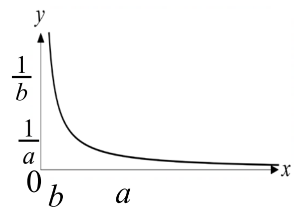
So we have $-\dfrac{2}{1} < 1$ but $2 > -1$.

(f) $a > b > 0 \quad \Rightarrow \quad 0 < \dfrac{1}{a} < \dfrac{1}{b}$

True. It can be easily proven by considering the

graph of $y = \dfrac{1}{x}$

$0 < \dfrac{1}{a} < \dfrac{1}{b}$



$0 < b < a$

1.2 Solving inequalities by factorisation, completing the square, applying the quadratic formula and sketching simple graphs

Factorisation, completing the square and applying the quadratic formula are important techniques that can help us to solve inequalities.

Knowing how to sketch simple graphs, like straight lines, graphs involving quadratic, cubic or quartic expressions in x, and exponential and logarithmic graphs can also help us to solve the inequalities.

To solve $\mathrm{f}(x) > 0$ using graphical methods, we find the set of values of x such that the graph of $y = \mathrm{f}(x)$ is above the x-axis. Similarly, to solve $\mathrm{f}(x) < 0$, it is equivalent to finding the set of values of x such that the graph of $y = \mathrm{f}(x)$ is below the x-axis.

For the next example, we will solve *quadratic* inequalities.

Example 2

Without using a calculator, solve the following inequalities:

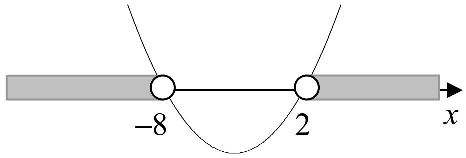
(a) $x^2 + 6x - 16 > 0$,

Solution:

(a) $x^2 + 6x - 16 > 0$

$(x + 8)(x - 2) > 0$

$x < -8$ or $x > 2$



(b) $-x^2 + 4x - 1 \geq 0$.

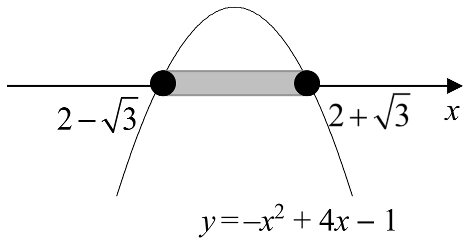
Solution:

(b) $-x^2 + 4x - 1 \geq 0$

Method 1: By using the quadratic formula

Let $-x^2 + 4x - 1 = 0$, i.e. $x^2 - 4x + 1 = 0$

$$x = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$$

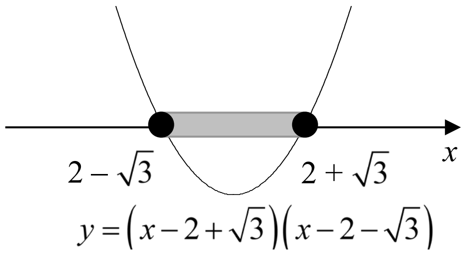


Using graph to solve $-x^2 + 4x - 1 \geq 0$,

$$2-\sqrt{3} \leq x \leq 2+\sqrt{3}$$

Method 2: By completing the square

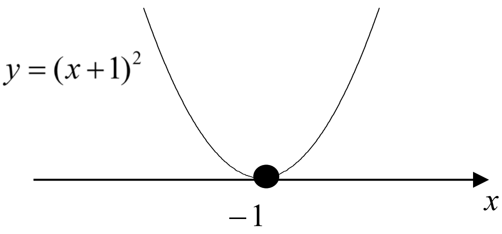
$$-x^2 + 4x - 1 \geq 0$$
$$x^2 - 4x + 1 \leq 0$$
$$x^2 - 4x + 2^2 - 3 \leq 0$$
$$(x-2)^2 - 3 \leq 0$$
$$\left(x-2+\sqrt{3}\right)\left(x-2-\sqrt{3}\right) \leq 0$$
$$2-\sqrt{3} \leq x \leq 2+\sqrt{3}$$



(c) $x^2 + 2x + 1 \le 0$.

Solution:

$$x^2 + 2x + 1 \le 0$$
$$(x+1)^2 \le 0$$
$$x = -1$$



Question: What is the solution if we are asked to solve $x^2 + 2x + 1 < 0$ instead?
No solution, so the solution set is $\varnothing$ or $\{\ \}$, *since there is no value of x that can satisfy the inequality.*

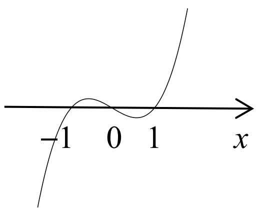
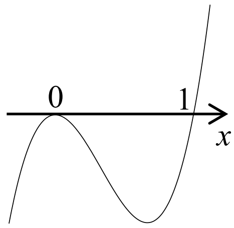
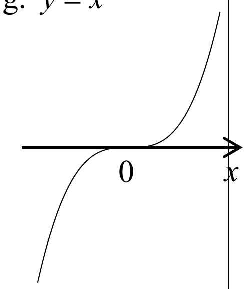
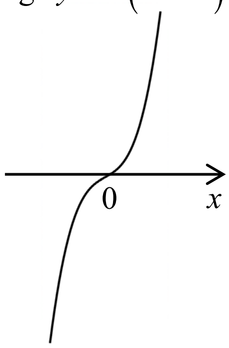
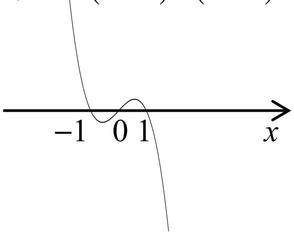
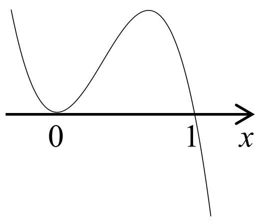
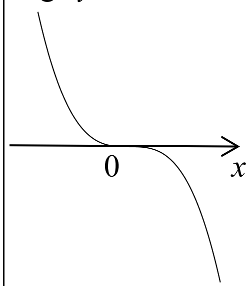
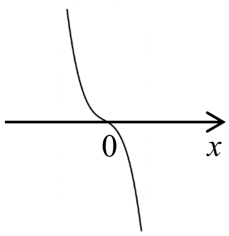
Question: How about solving $x^2 + 2x + 1 \ge 0$?
x *can be any real number. The solution set is* $\mathbb{R}$ or $(-\infty, \infty)$.

Recap on how to sketch graphs of polynomials

As seen from Example 2 above, solving inequalities involving polynomials depends largely on the behaviour of their graphs. When sketching the graph of a polynomial function with equation $y = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0$, it is often useful to factorise the polynomial function first. We will also need to consider the following factors:

- The sign of a_n (i.e. whether $a_n > 0$ or $a_n < 0$)
- The number of distinct roots of $y = 0$. In the presence of repeated roots, we are concerned with the multiplicity as well.

Let's illustrate the above with the graphs of some polynomial functions.

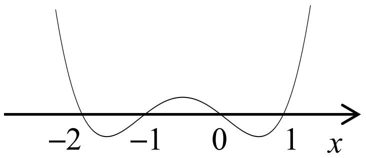
Cubic Curves of the form $y = a_3x^3 + a_2x^2 + a_1x + a_0$				
$a_3 > 0$	$y = 0$ has 3 distinct roots e.g. $y = (x-1)x(x+1)$ 	$y = 0$ has 2 distinct real roots, with 1 root being repeated twice e.g. $y = (x-1)x^2$ 	$y = 0$ has 1 real root, which is repeated thrice e.g. $y = x^3$ 	$y = 0$ has exactly 1 real root e.g. $y = x(x^2+1)$ 
$a_3 < 0$	$y = 0$ has 3 distinct roots e.g. $y = -(x-1)x(x+1)$ 	$y = 0$ has 2 distinct real roots, with 1 root being repeated twice e.g. $y = (1-x)x^2$ 	$y = 0$ has 1 real root, which is repeated thrice e.g. $y = -x^3$ 	$y = 0$ has exactly 1 real root e.g. $y = -x(x^2+1)$ 

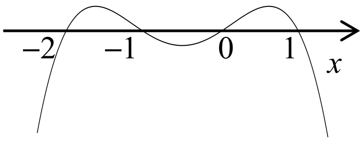
Quartic Curves of the form $y = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$.

Note that, unlike cubic curves which must have at least one real x–intercept, it is possible for quartic curves to have no x–intercept. We will consider the cases with at least one x–intercept below.

Case 1: $y = 0$ has 4 distinct roots

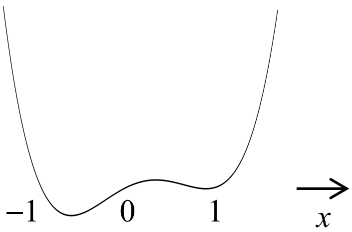
e.g. $y = (x+2)(x-1)x(x+1)$ or $y = -(x+2)(x-1)x(x+1)$

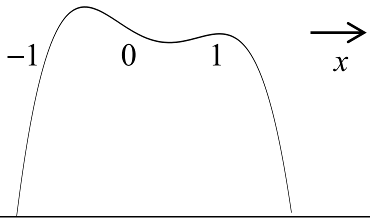




Case 2: $y = 0$ has 3 distinct roots, where 1 root is repeated twice

e.g. $y = (x-1)^2 x(x+1)$ or $y = -(x-1)^2 x(x+1)$

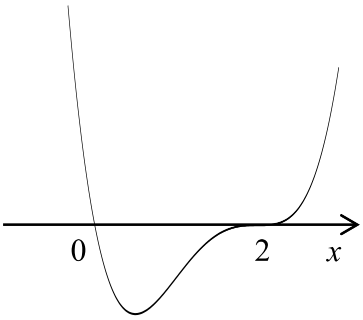


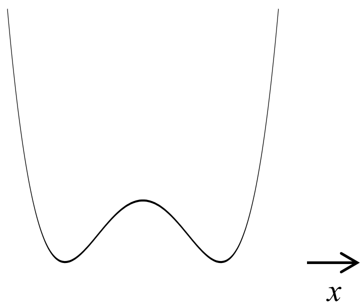


Case 3: $y = 0$ has 2 distinct roots,

where 1 root is repeated thrice or both roots are each repeated twice

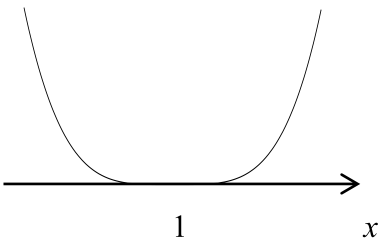
e.g. $y = (x-2)^3 x$ $\quad$ $y = (x-1)^2(x+1)^2$





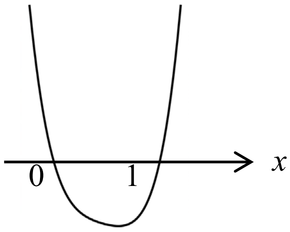
Case 4: $y = 0$ has only 1 root which is repeated four times

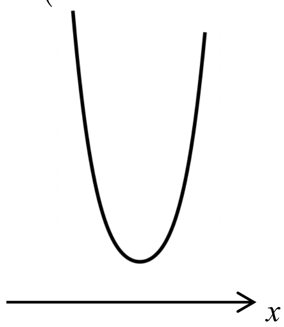
e.g. $y = (x-1)^4$



Case 5: $y = 0$ has exactly 2 real roots or no real roots

e.g. $y = x(x-1)\left(x^2+1\right)$ or $y = \left(x^2+1\right)^2$





In general, when sketching graphs of polynomial functions of the form $y = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0$ to solve inequalities, we pay special attention to the roots and the tail ends of the graph.

(a) If the polynomial $a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0$ has a repeated factor of the form $(x-k)^m$ where $m \in \mathbb{Z}^+$ and $m \geq 2,$ the graph has a stationary point at the x-intercept $x = k$.
Note: There could be other stationary point(s) as well.

(b) If n is even and $a_n > 0$, then the graph will go "up" on both ends.
If n is even and $a_n < 0$, then the graph will go "down" on both ends.
If n is odd and $a_n > 0$, then the graph will go "up" on the right-end and go "down" on the left-end.
If n is odd and $a_n < 0$, then the graph will go "down" on the right-end and go "up" on the left-end.

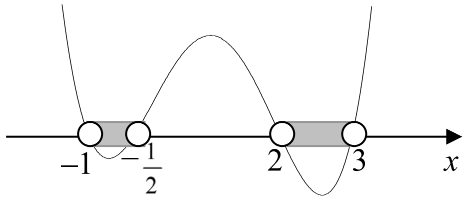
Example 3

(a) Solve the inequality $(x-2)(x+1)(2x+1)(x-3)<0$.

(b) Sketch the graph of $y=x^3-2x^2+x$. Hence, or otherwise, find the set of values of x such that $x^3-2x^2+x\le 0$.

Solution:

(a) $(x-2)(x+1)(2x+1)(x-3)<0$

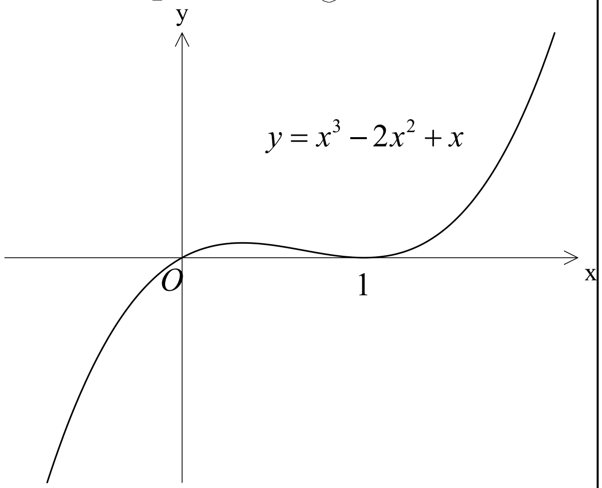


From graph, $-1<x<-\dfrac{1}{2}$ or $2<x<3$.

(b) $y=x^3-2x^2+x$

$=x\left(x^2-2x+1\right)$

$=x\left(x-1\right)^2$



The graph has x-intercepts at $x=0$ and $x=1$.
There is a repeated root at $x=1$ (stationary point).

For $x^3-2x^2+x\le 0$,
from the graph, set of values of x are $(-\infty,0]\cup\{1\}$

Otherwise Method:

$x^3-2x^2+x\le 0 \quad\Leftrightarrow\quad x\left(x-1\right)^2\le 0$

Since $\left(x-1\right)^2\ge 0$ for all $x\in\mathbb{R}$, $x\le 0$ or $\left(x-1\right)=0$.

$\Rightarrow \quad x\le 0 \quad$ or $\quad x=1$.

Set of values of x is $(-\infty,0]\cup\{1\}$.

Note: For **(b)**, the question asks for the solution to be "*set of values*". Thus, the solution should be given in **set notation** form.

1.3 Solving inequalities involving rational functions by reducing the inequalities to one involving polynomials

We shall now look at inequalities of the form $\dfrac{\mathrm{f}(x)}{\mathrm{g}(x)}>0$ (or cases involving $\ge, <, \le$) where f(x) and g(x) are linear expressions or quadratic expressions that are either factorisable or always positive. Such inequalities can be simplified to inequalities that involve polynomials of degree at most four.

Also, recall that $\dfrac{a}{b}>0 \Leftrightarrow ab>0$.

In particular, if $\dfrac{a}{b}>0$ and $a>0$ (or $b>0$), then $b>0$ (or $a>0$ respectively).

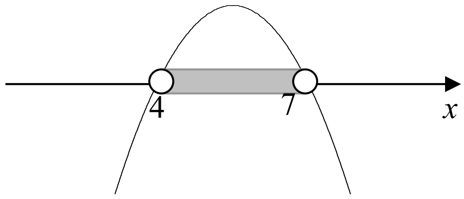
Example 4

Solve each of the following inequalities exactly.

(a) $\dfrac{7-x}{x-4} > 0,$

Solution:

(a) $\dfrac{7-x}{x-4} > 0$



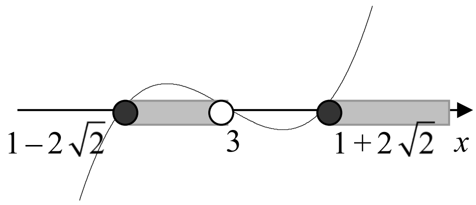
$(7-x)(x-4) > 0,\ x \neq 4$

$4 < x < 7$

(b) $\dfrac{4}{x-3} \leq x+1,$

Solution:

(b)
$$\frac{4}{x-3} \leq x+1, \quad x \neq 3$$
$$\frac{4}{x-3} - (x+1) \leq 0$$
$$\frac{4-(x+1)(x-3)}{x-3} \leq 0$$
$$\frac{7+2x-x^2}{x-3} \leq 0$$
$$(x-3)(7+2x-x^2) \leq 0, \quad x \neq 3$$
$$(x-3)(x^2-2x-7) \geq 0$$
$$(x-3)\left[(x-1)^2-8\right] \geq 0$$
$$(x-3)(x-1+2\sqrt{2})(x-1-2\sqrt{2}) \geq 0$$

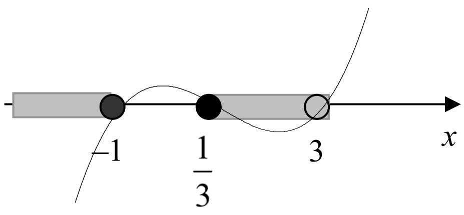


$1-2\sqrt{2} \leq x < 3$ or $x \geq 1+2\sqrt{2}$

(c) $\dfrac{4\left(x^2-1\right)}{x-3} \le x+1.$

Solution:

(c)
$$\frac{4\left(x^2-1\right)}{x-3} \le x+1, \quad x \neq 3$$
$$4\left(x^2-1\right)\left(x-3\right) \le \left(x+1\right)\left(x-3\right)^2$$
$$4\left(x+1\right)\left(x-1\right)\left(x-3\right)-\left(x+1\right)\left(x-3\right)^2 \le 0$$
$$\left(x+1\right)\left(x-3\right)\left[4\left(x-1\right)-\left(x-3\right)\right] \le 0$$
$$\left(x+1\right)\left(x-3\right)\left(3x-1\right) \le 0$$
$$x \le -1 \text{ or } \frac{1}{3} \le x < 3$$



Example 5

By completing the square, show that $4x^2-5x+3$ is always positive.

Hence solve the inequality $\dfrac{4x^2-5x+3}{(x-2)(x+1)} < 0.$

Deduce the solution of:

(a) $\dfrac{4(\ln x)^2-5\ln x+3}{(\ln x-2)(\ln x+1)} < 0,$ **(b)** $\dfrac{4x^4-5x^2+3}{(x^2-2)(x^2+1)} < 0,$

(c) $\dfrac{4\tan^2 x-5\tan x+3}{(\tan x-2)(\tan x+1)} < 0$ for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}.$

Solution:

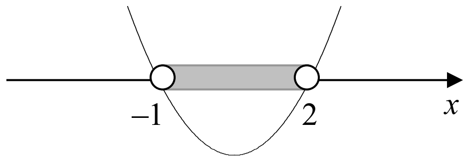
$4x^2 - 5x + 3 = 4\left(x^2 - \frac{5}{4}x\right) + 3 = 4\left[\left(x - \frac{5}{8}\right)^2 - \left(-\frac{5}{8}\right)^2\right] + 3 = 4\left(x - \frac{5}{8}\right)^2 + \frac{23}{16}$

Since $\left(x - \frac{5}{8}\right)^2 \geq 0 \ \ \forall x \in \mathbb{R}$, $4\left(x - \frac{5}{8}\right)^2 + \frac{23}{16} \geq \frac{23}{16} > 0$ so $4x^2 - 5x + 3$ is always positive.

For $\frac{4x^2 - 5x + 3}{(x-2)(x+1)} < 0$, since $4x^2 - 5x + 3$ is always positive, the inequality reduces to

$(x - 2)(x + 1) < 0$

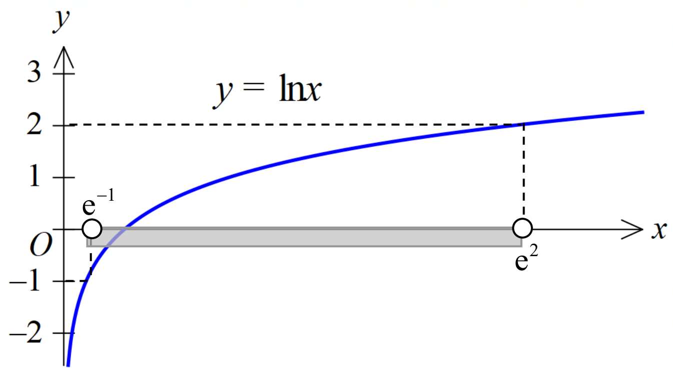
$-1 < x < 2$



(a) $\frac{4(\ln x)^2 - 5\ln x + 3}{(\ln x - 2)(\ln x + 1)} < 0$

By replacing x with $\ln x$:
$-1 < \ln x < 2$ (from first part)

From the graph of $y = \ln x$,
when $-1 < y < 2$,
we have $e^{-1} < x < e^2$.

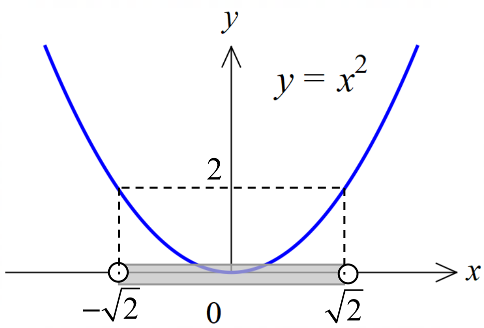


(b) $\frac{4x^4 - 5x^2 + 3}{(x^2 - 2)(x^2 + 1)} < 0$

$\frac{4\left(x^2\right)^2 - 5x^2 + 3}{(x^2 - 2)(x^2 + 1)} < 0$

By replacing x with x^2:
$-1 < x^2 < 2$ (from first part)

From the graph of $y = x^2$, when $-1 < y < 2$
we have $-\sqrt{2} < x < \sqrt{2}$.

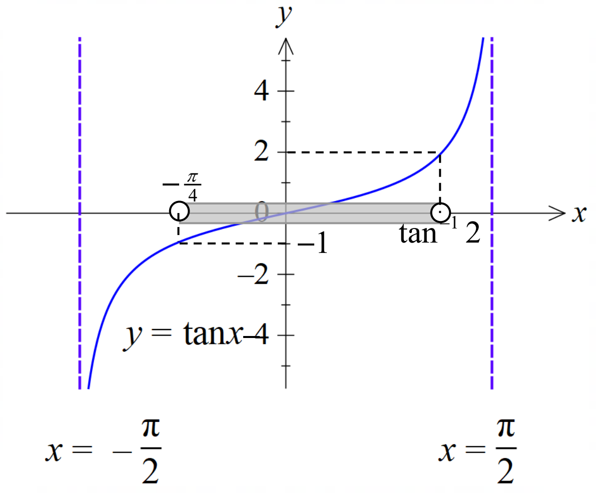


(c) $\dfrac{4\tan^2 x - 5\tan x + 3}{(\tan x - 2)(\tan x + 1)} < 0$ for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$.

By replacing x with $\tan x$:
$-1 < \tan x < 2$ (from first part)

From the graph of $y = \tan x$,
when $-1 < y < 2$:

we have $-\dfrac{\pi}{4} < x < \tan^{-1} 2$.



1.4 Solving inequalities involving modulus

Some inequalities appear in conjunction with the modulus, or absolute value symbol $| \quad |$.

The absolute value of a real number x, denoted by $|x|$, is defined by

$$|x| = \begin{cases} x, & x \geq 0, \\ -x, & x < 0. \end{cases}$$

Thus $y = |\mathrm{f}(x)| = \begin{cases} \mathrm{f}(x), & \mathrm{f}(x) \geq 0, \\ -\mathrm{f}(x), & \mathrm{f}(x) < 0. \end{cases}$

Some useful results involving modulus:

For any positive constant *a*,

1. $|x| < a \Leftrightarrow -a < x < a$.
2. $|x| > a \Leftrightarrow x < -a$ or $x > a$.
3. $|x| \leq a \Leftrightarrow -a \leq x \leq a$
4. $|x| \geq a \Leftrightarrow x \leq -a$ or $x \geq a$.

In general, for any positive constant *b*,

1. $|x-a| < b \Leftrightarrow -b < x-a < b$.
   i.e. $|x-a| < b \Leftrightarrow a-b < x < a+b$
2. $|x-a| > b \Leftrightarrow x-a < -b$ or $x-a > b$.
   i.e. $|x-a| > b \Leftrightarrow x < a-b$ or $x > a+b$

Example 6

Solve the following inequalities:

(a) $|2x-7| < 3$ **(b)** $|3x-2| \geq 4$

Solution:

(a)
$$|2x-7| < 3$$
$$-3 < 2x-7 < 3$$
$$2 < x < 5$$

(b)
$$|3x-2| \geq 4$$
$$3x-2 \leq -4 \quad \text{or} \quad 3x-2 \geq 4$$
$$x \leq -\frac{2}{3} \quad \text{or} \quad x \geq 2$$

To solve the inequality $f(x) \geq g(x)$ is equivalent to finding the range of values of x such that the graph of $y = f(x)$ is above or intersects the graph of $y = g(x)$.

Example 7

By sketching appropriate graphs on a single diagram, solve the following inequalities, leaving your answers in exact form.

(a) $|x-2| \geq 2x - \dfrac{11}{10}$,

Solution:

(a) $|x-2| \geq 2x - \dfrac{11}{10}$

Note that $|x-2| = \begin{cases} x-2, & x \geq 2 \\ -(x-2), & x < 2 \end{cases}$

From the graphs of $y = |x-2|$ and $y = 2x - \dfrac{11}{10}$, it can be seen that the x-coordinate of the point of intersection of the 2 graphs is less than 2. Thus, we write $|x-2| = -(x-2)$ at the point of intersection.

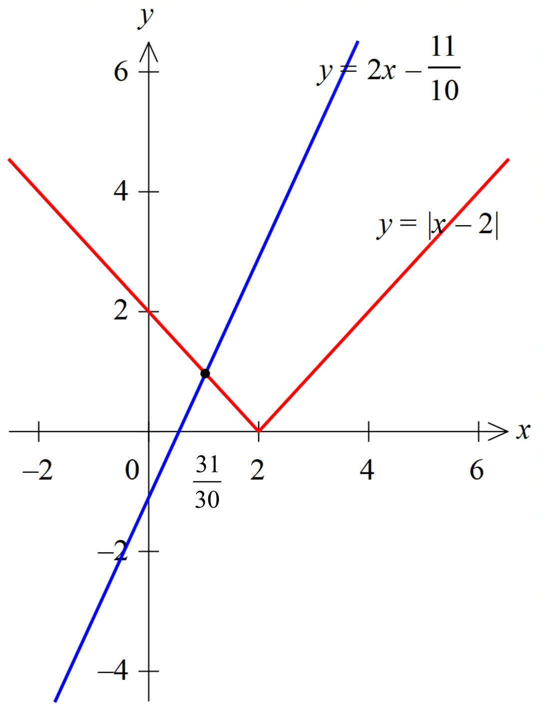
At the point of intersection,

$$-(x-2) = 2x - \frac{11}{10}$$

$$3x = \frac{31}{10}$$

$$x = \frac{31}{30}$$

The solution to the inequality is $x \leq \dfrac{31}{30}$



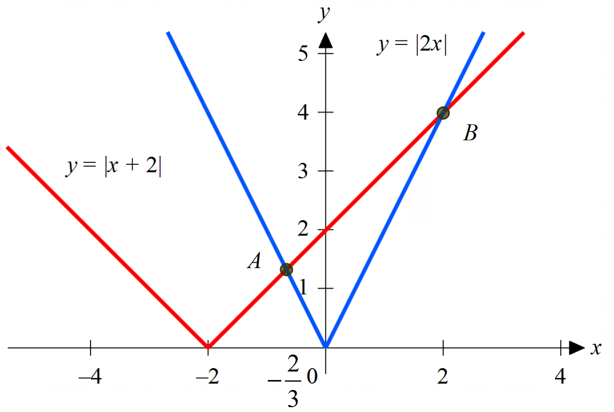
From the above graphs, it can be seen that when $x \leq \frac{31}{30}$, the graph of $y = |x-2|$ is "above" the graph of $y = 2x - \frac{11}{10}$.

(b) $|x+2| \le |2x|$.

Solution:

(b) $|x+2| \le |2x|$

Let A and B be the points of intersection of the two graphs.



At A, $x+2=-2x$

$3x=-2 \Rightarrow x=-\frac{2}{3}$.

At B, $x+2=2x \Rightarrow x=2$.

The solution to the inequality is $x \le -\frac{2}{3}$ or $x \ge 2$.

Example 8 [RJC JC1 Term 3 CT 9740/2006/Q4]

On a single clearly labeled diagram, sketch the graphs of $y=(x-1)(x-3)$ and $y=|x-4|$. Hence solve the inequality $(x-1)(x-3)>|x-4|$, giving your answer in exact form.

Solution:

Note that the relative position of the two graphs is crucial to solving the inequality, so do draw the graphs carefully.

At intersection points,

$(x-1)(x-3)=-(x-4)$

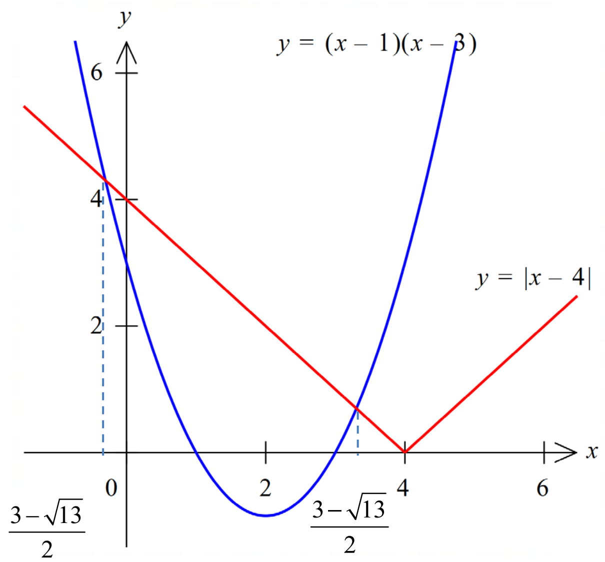
$x^2-4x+3=4-x$

$x^2-3x-1=0$

$$x=\frac{3\pm\sqrt{9-4(-1)}}{2}=\frac{3\pm\sqrt{13}}{2}.$$

For $(x-1)(x-3)>|x-4|$,

$x<\frac{3-\sqrt{13}}{2}$ or $x>\frac{3+\sqrt{13}}{2}$.



2 System of Linear Equations

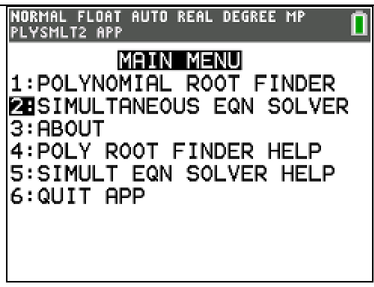
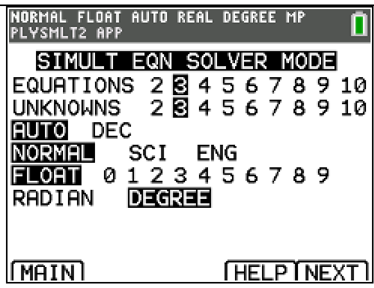
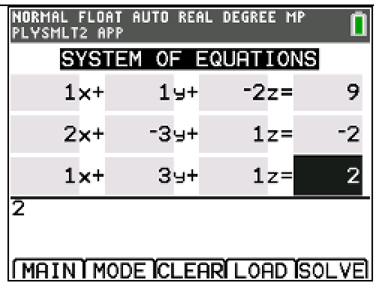
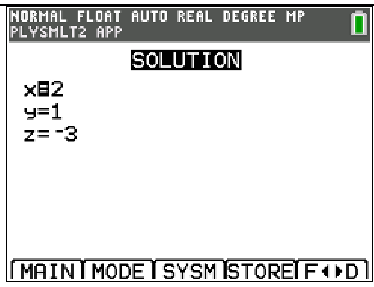
We already know how to solve a system of two linear equations in two unknowns by substitution or elimination. How about solving a system of three linear equations in three unknowns? Certainly the usual methods of substitution and elimination will work. Here the GC allows us to do it a little faster.

Consider the following equations:

$$\begin{aligned} x+y-2z&=9 \\ 2x-3y+z&=-2 \\ x+3y+z&=2 \end{aligned}$$

Using the **Simultaneous Eqn Solver** of the **PlySmlt2** app in the GC, we obtain

$x=2,\ y=1,\ z=-3.$

1. Press apps and select **PlySmlt2**. 2. Select **2: Simultaneous Eqn Solver**.	
3. Adjust the settings as depicted on the screen to solve 3 equations with 3 unknowns. 4. Select **NEXT** by pressing the graph button.	
5. Enter the coefficients into the equations. 6. Press graph to **SOLVE** the system of equations.	
7. Read off the answers. Hence $x=2,\ y=1,\ z=-3$.	

You will also be asked to formulate the equations from a problem situation.

Example 9
The graph of $y = x^3 + ax^2 + bx + c$ passes through $(-3, 1)$, $(1, -5)$ and $(2, -3)$. Find the values of a, b and c.

Solution
At $(-3, 1)$, we have $1 = (-3)^3 + a(-3)^2 + b(-3) + c \Rightarrow 9a - 3b + c = 28.$

At $(1, -5)$, we have $-5 = 1 + a + b + c \Rightarrow a + b + c = -6.$

At $(2, -3)$, we have $-3 = 8 + 4a + 2b + c \Rightarrow 4a + 2b + c = -11.$

From GC, $a = 0.7,\ b = -7.1,\ c = 0.4$.

Example 10 (SAJC 9740/2008/01/Q5)
Four housewives go to the supermarket together and each buy three different types of ham: picnic ham, honey-baked ham and turkey ham. The weights of the ham and the total amount paid by each of the four housewives are shown in the following table.

	Mrs Lee	Mrs Tan	Mrs Goh	Mrs Teo
Picnic Ham (kg)	1.50	0.50	2.45	1.30
Honey-baked Ham (kg)	0.60	2.00	1.45	0.90
Turkey Ham (kg)	2.25	1.65	0.80	2.10
Total amount paid in $	4.89	4.87	4.79	M

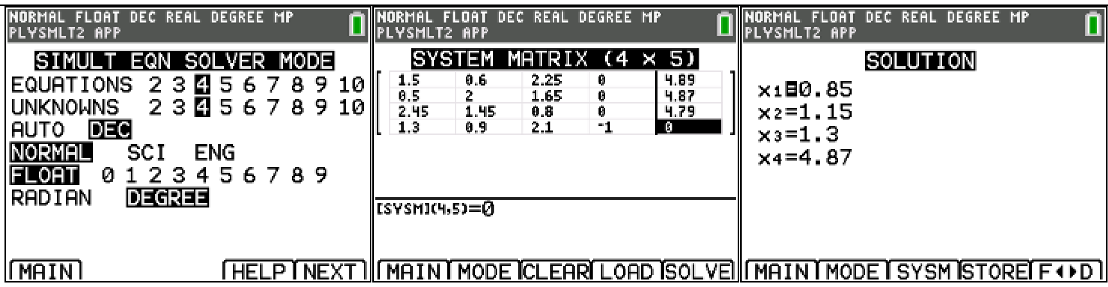
Assuming that, for each type of ham, the price per kilogram paid by each housewife is the same. Calculate the total amount M that Mrs Teo paid.

Solution
Let $\$P$, $\$H$, $\$T$ be the price of a kg of picnic ham, honey-baked ham and turkey ham respectively.

$$1.5P + 0.6H + 2.25T = 4.89 \quad \cdots(1)$$
$$0.5P + 2H + 1.65T = 4.87 \quad \cdots(2)$$
$$2.45P + 1.45H + 0.8T = 4.79 \quad \cdots(3)$$
$$1.3P + 0.9H + 2.1T = M \quad \Leftrightarrow \quad 1.3P + 0.9H + 2.1T - M = 0 \quad \cdots(4)$$

From GC, $M = 4.87$

$\therefore$ Total amount paid by Mrs Teo is \$4.87.



Example 11

A company wants to lease 25 trucks with a combined capacity of 800 cubic metres. Three different types of trucks are available: 4.2-metre trucks with capacity of 10 cubic metres, 5.3-metre trucks with capacity of 20 cubic metres, and 6.4-metre trucks with capacity of 40 cubic metres.

(i) What are the possible combinations of the number of each type of truck that the company could lease?

(ii) If the rent is \$115 per day for a 4.2-metre truck, \$150 per day for a 5.3-metre truck and \$200 per day for a 6.4-metre truck, how many of each type of truck should the company now lease to achieve a minimum daily leasing cost?

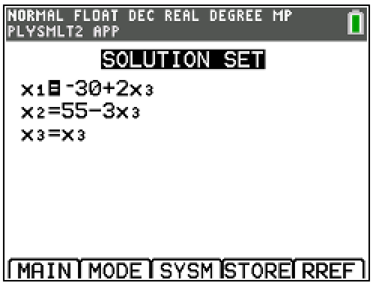
Solution

(i) If x, y and z represents the number of 4.2-metre, 5.3-metre and 6.4-metre trucks leased respectively, we set up the following pair of equations with the information provided.

$$x + y + z = 25$$
$$10x + 20y + 40z = 800$$

Solving in terms of z we obtain

$$x = 2z - 30$$
$$y = 55 - 3z$$



Since x, y and z are non-negative integers,

$x = 2z - 30 \geq 0 \quad \Rightarrow \quad z \geq 15$

and $y = 55 - 3z \geq 0 \quad \Rightarrow \quad z \leq 18$.

Hence $15 \leq z \leq 18$.

For these four values of z, use the above solution to get the corresponding values of x and y.

The possible combinations for (x, y, z) are $(0,10,15), (2,7,16), (4,4,17)$ and $(6,1,18)$.

(ii) To calculate the total rental charges for each type of truck:

Possible combination (x, y, z)	Total rental charge: $115x + 150y + 200z$
$(0,10,15)$	4500
$(2,7,16)$	4480
$(4,4,17)$	4460
$(6,1,18)$	4440

For the lowest rental charge, the company should lease 6, 1 and 18 of the 4.2-metre, 5.3-metre and 6.4-metre trucks respectively.

SUMMARY