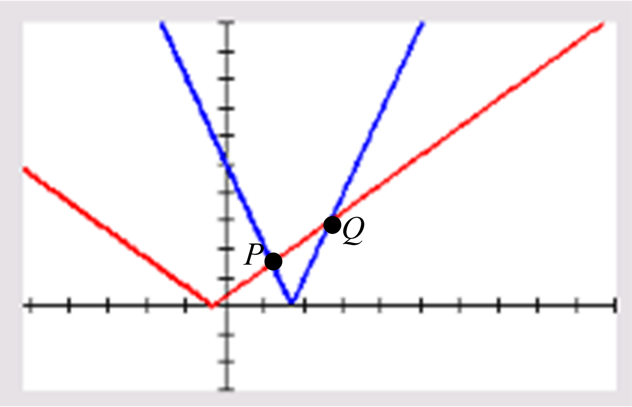
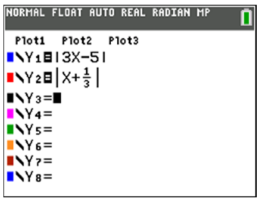
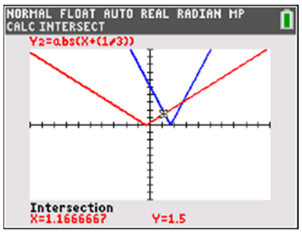
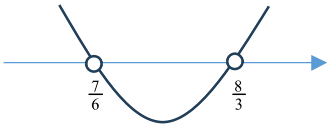
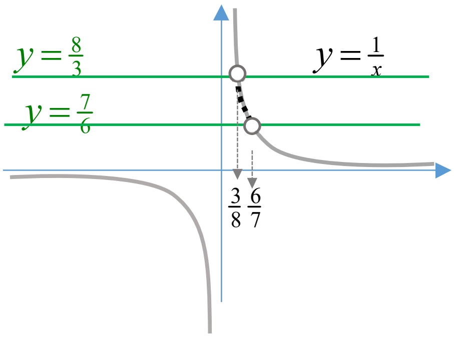
1 Solve the inequality $|3x-5| > \left|x+\frac{1}{3}\right|$. [3]

Hence solve the inequality $\left|\frac{3-5x}{x}\right| > \left|\frac{3+x}{3x}\right|$. [2]

Qn1 **[5]**	**Suggested Solutions**	**Comments** **[Re-visit Tut 3 Q2b, Q6]**
(a)	**Method 1A: Graphical Approach** $\lvert 3x-5\rvert > \left\lvert x+\frac{1}{3}\right\rvert$  To find *P*: $-(3x-5) = x+\frac{1}{3} \Rightarrow x = \frac{7}{6}$ To find *Q*: $3x-5 = x+\frac{1}{3} \Rightarrow x = \frac{8}{3}$ Therefore, from the sketch, $\lvert 3x-5\rvert > \left\lvert x+\frac{1}{3}\right\rvert$ $x < \frac{7}{6}$ or $x > \frac{8}{3}$	This question was not well done. Many students attempted to solve the question by considering the various cases (avoid this approach) and made many errors with inequality signs and/or algebraic manipulations. A **graphical approach** would have been the most straightforward way to handle the two modulus functions.  NORMAL FLOAT AUTO REAL RADIAN MP Plot1 Plot2 Plot3 \Y1=|3X−5| \Y2=|X+1/3| \Y3= \Y4= \Y5= \Y6= \Y7= \Y8= The graphs visually provide us with the correct inequality after obtaining the correct point of intersection either through GC or algebraic manipulation of an equation.  NORMAL FLOAT AUTO REAL RADIAN MP CALC INTERSECT Y2=abs(X+(1/3)) Intersection X=1.1666667 Y=1.5
	Method 2: Algebraical Approach $\lvert 3x-5\rvert > \left\lvert x+\frac{1}{3}\right\rvert$ $(3x-5)^2 > \left(x+\frac{1}{3}\right)^2$ $9x^2-30x+25 > x^2+\frac{2}{3}x+\frac{1}{9}$ $8x^2-\frac{92}{3}x+\frac{224}{9} > 0$ $72x^2-276x+224 > 0$	This question allows the flexibility to **solve algebraically** and since both sides of the inequalities are always positive, we can square both sides to "remove" the modulus.

Qn1 **[5]**	**Suggested Solutions**	**Comments** **[Re-visit Tut 3 Q2b, Q6]**
	$18x^2-69x+56>0$ $(3x-8)(6x-7)>0$ Critical Values: $x=\frac{-(-276)\pm\sqrt{(-276)^2-4(72)(224)}}{2(72)}$ $x=\frac{276\pm\sqrt{11664}}{144}$ $x=\frac{8}{3},\frac{7}{6}$  $x<\frac{7}{6}$ or $x>\frac{8}{3}$	
(b)	Replace x in previous part with $\frac{1}{x}$, $\left\lvert 3\left(\frac{1}{x}\right)-5\right\rvert>\left\lvert\left(\frac{1}{x}\right)+\frac{1}{3}\right\rvert$ $\left\lvert\frac{3-5x}{x}\right\rvert>\left\lvert\frac{3+x}{3x}\right\rvert$ **<u>Method 1</u>**: Use Sketch  $\frac{1}{x}<\frac{7}{6}$ or $\frac{1}{x}>\frac{8}{3}$ $x<\frac{3}{8}$ or $x>\frac{6}{7}$, $x\neq 0$ **<u>Method 2</u>**:	For the 'hence' part, a large number of students did not read the question carefully and tried to **solve this part from scratch and thus did not earn any credit**. It is important to note that inequalities found included $x=0$ but our replacement involves replacing x with $\frac{1}{x}$ where $x\neq 0$ Most students therefore **left out** $x\neq 0$ from their solution and hence mark was lost.

Qn1 **[5]**	**Suggested Solutions**	**Comments** **[Re-visit Tut 3 Q2b, Q6]**
	$\frac{1}{x}<\frac{7}{6}$ or $\frac{1}{x}>\frac{8}{3}$ $\frac{1}{x}-\frac{7}{6}<0$ or $\frac{1}{x}-\frac{8}{3}>0$ $\frac{6-7x}{6x}<0$ or $\frac{3-8x}{3x}>0$ $x<0$ or $x>\frac{6}{7}$ or $0<x<\frac{3}{8}$ $x<\frac{3}{8}$ or $x>\frac{6}{7}$, $x\neq 0$	

2 The curve C has equation $y^3+2x^2y=1$ and it cuts the y-axis at point N.

(a) Show that

$$\left(\alpha y^2+\beta x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\mu y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+\lambda x\frac{\mathrm{d}y}{\mathrm{d}x}+\gamma y=0$$

where α, β, μ, λ and γ are constants to be determined. [3]

(b) Find the values of $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ at N and comment on the nature of N. [3]

Qn2 **[6]**	**Suggested Solutions**	**Comments** **[Re-visit Tut 5A Q5]**
(a)	$y^3+2x^2y=1$ --- (1) Differentiate (1) throughout w.r.t x : $3y^2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)+2x^2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)+4xy=0$ --- (2) Differentiate (2) throughout w.r.t x : $3y^2\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)+6y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+2x^2\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)+4x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)+4x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)+4y=0$ --- (3) $\left(3y^2+2x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+6y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+8x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)+4y=0$ $\alpha=3, \beta=2, \mu=6, \lambda=8$ and $\gamma=4$ (shown)	Most students were able to differentiate equation (1) using **implicit differentiation** and **product rule**, despite some careless mistakes. Some students tried to **express $\frac{\mathrm{d}y}{\mathrm{d}x}$ as the subject** (this **must be avoided** as question did not require) and **use quotient rule** for further differentiation. The algebraic manipulation will then be **too tedious** for them to get the answer. Students should take note that there is **no need to have $\frac{\mathrm{d}y}{\mathrm{d}x}$ as the subject** before further

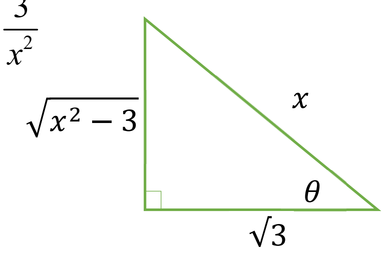
Qn2 **[6]**	**Suggested Solutions**	**Comments** **[Re-visit Tut 5A Q5]**
		differentiation. It is advised to use product rule rather than quotient rule for further differentiation. Some students had problems differentiating $3y^2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ and $2x^2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ using **product rule** and **implicit differentiation**.
	When curve cuts the y-axis at N, $x=0$ $y^3=1 \quad \Rightarrow \quad y=1$ Substitute $(0,1)$ into (2): $3\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=0 \quad \Leftrightarrow \quad \frac{\mathrm{d}y}{\mathrm{d}x}=0$ Substitute $(0,1)$ and $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ into (3): $3\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)+4=0$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{4}{3}<0$ Therefore $N(0,1)$ is a maximum point.	Many students did not substitute $x=0$ into $y^3+2x^2y=1$ to find the y-intercepts and therefore could not find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ and hence could not find the value of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

3 **(a)** **(i)** Write down $\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^{\cos 2x}$. [1]

(ii) Hence find $\int (\sin 2x)\,\mathrm{e}^{\cos 2x+\mathrm{e}^{\cos 2x}}\,\mathrm{d}x$. [2]

(b) Use the substitution $x=\sqrt{3}\sec\theta$ to find $\int \frac{1}{x^2\sqrt{x^2-3}}\,\mathrm{d}x$ where $0\le\theta<\frac{\pi}{2}$. [4]

Qn3 **[7]**	**Suggested Solutions**	**Comments** **[Re-visit Tut 6A Q 3e, Q4]**
(a) **(i)**	$\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^{\cos 2x}=(-2\sin 2x)\,\mathrm{e}^{\cos 2x}$	Generally, well done. Most mistakes are careless in the chain rule, omitting either the negative or the '2'.
(ii)	$\int (\sin 2x)\,\mathrm{e}^{\cos 2x+\mathrm{e}^{\cos 2x}}\,\mathrm{d}x$ $=-\frac{1}{2}\int\left[-2(\sin 2x)\mathrm{e}^{\cos 2x}\right]\mathrm{e}^{\mathrm{e}^{\cos 2x}}\,\mathrm{d}x$ $=-\frac{1}{2}\mathrm{e}^{\mathrm{e}^{\cos 2x}}+C$	Most common mistake is taking the **wrong approach**: students attempting to **do by-parts✗ or substitution✗.** There is **no credit** for any **attempt** to do **using the wrong method/approach**. When attempting integration, the first thing should be to consider if it's of the form $\mathrm{f}'(x)\left[\mathrm{f}(x)\right]^n$ or in this case, $\mathrm{f}'(x)\mathrm{e}^{\mathrm{f}(x)}$. **The number of marks can give a clue.** By-parts or substitution Questions will usually carry more marks.
(b)	$x=\sqrt{3}\sec\theta$ $\frac{\mathrm{d}x}{\mathrm{d}\theta}=\sqrt{3}\sec\theta\tan\theta$ $\int\frac{1}{x^2\sqrt{x^2-3}}\,\mathrm{d}x$ $=\int\frac{\cos^2\theta}{3\sqrt{3}\sqrt{\sec^2\theta-1}}\left(\sqrt{3}\sec\theta\tan\theta\right)\mathrm{d}\theta$ $=\int\frac{\cos^2\theta}{3\sqrt{3}\sqrt{\tan^2\theta}}\left(\sqrt{3}\sec\theta\tan\theta\right)\mathrm{d}\theta$ $=\int\frac{\cos^2\theta}{3\tan\theta}(\sec\theta\tan\theta)\,\mathrm{d}\theta$ Since $0\le\theta<\frac{\pi}{2}$, $\sqrt{\tan^2\theta}=\lvert\tan\theta\rvert=\tan\theta$	This question was generally **not well done**. A number of students were **unaware that the derivative of** $\sec\theta$ **is provided in MF27**. Some attempted to differentiate $\sec\theta=(\cos\theta)^{-1}$ using **chain rule** (which is acceptable), but **many omitted the chain rule**, leading to incorrect subsequent steps. Several students also **could not recall the trigonometric identity** $\sec^2\theta-1=\tan^2\theta$ ✓, or recall it wrongly as $\sec^2\theta-1=\cos ec^2\theta$ ✗

Qn3 **[7]**	**Suggested Solutions**	**Comments** **[Re-visit Tut 6A Q 3e, Q4]**
	$=\int \frac{1}{3}\cos\theta \, \mathrm{d}\theta$ $=\frac{1}{3}\sin\theta + C$ Since $x=\sqrt{3}\sec\theta \Leftrightarrow \cos\theta=\frac{\sqrt{3}}{x}$ $\sin\theta=\sqrt{1-\cos^2\theta}=\sqrt{1-\frac{3}{x^2}}$  $\therefore \int \frac{1}{x^2\sqrt{x^2-3}} \, \mathrm{d}x$ $=\frac{1}{3}\sqrt{1-\frac{3}{x^2}}+C$ $=\frac{\sqrt{x^2-3}}{3x}+C$	and was thus unable to continue after doing the substitution. In addition, many students **wrongly obtained** $\int \sec\theta \, \mathrm{d}\theta$ ✗ and did not know what to do subsequently. Some **common errors**✗: • $\int \frac{1}{\sec\theta} \mathrm{d}\theta = \int \frac{\sqrt{3}}{x} \mathrm{d}x = \sqrt{3}\ln\lvert x\rvert + c$ ✗ • **wrongly changed the given conditions or misread the question**✗ by changing it into a **definite integral** question using the values 0 and $\frac{\pi}{2}$ in the question as limits • **wrongly** leaving answer in terms of inverse trigonometric function✗ $\frac{1}{3}\sin\left(\cos^{-1}\left(\frac{\sqrt{3}}{x}\right)\right)+C$ ✗

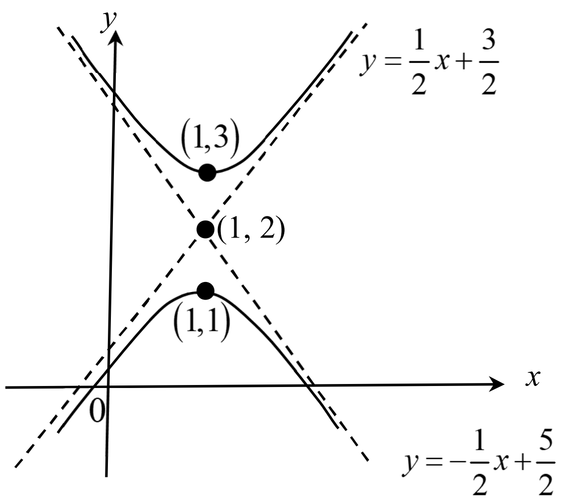
4 A curve C_1 has equation $(ky)^2 - 8ky - x^2 + 2x = -11$, where $k > 0$.

(a) Show that the equation of C_1 can be expressed as $\alpha\left(y-\frac{4}{k}\right)^2 - \beta(x-1)^2 = 1$, where α is a constant in terms of k and β is a constant to be determined. [3]

(b) It is given that one of the asymptotes of C_1 has equation $y=\frac{1}{2}x+\frac{3}{2}$. Find the value of k. Hence write down the equation of the other asymptote. [3]

(c) Using the value of k found in part **(b)**, sketch C_1, stating clearly the equations of its asymptotes and the coordinates of any turning points. [3]

Qn4 **[9]**	**Suggested Solutions**	**Comments** **[Re-visit Tut 2A Q8]**
(a)	$k^2y^2-8ky-x^2+2x=-11$ $k^2\left(y^2-\frac{8}{k}y\right)-x^2+2x=-11$ $k^2\left[y^2-\frac{8}{k}y+\left(\frac{-4}{k}\right)^2-\left(\frac{-4}{k}\right)^2\right]-\left[x^2-2x+(-1)^2-(-1)^2\right]=-11$ $(ky)^2-8ky-x^2+2x=-11$ $k^2\left(y-\frac{4}{k}\right)^2-16-(x-1)^2+1=-11$ $k^2\left(y-\frac{4}{k}\right)^2-(x-1)^2=4$ $\frac{k^2}{4}\left(y-\frac{4}{k}\right)^2-\frac{1}{4}(x-1)^2=1$ (Shown) $\alpha=\frac{k^2}{4}$, $\beta=\frac{1}{4}$	Students recognised the need to **complete the square**. However, for those who did not use brackets failed to obtain the correct constants $k^2\left[-\left(\frac{-4}{k}\right)^2\right]=-16$ and/or $-\left[-(-1)^2\right]=1$. Quite a few students **wrongly factorised k to obtain** $\frac{1}{k^2}$ ✗ instead of **factorising** k^2. For example, $(ky)^2-8ky=\frac{1}{k^2}\left(y^2-\frac{8y}{k}\right)$ ✗ $(ky)^2-8ky=k^2\left(y^2-\frac{8y}{k}\right)$ ✓
(b)	**Asymptotes**: As $x\to\pm\infty$, $\frac{k^2}{4}\left(y-\frac{4}{k}\right)^2\to\frac{1}{4}(x-1)^2$ $\left(y-\frac{4}{k}\right)^2\to\frac{1}{k^2}(x-1)^2$ $\left(y-\frac{4}{k}\right)\to\pm\frac{1}{k}(x-1)$ $\therefore y=\frac{4}{k}\pm\frac{1}{k}(x-1)$ $y=\frac{x}{k}+\frac{3}{k}$ or $y=-\frac{x}{k}+\frac{5}{k}$ Comparing the gradient of the given asymptote: $y=\frac{x}{k}+\frac{3}{k}$ to $y=\frac{1}{2}x+\frac{3}{2}$, $\therefore k=2$ Equation of the other asymptote is $y=-\frac{x}{2}+\frac{5}{2}$	**Common mistake** made when students **substituted the equation of the given asymptote** $y=\frac{1}{2}x+\frac{3}{2}$ **into the relation of the hyperbola**✗. Thereafter, setting the discriminant of the resultant quadratic equation < 0. By doing so, they could not obtain the correct the value of *k*. A small number of students could not distinguish between an **equation** and an **expression** and **wrongly gave the equation of the other asymptote** as $-\frac{x}{2}+\frac{5}{2}$ ✗instead of $y=-\frac{x}{2}+\frac{5}{2}$ ✓.
	Alternatively, when centre of hyperbola $\left(1,\frac{4}{k}\right)$	

Qn4 **[9]**	**Suggested Solutions**	**Comments** **[Re-visit Tut 2A Q8]**
	Since the two asymptotes pass through $\left(1,\frac{4}{k}\right)$ and one of them has gradient $\frac{1}{2}$, Using $y=\frac{1}{2}x+\frac{3}{2}$ where $\frac{4}{k}=\frac{1}{2}(1)+\frac{3}{2}$ Therefore $k=2$.	Students who used this method are more successful in obtaining the correct value of k and the equation of the other asymptote.
	Consider $y=-\frac{1}{2}x+c$ where $(1,2)$ is the centre of the hyperbola $2=-\frac{1}{2}(1)+c \quad \Rightarrow c=\frac{5}{2}$ Equation of the other asymptote is $y=-\frac{x}{2}+\frac{5}{2}$	A small number of students could not distinguish between an **equation** and an **expression** and **wrongly gave the equation of the other asymptote** as $-\frac{x}{2}+\frac{5}{2}$ ✗instead of $y=-\frac{x}{2}+\frac{5}{2}$ ✓.
(c)	Graph of C 	Many students **did not label the centre of the graph** (1, 2). From $\alpha\left(y-\frac{4}{k}\right)^2-\beta(x-1)^2=1$, students should observe that the centre of the hyperbola is at $x=1$, which is a way to check that the other asymptote should intersect the given asymptote $y=\frac{1}{2}x+\frac{3}{2}$ at $x=1$. Many students could not obtain the correct minimum and maximum points despite follow through marks given for the value of k found in part **(b)**.

5 **(a)** Using integration by parts, show that

$$\int e^{2x}\sin x\,dx = e^{2x}\left[A\sin x + B\cos x\right] + C,$$

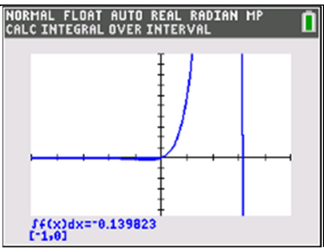
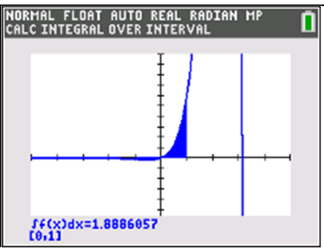
where A, B and C are arbitrary constants to be determined. [4]

(b) A curve C_2 has equation $y = e^{2x}\sin x$. Show that the exact area of the region bounded by C_2, the x-axis and the lines $x = -1$ and $x = 1$ can be expressed as

$$\alpha + \beta\sin(1) + \delta\cos(1),$$

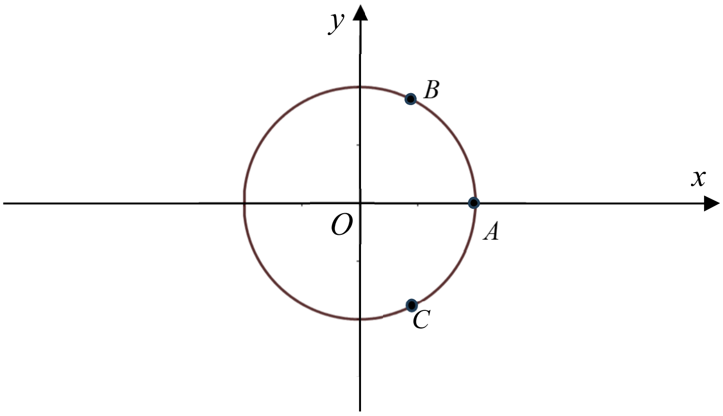
where α, β and δ are exact constants to be determined. [3]

Qn5 [7]	**Suggested Solutions**	**Comments [Re-visit Tut 6A Q3g,k; Tut 6B Q1]**
(a)	**Method 1**: $\int e^{2x}\sin x\,dx$ Let $u = e^{2x}$ and $v' = \sin x$ $u' = 2e^{2x}$, $v = -\cos x$ $\int e^{2x}\sin x\,dx = -e^{2x}\cos x - \int -2e^{2x}\cos x\,dx$ $\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2\int e^{2x}\cos x\,dx$ Let $u = e^{2x}$ and $v' = \cos x$ $u' = 2e^{2x}$, $v = \sin x$ $\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2\left[e^{2x}\sin x - \int 2e^{2x}\sin x\,dx\right]$ $5\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2e^{2x}\sin x + K$ $\int e^{2x}\sin x\,dx = -\frac{1}{5}e^{2x}\cos x + \frac{2}{5}e^{2x}\sin x + C$ $\int e^{2x}\sin x\,dx = e^{2x}\left[\frac{2}{5}\sin x - \frac{1}{5}\cos x\right] + C$ Where $A = \frac{2}{5}$ and $B = -\frac{1}{5}$	- Incorrect formula for **integration by parts** was used e.g. $\int uv'\,dx = uv + \int u'v\,dx$ ✗ **is wrong** Students should remember it is $\int uv'\,dx = uv - \int u'v\,dx$ ✓ (or derive by differentiation of uv with respect to x using product rule) - Errors were observed in the **integration of** v': Method 1: $v' = \sin x \Rightarrow v = \cos x$ ✗ Method 2 $v' = e^{2x} \Rightarrow v = 2e^{2x}$ ✗ - **Inconsistent use** u and v' When applying **integration by parts the second time** — students should ensure the same pair of functions is chosen for both steps. - Some students failed to notice that the same integral reappears after the second round of integration by parts. At that point, the integral should be transferred to the

Qn5 [7]	Suggested Solutions	Comments [Re-visit Tut 6A Q3g,k; Tut 6B Q1]
		LHS and solved algebraically, not integrated by parts again. - Unable to obtain the expressions for $5\int e^{2x}\sin x\,dx$ (Method 1) or $\frac{5}{4}\int e^{2x}\sin x\,dx$ (Method 2) due to **algebraic or computation errors**.
	Method 2: $\int e^{2x}\sin x\,dx$ Let $u=\sin x$ and $v'=e^{2x}$ $u'=\cos x,\quad v=\frac{1}{2}e^{2x}$ $\int e^{2x}\sin x\,dx=\frac{1}{2}e^{2x}\sin x-\int\frac{1}{2}e^{2x}\cos x\,dx$ Let $u=\cos x$ and $v'=\frac{1}{2}e^{2x}$ $u'=-\sin x,\quad v=\frac{1}{4}e^{2x}$ $\int e^{2x}\sin x\,dx=\frac{1}{2}e^{2x}\sin x-\frac{1}{4}e^{2x}\cos x-\int\frac{1}{4}e^{2x}\sin x\,dx$ $\frac{5}{4}\int e^{2x}\sin x\,dx=\frac{1}{2}e^{2x}\sin x-\frac{1}{4}e^{2x}\cos x+K$ $\int e^{2x}\sin x\,dx=e^{2x}\left[\frac{2}{5}\sin x-\frac{1}{5}\cos x\right]+C$ Where $A=\frac{2}{5}$ and $B=-\frac{1}{5}$	
	 CALC INTEGRAL OVER INTERVAL ∫f(x)dx=⁻0.139823 [⁻1,0]  CALC INTEGRAL OVER INTERVAL ∫f(x)dx=1.8886057 [0,1]	When finding **areas or volumes**, students should **first sketch the graph** to identify the regions below or above the x- axis and the limits **before integrating with respect to** x.

Qn5 [7]	**Suggested Solutions**	**Comments [Re-visit Tut 6A Q3g,k; Tut 6B Q1]**
(b)	Area of required region $= -\int_{-1}^{0} \mathrm{e}^{2x} \sin x \, \mathrm{d}x + \int_{0}^{1} \mathrm{e}^{2x} \sin x \, \mathrm{d}x$ $= \left[\mathrm{e}^{2x} \left[\frac{2}{5} \sin x - \frac{1}{5} \cos x \right] \right]_0^{-1} + \left[\mathrm{e}^{2x} \left[\frac{2}{5} \sin x - \frac{1}{5} \cos x \right] \right]_0^{1}$ $= \frac{1}{5} - \frac{1}{5} \mathrm{e}^{-2} \cos(-1) + \frac{2}{5} \mathrm{e}^{-2} \sin(-1) + \frac{2}{5} \mathrm{e}^{2} \sin(1) - \frac{1}{5} \mathrm{e}^{2} \cos(1) + \frac{1}{5}$ $=$ $\frac{2}{5} - \frac{1}{5} \mathrm{e}^{-2} \cos(1) - \frac{2}{5} \mathrm{e}^{-2} \sin(1) + \frac{2}{5} \mathrm{e}^{2} \sin(1) - \frac{1}{5} \mathrm{e}^{2} \cos(1)$ $= \frac{2}{5} + \frac{2}{5} \mathrm{e}^{2} \sin(1) - \frac{2}{5} \mathrm{e}^{-2} \sin(1) - \frac{1}{5} \mathrm{e}^{-2} \cos(1) - \frac{1}{5} \mathrm{e}^{2} \cos(1)$ $= \frac{2}{5} + \frac{2\left(\mathrm{e}^{2} - \mathrm{e}^{-2}\right)}{5} \sin(1) - \frac{\left(\mathrm{e}^{2} + \mathrm{e}^{-2}\right)}{5} \mathrm{e}^{-2} \cos(1)$ $\alpha = \frac{2}{5}$, $\beta = \frac{2\left(\mathrm{e}^{2} - \mathrm{e}^{-2}\right)}{5}$ and $\delta = -\frac{\left(\mathrm{e}^{2} + \mathrm{e}^{-2}\right)}{5}$	- Some students **didn't separate the integration into different regions**, and instead evaluated $\int_{-1}^{1} \mathrm{e}^{2x} \sin x \, \mathrm{d}x$ ✖ directly, which led to **incorrect** results. - **Errors were made in applying trigonometric properties**, particularly: $\sin(-\theta) = -\sin\theta$ and $\cos(-\theta) = \cos\theta$

6



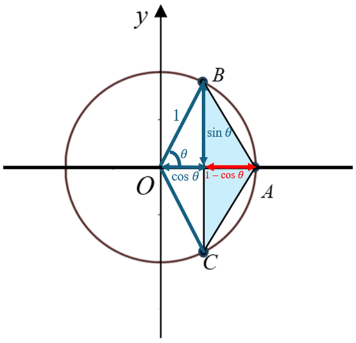
The diagram above shows a unit circle with centre at the origin *O*. The point *A* lies on the circumference of the circle and on the positive *x*-axis. A variable point *B* lies on the circumference of the circle such that the angle between the line segment *OB* and the positive *x*-axis is θ , where $0 < \theta < \pi$. The point *C* is a point of reflection of *B* about the *x*-axis.

Let T denote the area of the triangle *ABC*.

(a) Show that $T = p\sin\theta + q\sin 2\theta$, where p and q are constants to be determined. [2]

(b) Using calculus, find the exact value of θ such that T is maximum as θ varies. [4]

(c) Using your answer found in part **(b)**, find the ratio of the area of the triangle *OBC* to the area of the triangle *ABC*. [2]

Qn6 **[8]**	**Suggested Solutions**	**Comments** **[Re-visit Tut 5B Q6]**
(a)	 **Method 1**: The coordinates of *B* is $\left(\cos\theta, \sin\theta\right)$. The coordinates of *C* is $\left(\cos\theta, -\sin\theta\right)$. Base of triangle *ABC* $= 2\sin\theta$ Height of triangle *ABC* $= 1 - \cos\theta$	This part was not well done. Many students didn't interpret the question correctly or **wrongly assumed** the following: - Area of $\triangle ABC = \triangle OBC$ ✗ - $\measuredangle BAC = 2\theta$ ✗. - $AB = OB$ ✗ **These are not true** as θ **can vary**. The easiest method is to prove this using Area of $\triangle ABC =$ $\frac{1}{2}\left(\text{base}\right)\left(\perp\text{ height}\right)$.

Qn6 [8]	**Suggested Solutions**	**Comments [Re-visit Tut 5B Q6]**
	$T=\frac{1}{2}(2\sin\theta)(1-\cos\theta)$ $=\sin\theta(1-\cos\theta)$ $=\sin\theta-\sin\theta\cos\theta$ $=\sin\theta-\frac{1}{2}\sin 2\theta$ $p=1,\ q=-\frac{1}{2}$	There seemed to be a lack of understanding of the unit circle (or perhaps some missed out the fact that it is a unit circle).
	Method 2: Area of triangle $OAB=\frac{1}{2}\sin\theta$ Let F be the foot of perpendicular from B to the positive x-axis. Area of triangle OBF $=\frac{1}{2}\sin\theta\cos\theta$ $T=2\times\frac{1}{2}(\sin\theta-\sin\theta\cos\theta)$ $=\sin\theta-\sin\theta\cos\theta$ $=\sin\theta-\frac{1}{2}\sin 2\theta$ $p=1,\ q=-\frac{1}{2}$	
(b)	$T=\sin\theta-\frac{1}{2}\sin 2\theta$ $\frac{\mathrm{d}T}{\mathrm{d}\theta}=\cos\theta-\cos 2\theta$ $=\cos\theta-\left[2\cos^2\theta-1\right]$ $=1+\cos\theta-2\cos^2\theta$ $=(1-\cos\theta)(1+2\cos\theta)$ Let $\frac{\mathrm{d}T}{\mathrm{d}\theta}=0$ $\Rightarrow(1-\cos\theta)(1+2\cos\theta)=0$ $\Rightarrow\cos\theta=1$ or $\cos\theta=-\frac{1}{2}$ $\Rightarrow\theta=0$ (rej) or $\theta=\frac{2\pi}{3}$	Most students were able to determine the correct approach: find $\frac{\mathrm{d}T}{\mathrm{d}\theta}$ and let $\frac{\mathrm{d}T}{\mathrm{d}\theta}=0$. Some **students skipped** $\frac{\mathrm{d}T}{\mathrm{d}\theta}=0$ and directly jump to $\frac{\mathrm{d}^2T}{\mathrm{d}\theta^2}<0$. This is **incorrect concept.** The **second derivative test only comes after we solve the first derivative for the stationary point**. A small number of students **forgot to prove the stationary point is a maximum.** This **mark** is **only given** if they can solve for the **correct** $\theta=\frac{2\pi}{3}$.

<table>
<tr><th>Qn6
[8]</th><th>Suggested Solutions</th><th>Comments
[Re-visit Tut 5B Q6]</th></tr>
<tr><td></td><td>

$$\frac{\mathrm{d}T}{\mathrm{d}\theta}=\cos\theta-\cos 2\theta$$

$$\frac{\mathrm{d}^2T}{\mathrm{d}\theta^2}=-\sin\theta+4\cos\theta\sin\theta$$

$$\left.\frac{\mathrm{d}^2T}{\mathrm{d}\theta^2}\right|_{\theta=\frac{2\pi}{3}}=-\frac{\sqrt{3}}{2}+4\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)<0$$

Hence T is maximum when $\theta=\frac{2\pi}{3}$.

</td><td>If the values of p,q are not obtained correctly, students will only be able to score 2 marks at most.</td></tr>
<tr><td></td><td>

Alternatively (1st derivative test)

<table>
<tr><td>θ</td><td>$\theta=\frac{2\pi}{3}^{-}$</td><td>$\theta=\frac{2\pi}{3}$</td><td>$\theta=\frac{2\pi}{3}^{+}$</td></tr>
<tr><td>$\frac{\mathrm{d}T}{\mathrm{d}\theta}$</td><td>+</td><td>0</td><td>-</td></tr>
</table>

Hence T is maximum when $\theta=\frac{2\pi}{3}$.

</td><td></td></tr>
<tr><td>(c)</td><td>

Method 1:
Since triangles *OBC* and *ABC* share the same base, we can compare their heights to find the ratio.

When $\theta=\frac{2\pi}{3},\cos\theta=-\frac{1}{2}$

Height of triangle $OBC=\left|\cos\frac{2\pi}{3}\right|=\frac{1}{2}$

Height of triangle $ABC=\left|1-\cos\frac{2\pi}{3}\right|=\frac{3}{2}$

Hence ratio is $1:3$

</td><td>

This part was poorly attempted, mainly because students couldn't do part **(b)** and left this part blank.

The few who attempted were generally able to find the ratio using area formula.

Some **students left the answer as $\frac{1}{3}$ instead of giving it in ratio form**.

A **ratio** compares two quantities.
A **fraction** shows a part of a single whole.

</td></tr>
<tr><td></td><td>

Method 2:

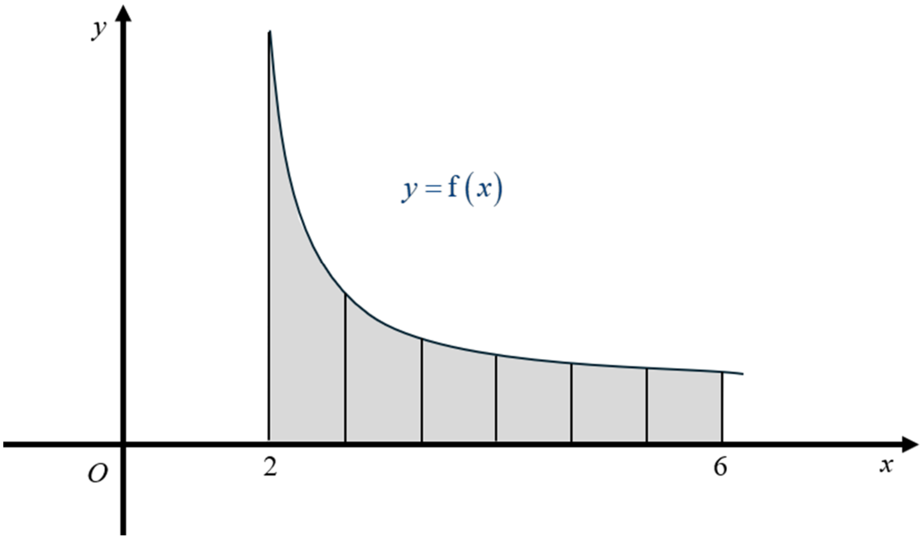
When $\theta=\frac{2\pi}{3}$,

Area of triangle $OBC=\frac{1}{2}(1)^2\sin\frac{2\pi}{3}=\frac{\sqrt{3}}{4}$

</td><td></td></tr>
</table>

Qn6 **[8]**	**Suggested Solutions**	**Comments** **[Re-visit Tut 5B Q6]**
	Area of triangle $ABC = \sin\left(\frac{2\pi}{3}\right)\left(1-\cos\left(\frac{2\pi}{3}\right)\right)$ $= \frac{\sqrt{3}}{2}\left(1+\frac{1}{2}\right)$ $= \frac{3\sqrt{3}}{4}$ Hence ratio is $1:3$	

7 The diagram below shows a sketch of part of the curve $y=\mathrm{f}(x)$. The area of the shaded region under the curve between $x=2$ and $x=6$ is A units^2. This shaded region is split into 6 vertical strips of equal width, h.



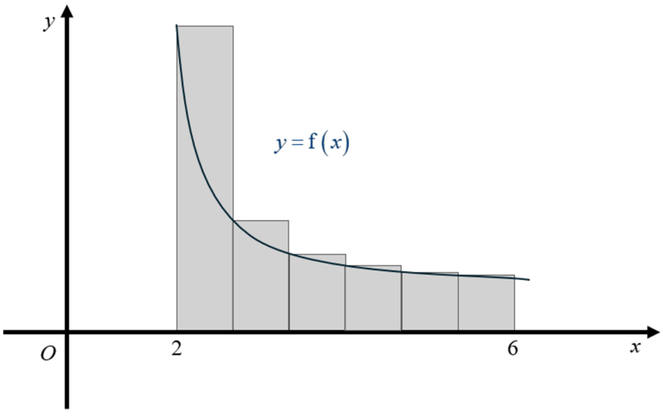
(a) **(i)** State the value of h. [1]

(ii) Using a sketch, explain whether $\sum_{n=0}^{5} h\mathrm{f}(2+hn)$ is an overestimate or underestimate of A. [2]

(iii) Comment on how a better estimate in part **(a)(ii)** could be found. [1]

(b) The region bounded by the curve $y=\sqrt{\dfrac{2}{x\ln x^2}}$, where $x>1$, the x-axis and the lines $x=2$ and $x=6$, is rotated through 2π radians about the x-axis. Using calculus, find the exact volume of the solid obtained, simplifying your answer. [3]

Qn 7 [7]	**Suggested Solutions**	**Comments** **[Re-visit Tut 6B Q3]**
(a) (i)	$h=\dfrac{6-2}{6}=\dfrac{4}{6}=\dfrac{2}{3}$	Some students counted the **number of strips wrongly** as **5✗**. Some students did not **simplify** the **fraction** to $\dfrac{2}{3}$.
(a) (ii)	$\sum_{n=0}^{5} h\mathrm{f}(2+hn)$ where $h=\dfrac{2}{3}$	Many students were unable to **interpret the summation** as the sum of area of 6 rectangles.

Qn 7 **[7]**	**Suggested Solutions**	**Comments** **[Re-visit Tut 6B Q3]**
	$\frac{2}{3}\sum_{n=0}^{5} \mathrm{f}\left(2+\frac{2}{3}n\right)$ $=\frac{2}{3}\left[\mathrm{f}(2)+\mathrm{f}\left(2+\frac{2}{3}\right)+\mathrm{f}\left(2+2\left(\frac{2}{3}\right)\right)+\ldots+\mathrm{f}\left(2+5\left(\frac{2}{3}\right)\right)\right]$ = Total area of 6 rectangles as shown:  Hence $\sum_{n=0}^{5} h\mathrm{f}(2+hn)$ gives an overestimate of A.	Some students interpret the rectangles wrongly as an underestimate of the area under the curve. Students need to **pay attention** to what the **<u>height of each rectangle</u>** is.
(a) **(iii)**	Split into **more than** 6 vertical strips of equal width, h. When the number of vertical strips (rectangles) **increases**, the approximation of the area approaches closer to the value of A.	Many students wrongly wrote the answer as "**use integration to find the actual area**✗ under the curve". They **<u>misunderstood the intention</u>** of the question. The question is **asking for a better <u>estimate</u>** and not how to get the exact area under the curve.
(b)	Exact volume generated $= \pi\int_2^6 \left(\sqrt{\frac{2}{x\ln x^2}}\right)^2 \mathrm{d}x$ $= \pi\int_2^6 \frac{2}{x\ln x^2}\,\mathrm{d}x$ $=\pi\int_2^6 \frac{\frac{2}{x}}{\ln x^2}\,\mathrm{d}x$ $=\pi\left[\ln\left\lvert\ln x^2\right\rvert\right]_2^6$ $=\pi\left[\ln\left(\ln 6^2\right)-\ln\left(\ln 2^2\right)\right]$	Many students could not **recall the correct formula** to find **volume of revolution**. Many students could not recognise the standard formula $\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln\lvert\mathrm{f}(x)\rvert + C$ and therefore could not integrate the function correctly.

Qn 7 [7]	Suggested Solutions	Comments [Re-visit Tut 6B Q3]
	$= \pi\left[\ln\left(\frac{\ln 6}{\ln 2}\right)\right]$ units^3	Students need to take note that exact answer is required for this question. Hence, answer from GC is not accepted.

7 The function f and g are defined as

$$\mathrm{f}: x \mapsto \pi x, \quad x \in \mathbb{R}.$$

$$\mathrm{g}: x \mapsto \begin{cases} \sin x & \text{for } 0 \le x \le \dfrac{\pi}{2}, \\ \cos x + 1 & \text{for } \dfrac{\pi}{2} < x \le \pi. \end{cases}$$

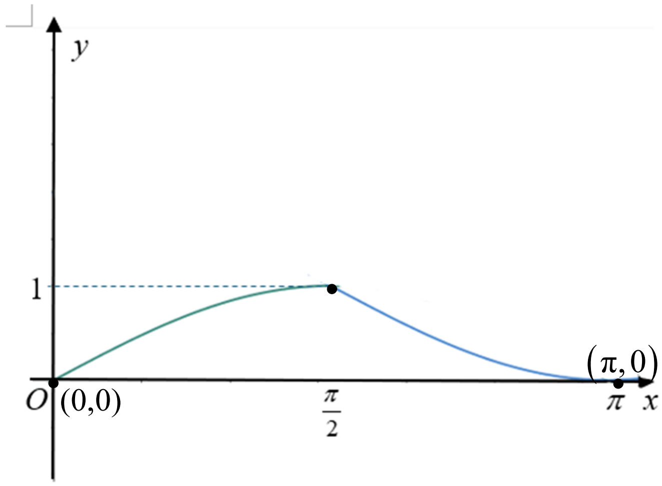
(a) Find $\mathrm{f}^{25}(x)$. [1]

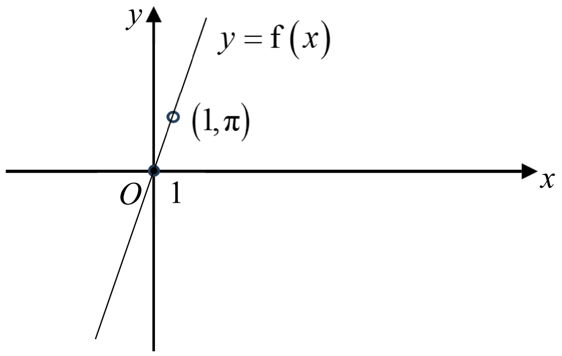
(b) Sketch the graph of $y = \mathrm{g}(x)$ for $0 \le x \le \pi$. [2]

(c) If the domain of g is restricted to $\mathrm{D} = (a, \pi]$ such that g^{-1} exists, state the smallest exact value of a. Hence find $\mathrm{g}^{-1}(x)$ and state the domain of g^{-1}. [3]

(d) For the domain D found in part **(b)**, show that the composite function fg exists and find the exact range of fg . [3]

(e) Without finding f^{-1} and fg , find the exact solution of $\mathrm{f}^{-1}(x) = \mathrm{g}\left(\dfrac{3\pi}{4}\right)$. [3]

Qn8 [12]	**Suggested Solutions**	**Comments [Re-visit Tut 4 Q9]**
(a)	$\mathrm{f}(x) = \pi x$ $\mathrm{f}^2(x) = \mathrm{f}[\mathrm{f}(x)] = \mathrm{f}(\pi x) = \pi(\pi x) = \pi^2 x$ $\mathrm{f}^3(x) = \mathrm{f}[\mathrm{f}^2(x)] = \mathrm{f}(\pi^2 x) = \pi(\pi^2 x) = \pi^3 x$ … $\mathrm{f}^{25}(x) = \pi^{25} x$ or $2.68 \times 10^{12} x$ (3 s.f.)	Various **wrong answers** presented such as $\mathrm{f}^{25}(x) = 25\pi x$ ✘ or $\mathrm{f}^{25}(x) = (\pi x)^{25}$ ✘
(b)		$\left(\dfrac{\pi}{2}, 1\right)$ is a. **sharp point** Some students drew it as a turning (rounded) point. $y = 1$ must be indicated to **show the range** of g. For $0 \le x \le \dfrac{\pi}{2}$, curve must **concave downwards**. For $\dfrac{\pi}{2} < x \le \pi$, curve must **concave upwards**. The **2 ends points** must be **clearly labelled** to show the domain of g.

Qn8 **[12]**	**Suggested Solutions**	**Comments** **[Re-visit Tut 4 Q9]**
(c)	$g: x \mapsto \begin{cases} \sin x & \text{for } 0 \le x \le \frac{\pi}{2}, \\ \cos x + 1 & \text{for } \frac{\pi}{2} < x \le \pi. \end{cases}$ $D_g = (a, \pi]$ or $a < x \le \pi$ Smallest $a = \frac{\pi}{2}$ Let $y = \cos x + 1 \Rightarrow x = \cos^{-1}(y-1)$ $g^{-1}(x) = \cos^{-1}(x-1)$ $D_{g^{-1}} = R_g = [0,1)$	**Common error 1**: $a = 0$ ✗. Students who made this mistake have probably forgotten that g must be one-to-one for g^{-1} to exist. **Common error 2:** Some have forgotten to replace *y* by *x*, leaving answer as $g^{-1}(x) = \cos^{-1}(y-1)$ **Common error 3:** $D_{g^{-1}} = R_g = [0,1]$ ✗. Students probably **did not consider excluding** $x = \frac{\pi}{2}$.
(d)	$R_g = [0,1)$ & $D_f = \mathbb{R}$ Since $R_g \subset D_f$. Hence fg exists.  $\left(\frac{\pi}{2}, \pi\right] \xrightarrow{g} [0,1) \xrightarrow{f} [0, \pi)$ $R_{fg} = [0, \pi)$	**Common error 1:** $R_{fg} = [0, \pi]$ ✗ **Common error 2:** Many students are very careless with the **use of round and square brackets** and use them interchangeably

Qn8 **[12]**	**Suggested Solutions**	**Comments** **[Re-visit Tut 4 Q9]**
(e)	$\mathrm{f}^{-1}(x)=\mathrm{g}\left(\frac{3\pi}{4}\right)$ $\mathrm{f}\left[\mathrm{f}^{-1}(x)\right]=\mathrm{f}\left[\mathrm{g}\left(\frac{3\pi}{4}\right)\right]$ $x=\mathrm{f}\left[\mathrm{g}\left(\frac{3\pi}{4}\right)\right]$ $x=\mathrm{f}\left(\cos\frac{3\pi}{4}+1\right)=\mathrm{f}\left(1-\frac{1}{\sqrt{2}}\right)=\pi\left(1-\frac{1}{\sqrt{2}}\right)$	**Common error 1**: Some students did not see the link with **part (d)** and did not use the restricted domain $\mathrm{D_g}=\left(\frac{\pi}{2},\pi\right]$ and hence use the wrong rule. **Common error 2:** Some students did not read the question carefully and did not evaluate $\cos\frac{3\pi}{4}$ as required. **Common error 3**: Some students thought that $\mathrm{f}^{-1}(x)=\mathrm{g}\left(\frac{3\pi}{4}\right)$ implies $\mathrm{f}(x)=\mathrm{g}^{-1}\left(\frac{3\pi}{4}\right)$ ✗ when it should be $\mathrm{f}\left[\mathrm{f}^{-1}(x)\right]=\mathrm{f}\left[\mathrm{g}\left(\frac{3\pi}{4}\right)\right]$ ✓ $x=\mathrm{f}\left[\mathrm{g}\left(\frac{3\pi}{4}\right)\right]$ ✓

9 **(a)** Given that **a** and **b** are non-zero vectors such that $\mathbf{a}\bullet\mathbf{b}=|\mathbf{a}||\mathbf{b}|$, find the relationship between **a** and **b**. Justify your answer. [2]

(b) A solar panel array has been installed on the rooftop of a building. Points $(x,\ y,\ z)$ are defined relative to the origin $(0,\ 0,\ 0)$ on the horizontal ground modelled by the *x*-*y* plane. The thickness of the solar panel array can be neglected.

(i) The point $A(2,\ 3,\ 4)$, $B(6,\ 3,\ 3)$ and $C(14,\ 4,\ 1)$ lie on the solar panel array modelled by the plane π. Find the cartesian equation of π. [3]

(ii) The solar panel array π at the rooftop is tilted at an acute angle of $\theta°$ from the horizontal ground. Find θ. [2]

Mounted above the solar panel array is an inspection device. As it moves, the motion of the inspection device can be modelled by another plane π_1 parallel to π. The thickness of π_1 can be neglected.

(iii) Given that the inspection device is $\frac{1}{\sqrt{17}}$ units from the solar panel array, find the two possible equations of π_1 in scalar product form. Give a reason for rejecting one of the equations. [3]

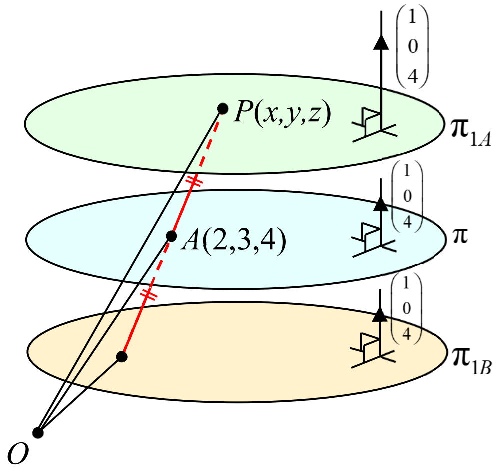
(iv) The inspection device is now at point $(3,0,4)$. It emits a ray of light that is perpendicular to π. Find the position vector of the point where the ray of light hits π. [2]

Qn **9**	**Suggested Solutions**	**Comments** **[Re-visit Tut 7c Q11]**
	Several students lacked understanding in notation for vectors (marks are **not deducted <u>this time only</u>**) — for example, not distinguishing between vector symbols $\underset{\sim}{a}$, $\underset{\sim}{b}$, $\overrightarrow{AB}$ and $\underset{\sim}{r}\bullet\underset{\sim}{n}=\underset{\sim}{a}\bullet\underset{\sim}{n}=d$. Students **should ensure symbols are written accurately** to convey correct meaning where d and AB are scalar quantities.	
(a) **[2]**	Given $\underset{\sim}{a}\bullet\underset{\sim}{b}=\lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{b}\rvert$ Let θ be the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ $\lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{b}\rvert\cos\theta=\lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{b}\rvert$ $\cos\theta=1$ $\theta=0°$	Many students lost marks for not stating the vectors are in the ***<u>same direction</u>***. Students **must show that the angle between the 2 vectors** is $0°$ using the dot product definition $\lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{b}\rvert\cos\theta=\lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{b}\rvert$.

Qn **9**	**Suggested Solutions**	**Comments** **[Re-visit Tut 7c Q11]**
	$\underset{\sim}{a}$ and $\underset{\sim}{b}$ are **parallel vectors.** When $\theta = 0°$, they are in the ***same direction***.	A few students made algebraic errors, writing $\cos\theta = 0$ instead of $\cos\theta = 1$. Some correctly found $\cos\theta = 1$ but gave the **wrong special angle** (e.g. $\theta = 180°, 90°, 45°$ ✗). Remember to check such values carefully or use a calculator if unsure.
(bi) **[3]**	The point $A(2,\ 3,\ 4)$, $B(6,\ 3,\ 3)$ and $C(14,\ 4,\ 1)$ lie on the solar panel array modelled by the plane π. $\overrightarrow{AB} = \begin{pmatrix} 6 \\ 3 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} 14 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 12 \\ 1 \\ -3 \end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix} 14 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 6 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 8 \\ 1 \\ -2 \end{pmatrix}$ A normal vector parallel to π is: $\underset{\sim}{n} = \begin{pmatrix} 12 \\ 1 \\ -3 \end{pmatrix} \times \begin{pmatrix} 8 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}$ Equation of π in **Scalar Product form**: $\underset{\sim}{r} \bullet \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} = 18$ **Cartesian Equation** of π: $x + 4z = 18,\ y \in \mathbb{R}$	A handful of students lack the ability to **distinguish clearly** between a **direction vector** (e.g. $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{BC}$) and a **position vector. E.g.** $\underset{\sim}{a} = \overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 6 \\ 3 \\ 3 \end{pmatrix}, \overrightarrow{OC} = \begin{pmatrix} 14 \\ 4 \\ 1 \end{pmatrix}$ While most were able to find the **direction vectors** parallel to the plane, many stopped at $\underset{\sim}{r} = \underset{\sim}{a} + \lambda\overrightarrow{AB} + \mu\overrightarrow{BC}$, where $\lambda, \beta \in \mathbb{R}$. This is the vector equation of a plane in parametric form and it is **not** a **Cartesian equation**. Unfortunately, it's disappointing to see **many careless mistakes when performing the cross products of two vectors** and also when the cross product formula **can also be found in MF27**. It is **wrong**✗ **to write** $\pi = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 8 \\ 1 \\ -2 \end{pmatrix}$ ✗ Instead, students should write

Qn **9**	**Suggested Solutions**	**Comments** **[Re-visit Tut 7c Q11]**
		$\underset{\sim}{r}=\begin{pmatrix}2\\3\\4\end{pmatrix}+\lambda\begin{pmatrix}4\\0\\-1\end{pmatrix}+\mu\begin{pmatrix}8\\1\\-2\end{pmatrix}$ ✓ Where $\underset{\sim}{r}$ represents the **position vector** of any other points that lie on the plane Hence students should use the **cross product** of two non-parallel **direction vectors** to find the normal vector of the plane, $\underset{\sim}{n}$. Therefore $\underset{\sim}{n}\neq\overrightarrow{OA}\times\overrightarrow{OB}$ since the **<u>origin</u> does <u>not</u> lie on the plane**. When using the scalar product form of the equation of plane, $\underset{\sim}{r}\bullet\underset{\sim}{n}=\underset{\sim}{a}\bullet\underset{\sim}{n}$, students should note that $\underset{\sim}{a}$ must be the **position vector** of any point that lies on the plane (*do <u>not</u> use direction vector*) and the scalar $d=\underset{\sim}{a}\bullet\underset{\sim}{n}$ Then students can let $\underset{\sim}{r}=\begin{pmatrix}x\\y\\z\end{pmatrix}$ to convert to the **<u>Cartesian equation</u>**. $x+4z=18$ Many students also **wrongly wrote** $x+4z=18,\ y=0$ ✗ when it should be $x+4z=18,\ y\in\mathbb{R}$ ✓.
	Avoid this method: $\underset{\sim}{r}=\underset{\sim}{a}+\lambda\overrightarrow{AB}+\beta\overrightarrow{AC}$ $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}2+4\lambda+8\mu\\3+\mu\\4-\lambda-2\mu\end{pmatrix}$ $x=2+4\lambda+8\mu$ --- (1) $y-3=\mu$ --- (2)	Alternatively, students will need to eliminate the scalars $\lambda,\beta\in\mathbb{R}$ in $\underset{\sim}{r}=\underset{\sim}{a}+\lambda\overrightarrow{AB}+\beta\overrightarrow{AC}$ to find the cartesian equation of the plane which is tedious.

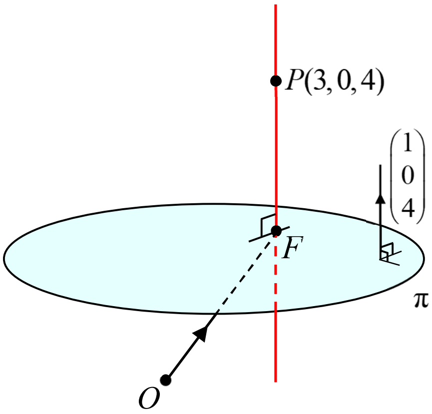
Qn **9**	**Suggested Solutions**	**Comments** **[Re-visit Tut 7c Q11]**
	$\lambda = 4 - z - 2\mu$ --- (3) Substitute (2) into (3): $\lambda = 4 - z - 2\left(y-3\right) = 10 - z - 2y$ From (1): $x = 2 + 4\left(10 - z - 2y\right) + 8\left(y-3\right)$ $x = 2 + 40 - 4z - 8y + 8y - 24$ $x = 18 - 4z$ $x + 4z = 18$	
(bii)	The **normal vector** of the horizontal ground modelled by the x-y plane $= \underset{\sim}{n}_1 = \begin{pmatrix}0\\0\\1\end{pmatrix}$ The normal vector parallel to π is $\underset{\sim}{n}_2 = \begin{pmatrix}12\\1\\-3\end{pmatrix} \times \begin{pmatrix}8\\1\\-2\end{pmatrix} = \begin{pmatrix}1\\0\\4\end{pmatrix}$ Let θ be the acute angle between π and the horizontal ground. $\cos\theta = \dfrac{\left\lvert\begin{pmatrix}0\\0\\1\end{pmatrix} \bullet \begin{pmatrix}1\\0\\4\end{pmatrix}\right\rvert}{\sqrt{17}}$ $\theta = 14.03624347$ $\theta = 14.0 (1 \text{ d.p.})$	In Tut 7C,Qn 11, the horizontal ground is also the x-y plane and hence the normal vector of the x-y plane is the z-axis. A handful of students wasted some time to apply cross product to find the normal vector of the x-y plane using $\begin{pmatrix}1\\0\\0\end{pmatrix} \times \begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix}$. However, it is important to note that the formula for finding the **angle between two planes** involves the **2 normal vectors**✓ and **not** the vectors **parallel**✘ to the respective planes. The correct formula to be applied is $\cos\theta = \dfrac{\lvert \underset{\sim}{n}_1 \bullet \underset{\sim}{n}_2 \rvert}{\lvert \underset{\sim}{n}_1 \rvert \lvert \underset{\sim}{n}_2 \rvert}$ ✓ **Wrong workings** ✘ include the following: **(a)** $\cos\theta = \dfrac{\lvert \underset{\sim}{n}_1 \bullet \underset{\sim}{d}_2 \rvert}{\lvert \underset{\sim}{n}_1 \rvert \lvert \underset{\sim}{d}_2 \rvert}$ ✘ where $\underset{\sim}{d}_2 = \begin{pmatrix}1\\0\\0\end{pmatrix}$ ✘, $\begin{pmatrix}0\\1\\0\end{pmatrix}$ ✘ or $\begin{pmatrix}1\\1\\0\end{pmatrix}$ ✘ etc. **(b)** $\cos\theta = \dfrac{\lvert \underset{\sim}{d}_1 \bullet \underset{\sim}{d}_2 \rvert}{\lvert \underset{\sim}{d}_1 \rvert \lvert \underset{\sim}{d}_2 \rvert}$ ✘ where $\underset{\sim}{d}_1 = \overrightarrow{AB}$ ✘, $\overrightarrow{AC}$ ✘ or $\overrightarrow{BC}$ ✘ etc.

Qn **9**	**Suggested Solutions**	**Comments** **[Re-visit Tut 7c Q11]**
		Some also identified the wrong angle between the 2 planes $\sin\theta = \frac{\lvert \underset{\sim}{n}_1 \bullet \underset{\sim}{n}_2 \rvert}{\lvert \underset{\sim}{n}_1 \rvert \lvert \underset{\sim}{n}_2 \rvert}$ ✘ **Or** $\cos\left(90° - \theta°\right) = \frac{\lvert \underset{\sim}{n}_1 \bullet \underset{\sim}{n}_2 \rvert}{\lvert \underset{\sim}{n}_1 \rvert \lvert \underset{\sim}{n}_2 \rvert}$ A handful of students did not follow the question's instructions, which required **the angle to be given in degrees**. Standard conventions — such as rounding non-exact angles in degrees to one decimal place — were also often not observed.
(biii)		Let the point on π_1 be P. We have $P\left(x, y, z\right)$ or $\overrightarrow{OP} = \begin{pmatrix}x\\y\\z\end{pmatrix}$ **Equation of plane in Scalar Product form**: $\underset{\sim}{r} \bullet \underset{\sim}{n} = \underset{\sim}{a} \bullet \underset{\sim}{n}$
	Method 1: Using the Vector equations of the planes in scalar product form since all three planes are parallel (Use the **same normal vector for all three planes** to compare shortest distance relative to the Origin) From part **(b)(i)** $\pi: \quad \underset{\sim}{r} \bullet \begin{pmatrix}1\\0\\4\end{pmatrix} = 18$ $\pi: \quad \underset{\sim}{r} \bullet \frac{\begin{pmatrix}1\\0\\4\end{pmatrix}}{\sqrt{17}} = \frac{18}{\sqrt{17}}$ Let	In this question, students are expected to present **both possible plane equations** and **not** use the given point from part **(iv)** to verify or determine the correct one. Most students know that the key idea is to apply the **length of projection formula. But many either did not understand the formula or recall it wrongly.** Using the standard form of vector equation of a plane in the scalar product form $\underset{\sim}{r} \bullet \underset{\sim}{\hat{n}} = \underset{\sim}{a} \bullet \underset{\sim}{\hat{n}}$

Qn **9**	**Suggested Solutions**	**Comments** **[Re-visit Tut 7c Q11]**
	$\pi_{1A/B}:\ \underset{\sim}{r}\cdot\frac{\begin{pmatrix}1\\0\\4\end{pmatrix}}{\sqrt{17}}=\frac{k}{\sqrt{17}}$ Distance between π_1 and π: $\left\lvert\frac{18}{\sqrt{17}}-\frac{k}{\sqrt{17}}\right\rvert=\frac{1}{\sqrt{17}}$ $\lvert 18-k\rvert=1$ $18-k=\pm1$ $k=17,\quad 19$ $\pi_1:\ \underset{\sim}{r}\cdot\begin{pmatrix}1\\0\\4\end{pmatrix}=19$ or $\underset{\sim}{r}\cdot\begin{pmatrix}1\\0\\4\end{pmatrix}=17$ Alternatively, write $\pi_1:\ \underset{\sim}{r}\cdot\frac{\begin{pmatrix}1\\0\\4\end{pmatrix}}{\sqrt{17}}=\frac{19}{\sqrt{17}}$ or $\pi_1:\ \underset{\sim}{r}\cdot\frac{\begin{pmatrix}1\\0\\4\end{pmatrix}}{\sqrt{17}}=\frac{19}{\sqrt{17}}$	The **perpendicular distances** from the origin to the planes π and π_1 are $\frac{18}{\sqrt{17}}$ and $\frac{k}{\sqrt{17}}$ respectively. However, many students **wrongly wrote** $\pi_1:\ \underset{\sim}{r}\cdot\begin{pmatrix}1\\0\\4\end{pmatrix}=18\pm\frac{1}{\sqrt{17}}$ ✗ **When it should be** $\pi_1:\ \underset{\sim}{r}\cdot\frac{\begin{pmatrix}1\\0\\4\end{pmatrix}}{\sqrt{17}}=\frac{18}{\sqrt{17}}\pm\frac{1}{\sqrt{17}}$ ✓ The two possible solutions arise because we must take the **modulus of the difference** in these perpendicular distances — hence, $\lvert 18-k\rvert=1$ A handful of students still struggle with equations involving modulus. Some mistakenly placed the modulus incorrectly and wrongly wrote the formula as $\frac{\lvert D\rvert}{\sqrt{17}}-\frac{\lvert D_1\rvert}{\sqrt{17}}=\frac{1}{\sqrt{17}}$ ✗ when it should have been $\left\lvert\frac{D}{\sqrt{17}}-\frac{D_1}{\sqrt{17}}\right\rvert=\frac{1}{\sqrt{17}}$ ✓. The modulus is necessary to ensure that the difference in the shortest distances between the respective planes and the origin is expressed as a positive value, regardless of which plane is further away.

Qn 9	Suggested Solutions	Comments [Re-visit Tut 7c Q11]
(biii)	**Method 2:** Find the two cartesian equations of π_1 using length of projection formula. Let $P(x, y, z)$ be the points on π_1 Given $\left\|\overrightarrow{AP}\bullet\underset{\sim}{\hat{n}}\right\| = \frac{1}{\sqrt{17}}$ $\frac{\left\|\left[\begin{pmatrix}x\\y\\z\end{pmatrix}-\begin{pmatrix}2\\3\\4\end{pmatrix}\right]\bullet\begin{pmatrix}1\\0\\4\end{pmatrix}\right\|}{\sqrt{17}}=\frac{1}{\sqrt{17}}$ $\frac{\left\|\begin{pmatrix}x-2\\y-3\\z-4\end{pmatrix}\bullet\begin{pmatrix}1\\0\\4\end{pmatrix}\right\|}{\sqrt{17}}=\frac{1}{\sqrt{17}}$ $\left\|(x-2)+4(z-4)\right\|=1$ $\left\|x+4z-18\right\|=1$ $x+4z-18=1$ or $x+4z-18=-1$ $x+4z=19$ or $x+4z=17$ $\pi_1:\ \underset{\sim}{r}\bullet\begin{pmatrix}1\\0\\4\end{pmatrix}=19$ or $\underset{\sim}{r}\bullet\begin{pmatrix}1\\0\\4\end{pmatrix}=17$	In this case, the length of projection involves taking the **dot product** of a vector such as $\overrightarrow{AP}$ with the **unit normal vector** of the plane. Since the planes are parallel, their unit normal vectors are the same. Most of the students generally know $\left\|\overrightarrow{AP}\bullet\underset{\sim}{\hat{n}}\right\| = \frac{1}{\sqrt{17}}$ but most failed to let $\overrightarrow{OP}=\begin{pmatrix}x\\y\\z\end{pmatrix}$ to continue. Some students were also able to interpret $\left\|\overrightarrow{AP}\bullet\underset{\sim}{\hat{n}}\right\| = \frac{1}{\sqrt{17}}$ as $\frac{\left\|\begin{pmatrix}x\\y\\z\end{pmatrix}\bullet\begin{pmatrix}1\\0\\4\end{pmatrix}-18\right\|}{\sqrt{17}}=\frac{1}{\sqrt{17}}$ and obtained $\left\|x+4z-18\right\|=1$. The most **common mistakes** are the • lack of understanding **where to place the modulus** $\frac{\left\|\begin{pmatrix}x\\y\\z\end{pmatrix}\bullet\begin{pmatrix}1\\0\\4\end{pmatrix}\right\|-18}{\sqrt{17}}=\frac{1}{\sqrt{17}}$ ✘**correctly,** • **did not consider modulus,** • cannot find the **dot product** and **wrongly wrote** $\begin{pmatrix}x\\y\\z\end{pmatrix}\bullet\begin{pmatrix}1\\0\\4\end{pmatrix}=\begin{pmatrix}x\\0\\4z\end{pmatrix}$ ✘ when it

Qn **9**	**Suggested Solutions**	**Comments** **[Re-visit Tut 7c Q11]**
		should be **a scalar product** where $\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}1\\0\\4\end{pmatrix}=x+4z$ ✓
	Method 3: Find P on π_1 where P is the foot of the perpendicular from A to π_1 $\overrightarrow{OP}=\begin{pmatrix}2\\3\\4\end{pmatrix}+\lambda\begin{pmatrix}1\\0\\4\end{pmatrix}$ for some $\lambda\in\mathbb{R}$ $\left\lvert\overrightarrow{PA}\right\rvert=\left\lvert\lambda\begin{pmatrix}1\\0\\4\end{pmatrix}\right\rvert=\frac{1}{\sqrt{17}}$ $\lvert\lambda\rvert\sqrt{17}=\frac{1}{\sqrt{17}}$ $\lambda=\pm\frac{1}{17}$ $\overrightarrow{OP}=\begin{pmatrix}\frac{35}{17}\\3\\\frac{72}{17}\end{pmatrix}$ or $\overrightarrow{OP}=\begin{pmatrix}\frac{33}{17}\\3\\\frac{64}{17}\end{pmatrix}$ Therefore, the two possible equations of π_1 are $\underset{\sim}{r}\cdot\begin{pmatrix}1\\0\\4\end{pmatrix}=\begin{pmatrix}\frac{35}{17}\\3\\\frac{72}{17}\end{pmatrix}\cdot\begin{pmatrix}1\\0\\4\end{pmatrix}$ or $\underset{\sim}{r}\cdot\begin{pmatrix}1\\0\\4\end{pmatrix}=\begin{pmatrix}\frac{33}{17}\\3\\\frac{64}{17}\end{pmatrix}\cdot\begin{pmatrix}1\\0\\4\end{pmatrix}$ $\pi_1:\ \underset{\sim}{r}\cdot\begin{pmatrix}1\\0\\4\end{pmatrix}=19$ or $\underset{\sim}{r}\cdot\begin{pmatrix}1\\0\\4\end{pmatrix}=17$ (reject)	A small group of students used the idea of **unit vector of the normal vector** to write $\overrightarrow{OP}=\begin{pmatrix}2\\3\\4\end{pmatrix}\pm\frac{1}{\sqrt{17}}\hat{\underset{\sim}{n}}$ directly instead of working out the two possible values of λ. This is an elegant and efficient approach as it makes use of the geometric meaning of the unit normal vector — representing a step of fixed length along the perpendicular direction to the plane.
	Reject the equation $\underset{\sim}{r}\cdot\begin{pmatrix}1\\0\\4\end{pmatrix}=17$ as the inspection device is mounted above the solar panel array. Hence π_1 should be	When deciding which of the two equations applies in this context, students should explain/justify their choice using the information provided in the question.

Qn **9**	**Suggested Solutions**	**Comments** **[Re-visit Tut 7c Q11]**
	further from the origin (i.e. the scalar product should be more than 18) and hence $\underset{\sim}{r}\bullet\begin{pmatrix}1\\0\\4\end{pmatrix}=19$. Scalar Product form π_1 : $\underset{\sim}{r}\bullet\begin{pmatrix}1\\0\\4\end{pmatrix}=19$ Alternatively, the two equations of the planes could also be written as: π_1 : $\underset{\sim}{r}\bullet\begin{pmatrix}-1\\0\\-4\end{pmatrix}=-19$ or $\underset{\sim}{r}\bullet\begin{pmatrix}-1\\0\\-4\end{pmatrix}=-17$ Reject the equation $\underset{\sim}{r}\bullet\begin{pmatrix}-1\\0\\-4\end{pmatrix}=-17$ as the inspection device is mounted above the solar panel array with equation $\underset{\sim}{r}\bullet\begin{pmatrix}-1\\0\\-4\end{pmatrix}=-18$. Hence π_1 should be **further from the origin** (i.e. lesser than – 18) and hence $\underset{\sim}{r}\bullet\begin{pmatrix}-1\\0\\-4\end{pmatrix}=-19$.	Some students correctly rejected the equation $\underset{\sim}{r}\bullet\begin{pmatrix}1\\0\\4\end{pmatrix}=17$ but **incorrectly stating** the **inspection device was underground**✗. However, the device is positioned just below the solar panel. The key comparison should be based on the using of the following: a) **numerical value of the scalar product**, as it represents the perpendicular distance of the plane from the origin. Instead, students should reason **geometrically**, b) and the physical context (e.g., “above” or “below” the solar panel) to determine which equation corresponds to the correct position of the plane. (whether or not the plane is inside the building or not is beyond the information provided and hence no need to mention)
		

Qn **9**	**Suggested Solutions**	**Comments** **[Re-visit Tut 7c Q11]**
biv	**Method 1**: Find foot of perpendicular Let F be the foot of the perpendicular from P to π_1 $l_{PF}:\underset{\sim}{r}=\begin{pmatrix}3\\0\\4\end{pmatrix}+\lambda\begin{pmatrix}1\\0\\4\end{pmatrix}$, where $\lambda\in\mathbb{R}$ Since F lies on l_{PF}, $\overrightarrow{OF}=\begin{pmatrix}3+\lambda\\0\\4+4\lambda\end{pmatrix}$ Since F lies on π: $\underset{\sim}{r}\bullet\begin{pmatrix}1\\0\\4\end{pmatrix}=18$ $\begin{pmatrix}3+\lambda\\0\\4+4\lambda\end{pmatrix}\bullet\begin{pmatrix}1\\0\\4\end{pmatrix}=18$ $3+\lambda+16+16\lambda=18$ $\lambda=-\frac{1}{17}$ $\overrightarrow{OF}=\begin{pmatrix}\frac{50}{17}\\0\\\frac{64}{17}\end{pmatrix}=\frac{2}{17}\begin{pmatrix}25\\0\\32\end{pmatrix}$	**Method 1** is the most straight forward and easiest to score method with just a handful of students **wrongly writing** ✘ $\overrightarrow{PF}=\begin{pmatrix}3\\0\\4\end{pmatrix}+\lambda\begin{pmatrix}1\\0\\4\end{pmatrix}$ when it should be $\overrightarrow{OF}=\begin{pmatrix}3\\0\\4\end{pmatrix}+\lambda\begin{pmatrix}1\\0\\4\end{pmatrix}$ ✓ Many did not obtain full credit unfortunately due to calculation error of the normal vector in part **(b)(i)**. **<<NO ECF>>**
	Method 2: Projection Vector $P(3,0,4)$ Let F be the foot of the perpendicular from P to π_1 $\overrightarrow{AP}=\begin{pmatrix}3\\0\\4\end{pmatrix}-\begin{pmatrix}2\\3\\4\end{pmatrix}=\begin{pmatrix}1\\-3\\0\end{pmatrix}$ $\overrightarrow{FP}=\left(\overrightarrow{AP}\bullet\underset{\sim}{\hat{n}}\right)\underset{\sim}{\hat{n}}$ $\overrightarrow{FP}=\left(\begin{pmatrix}1\\-3\\0\end{pmatrix}\bullet\frac{\begin{pmatrix}1\\0\\4\end{pmatrix}}{\sqrt{17}}\right)\frac{\begin{pmatrix}1\\0\\4\end{pmatrix}}{\sqrt{17}}=\begin{pmatrix}\frac{1}{17}\\0\\\frac{4}{17}\end{pmatrix}$	It is difficult for students to score well using **Method 2** if they lack understanding of the **importance of direction in vectors**. If students find $\overrightarrow{PA}=\begin{pmatrix}-1\\3\\0\end{pmatrix}$, then the projection vector is $\overrightarrow{PF}=\left(\begin{pmatrix}-1\\3\\0\end{pmatrix}\bullet\frac{\begin{pmatrix}1\\0\\4\end{pmatrix}}{\sqrt{17}}\right)\frac{\begin{pmatrix}1\\0\\4\end{pmatrix}}{\sqrt{17}}=-\begin{pmatrix}\frac{1}{17}\\0\\\frac{4}{17}\end{pmatrix}$ $\overrightarrow{OF}=\overrightarrow{OP}+\overrightarrow{PF}$

Qn **9**	**Suggested Solutions**	**Comments** **[Re-visit Tut 7c Q11]**
	$\overrightarrow{OF}=\overrightarrow{OP}+\overrightarrow{PF}=\overrightarrow{OP}-\overrightarrow{FP}$ $\overrightarrow{OF}=\begin{pmatrix}3\\0\\4\end{pmatrix}-\begin{pmatrix}\frac{1}{17}\\0\\\frac{4}{17}\end{pmatrix}=\begin{pmatrix}\frac{50}{17}\\0\\\frac{64}{17}\end{pmatrix}=\frac{2}{17}\begin{pmatrix}25\\0\\32\end{pmatrix}$	Since **direction is crucial in finding the projection vector method**, students **should not use the modulus** when finding the **length of projection** using this approach to find the correct vector (and this is not the required position vector).
	Method 3: Use of unit vector Since $P(3,0,4)$ lies on π_1, the shortest distance is $\frac{1}{\sqrt{17}}$ $\overrightarrow{FP}=\pm\frac{1}{\sqrt{17}}\frac{\begin{pmatrix}1\\0\\4\end{pmatrix}}{\sqrt{17}}$ $\overrightarrow{FP}=\pm\frac{1}{17}\begin{pmatrix}1\\0\\4\end{pmatrix}$ $\overrightarrow{OF}=\begin{pmatrix}3\\0\\4\end{pmatrix}\pm\frac{1}{17}\begin{pmatrix}1\\0\\4\end{pmatrix}$ $\overrightarrow{OF}=\frac{1}{17}\begin{pmatrix}50\\0\\64\end{pmatrix}$ or $\overrightarrow{OF}=\frac{1}{17}\begin{pmatrix}52\\0\\72\end{pmatrix}$ **Check**: $\pi:\ \underset{\sim}{r}\bullet\begin{pmatrix}1\\0\\4\end{pmatrix}=18$, $\frac{1}{17}\begin{pmatrix}52\\0\\72\end{pmatrix}\bullet\begin{pmatrix}1\\0\\4\end{pmatrix}=20$ (reject) $\frac{1}{17}\begin{pmatrix}50\\0\\64\end{pmatrix}\bullet\begin{pmatrix}1\\0\\4\end{pmatrix}=18$ Therefore $\overrightarrow{OF}=\begin{pmatrix}\frac{50}{17}\\0\\\frac{64}{17}\end{pmatrix}=\frac{2}{17}\begin{pmatrix}25\\0\\32\end{pmatrix}$	Most students who received full credit (given the benefit of doubt) wrote $\overrightarrow{OF}=\begin{pmatrix}3\\0\\4\end{pmatrix}-\frac{1}{17}\begin{pmatrix}1\\0\\4\end{pmatrix}$. However, in reality, students should **explain or verify** why the subtraction sign is used — that is, why $\overrightarrow{OF}=\begin{pmatrix}3\\0\\4\end{pmatrix}-\frac{1}{17}\begin{pmatrix}1\\0\\4\end{pmatrix}$ and not $\overrightarrow{OF}=\begin{pmatrix}3\\0\\4\end{pmatrix}+\frac{1}{17}\begin{pmatrix}1\\0\\4\end{pmatrix}$ The choice of direction is essential when using the **unit vector method**. Students should justify their choice by checking which result satisfies the plane equation.

10 Yen and Mei independently embark on a multi-day open water swimming expedition along the same route. Each day, Yen and Mei begin swimming at 8 a.m. and swim continuously for 6 hours, with their distances recorded daily. Assume Yen and Mei remain at sea throughout the expedition and disregard their activities during the remaining 18 hours each day.

Let n represents the number of hours since 1 October at 8 a.m, where $n=1$ corresponds to the first hour on 1 October, $n=2$ corresponds to the second hour on 1 October, and so on.

(a) **(i)** Yen starts her journey on 1 October at 8 a.m. She completes 4 km in the first hour and the distance she covers in the next hour is 5% less than the previous hour. The distance continues to reduce in the same way for each subsequent day. Show that the distance covered in the first hour on 2 October is 2.94 km correct to 3 significant figures. [1]

(ii) Find the total distance Yen will swim by the end of 5 days. [2]

(iii) Find the theoretical total distance that Yen can swim over a long period of time. [2]

(b) **(i)** Due to unforeseen reasons, Mei starts her journey on 2 October at 8 a.m. She completes 5 km in the first hour and the distance she covers is 200 m less than the previous hour. The distance continues to reduce in the same way for each subsequent day. Show that the distance covered by Mei in the nth hour **from 1 October** at 8 a.m. is $6.4-0.2n$, where $n\geq 7$. [2]

(ii) Find the maximum total number of hours Mei can swim and the total distance covered by her. [4]

(c) Find the smallest integer value of n for which the total distance covered by Mei exceeds the total distance covered by Yen. [2]

Qn 10 [13]	**Suggested Solutions**	**Comments [Re-visit Tut 1 Q6]**
(a)(i)	<table><tr><td>n th hour</td><td>Distance covered</td></tr><tr><td>1</td><td>4</td></tr><tr><td>2</td><td>$(0.95)(4)$</td></tr><tr><td>3</td><td>$(0.95)^2(4)$</td></tr><tr><td>…</td><td>…</td></tr><tr><td>6</td><td>$(0.95)^5(4)$</td></tr></table> Yen swims a total of 6 hours each day.	Generally, well done.

Qn 10 **[13]**	**Suggested Solutions**	**Comments** **[Re-visit Tut 1 Q6]**
	On the first hour of Day 2 = 7th hour Distance covered in the 7th hour = $(0.95)^6(4)=2.940367563=2.94$ (3 s.f.)	
(a)(ii)	5 days = 30 hours Total distance travelled by Yen $=\frac{4\left(1-0.95^{30}\right)}{1-.095}$ or $\sum_{x=1}^{30}\left(4\times0.95^{x-1}\right)$ $=62.828899$ = 62.8 km (3 s.f.)	The following **incorrect GP sum** formula were often used: $\frac{4\left(1-0.95\right)^{30}}{1-.095}$, $\frac{4\left(1-0.95^{30-1}\right)}{1-.095}$ ✗
(a)(iii)	Total distance travelled by Yen over a long period of time $=\frac{4}{1-0.95}$ $=80$	• **Wrong approach**: Some students **used a table** to **observe the trend** for large values of *n*, while others substituted a large value of *n* into the sum formula and **rounded their results** to (e.g. 80). Both methods should only be used to **check** the answer, not to **determine** S_∞ . **Correct approach**: Use the correct formula $S_\infty=\frac{a}{1-r}$ or Consider the limit as *n* tends to infinity. e.g. $S_n=\frac{4\left(1-0.95^n\right)}{1-.095}$ As $n\to\infty$, $0.95^n\to0$. Thus $\frac{4\left(1-0.95^n\right)}{1-0.95}\to\frac{4(1-0)}{1-0.95}=\frac{4}{1-0.95}$ The sum approaches a finite value that can be found by evaluating the limit of the sum formula directly, rather than estimating numerically. • Students should also **avoid writing** $\frac{4\left(1-95\%^n\right)}{1-95\%}$.

<table>
<tr><th>Qn 10
[13]</th><th>Suggested Solutions</th><th>Comments
[Re-visit Tut 1 Q6]</th></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>(b)(i)</td><td>
<table>
<tr><th>n th hour</th><th>Distance covered</th></tr>
<tr><td>1</td><td>6.2</td></tr>
<tr><td>2</td><td>6.0</td></tr>
<tr><td>…</td><td></td></tr>
<tr><td>6</td><td>5.2</td></tr>
<tr><td>7</td><td>5</td></tr>
<tr><td>8</td><td>5 – 0.2</td></tr>
</table>
</td><td>Question asks to find AP nth term and not AP sum of n terms.

Students should keep consistently to km for distance i.e. 5km and 0.2 km rather than 5000m and 200m.</td></tr>
<tr><td></td><td><u>Method 1:</u> Consider the nth term where $n \geq 1$ (from 1st October)
Distance covered by Mei in the n th hour
$=6.2-0.2(n-1)$
$=6.4-0.2n$ (shown)

<u>Method 2</u>
Distance covered by May in the n th hour (from 1st October)
$=5+\left[(n-6)-1\right](-0.2)$
$=6.4-0.2n$ (shown)</td><td>Mei started swimming only on 2nd Oct 8am and completed 5km on her first hour. So 5km is distance covered in 7th hour of swimming. i.e $U_7=5$.
So if we project back to 1st Oct 1st hour, $U_1=6.2$.

Alternatively, if students consider first term $U_1=5$, then $U_n=5+(n-6-1)d$ with $d=-0.2$ as we need to take away the first 6 hours in 1st Oct.</td></tr>
<tr><td>(b)(ii)</td><td></td><td>Many students did not <u>consider an inequality</u>, ≥ 0 to find the maximum n.
Instead, they solve the equation $5-0.2(n-1)=0 \Rightarrow n=26$
Students need to use a table of values to justify that $n=$ 26 or 25 is the maximum n value.

The expression $6.4-0.2n$ in part (b)(i) was to help students to do the subsequent parts but unfortunately some students misinterpret 6.4 to be first term ✘ U_1 for the arithmetic progression.</td></tr>
</table>

<table>
<tr><th>Qn 10
[13]</th><th>Suggested Solutions</th><th>Comments
[Re-visit Tut 1 Q6]</th></tr>
<tr><td></td><td>

<u>**Method 1:**</u>
Consider the *n*th term from part **(bi)** where $n \geq 7$
$6.4-0.2n \geq 0$
$n \leq 32$

Maximum number of hours Mei swam = 32 – 6 = 26

When $n=32$, Distance swam = 0.
We can also write
Maximum number of hours Mei swam = 31 – 6 = 25

****
Sum of 25 (or 26) terms =
$\frac{25}{2}\left[2(5)+(25-1)(0.2)\right]$ or $\frac{25}{2}\left[5+0.2\right]$ = 65
$\frac{26}{2}\left[2(5)+(26-1)(0.2)\right]$ or $\frac{26}{2}\left[5+0.2\right]$ = 65

Or

Distance covered by May in 25 (or 26) hours
$=\frac{31}{2}\left[2(6.2)-0.2(31)\right]-\frac{6}{2}\left[2(6.2)-0.2(5)\right]$
= 99.2 – 34.2
=65

or

Use GC
$\sum_{r=7}^{31}(6.4-0.2n)=65$, $\sum_{r=7}^{32}(6.4-0.2n)=65$

</td><td>

<table>
<tr><td>n</td><td>$T_n = 6.4-0.2n$</td></tr>
<tr><td>31</td><td>0.2</td></tr>
<tr><td>32</td><td>0</td></tr>
<tr><td>33</td><td>−0.2</td></tr>
</table>

When $n=32$, distance swam = 0. So, the maximum n can be taken to be 31 or 32. And Mei swam max 25 or 26 hours respectively. But students should justify their answer with table or some explanation why Mei swam a maximum of 25 hours.

****

Many students mistook 6.4 to be first term U_1 . It is the term before 6.2. Can think of $U_0 = 6.4$

</td></tr>
<tr><td></td><td>

<u>**Method 2:**</u> Consider the *N*th term where $N \geq 1$
$5-0.2(N-1) \geq 0$
$N \leq 26$
So maximum number of hours Mei swam is 26.

OR state when $n=26$, Distance swam = 0.
We can also write
Maximum number of hours Mei swam = 25

</td><td>

<table>
<tr><td>N</td><td>$T_N = 5-0.2(N-1)$</td></tr>
<tr><td>25</td><td>0.2</td></tr>
<tr><td>26</td><td>0</td></tr>
<tr><td>27</td><td>−0.2</td></tr>
</table>

When $N=26$, distance swam = 0.

</td></tr>
</table>

Qn 10 [13]	Suggested Solutions	Comments [Re-visit Tut 1 Q6]

****

Sum of 25 (or 26) terms =

$$\frac{25}{2}[2(5)+(25-1)(0.2)] \text{ or } \frac{25}{2}[5+0.2] = 65$$

$$\frac{26}{2}[2(5)+(26-1)(0.2)] \text{ or } \frac{26}{2}[5+0.2] = 65$$

Can accept max hours Mei swam to be 25 or 26 hours respectively. But students should justify their answer with table or some explanation.

Method 3 : Consider the sum of mth term where $m \geq 1$

Let m be the number of hours Mei swam starting from 2nd Oct 8am.

$$S_m = \frac{m}{2}[2(5)+(m-1)(-0.2)]$$

$$= \frac{m}{2}[10.2-0.2m]$$

m	$S_m = \frac{m}{2}[10.2-0.2m]$
24	64.8
25	65
26	65
27	64.8

From table, max no of hours Mei swam is 25h and max distance is 65km.

(c)

Method 1A: Use Sum of AP

$$\frac{4(1-0.95^x)}{1-0.95} < \frac{n}{2}\left[2(6.2)-0.2(n-1)\right]-\frac{6}{2}\left[2(6.2)-0.2(5)\right]$$

n th hour	Yen , $Y_1 = \frac{4(1-0.95^x)}{1-0.95}$	Mei, $Y_2 = \frac{n}{2}\left[2(6.2)-0.2(n-1)\right]-\frac{6}{2}\left[2(6.2)-0.2(5)\right]$
19	49.8117	49.4
20	51.3211	51.8
21	52.7551	54

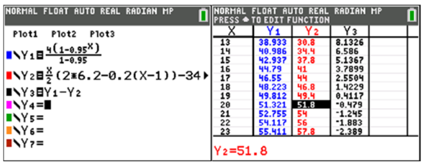
OR

$$\frac{4(1-0.95^x)}{1-0.95}-\frac{n}{2}\left[2(6.2)-0.2(n-1)\right]+\frac{6}{2}\left[2(6.2)-0.2(5)\right]<0$$

From GC, Mei will overtake Yen in the 20th hour.

To obtain credit for the smallest $n = 20$, students **must show**

- state the correct GP & AP sum formula used.
- **table of values** to compare the sum of distances of Yen and Mei.



```
NORMAL FLOAT AUTO REAL RADIAN MP
 Plot1  Plot2  Plot3
■\Y1■4(1-0.95^X)/(1-0.95)
■\Y2■X/2(2*6.2-0.2(X-1))-34▸
■\Y3■Y1-Y2
■\Y4=■
■\Y5=
■\Y6=
■\Y7=
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
PRESS ▲ TO EDIT FUNCTION
   X     Y1      Y2     Y3
  13   38.933  30.8   8.1326
  14   40.986  34.4   6.586
  15   42.937  37.8   5.1367
  16   44.79   41     3.7899
  17   46.55   44     2.5504
  18   48.223  46.8   1.4229
  19   49.812  49.4   0.4117
  20   51.321  51.8   -0.479
  21   52.755  54     -1.245
  22   54.117  56     -1.883
  23   55.411  57.8   -2.389
Y2=51.8
```

Method 1B: Use Sum of AP

$$\frac{4(1-0.95^x)}{1-0.95} < \frac{n-6}{2}\left[2(5)-0.2(n-7)\right]$$

<table>
<tr><th>Qn 10
[13]</th><th>Suggested Solutions</th><th>Comments
[Re-visit Tut 1 Q6]</th></tr>
<tr><td></td><td><u>Method 2</u>: Use Sigma Notation
<table>
<tr><th>n th hour</th><th>Yen , $Y_1 = \dfrac{4\left(1-0.95^x\right)}{1-0.95}$</th><th>Mei, $Y_2 = \sum_{r=7}^{x}(6.4-0.2r)$</th></tr>
<tr><td>19</td><td>49.8117</td><td>49.4</td></tr>
<tr><td>20</td><td>51.3211</td><td>51.8</td></tr>
<tr><td>21</td><td>52.7551</td><td>54</td></tr>
</table></td><td>Students can also learn to use the GC directly to obtain $\sum_{r=7}^{x}(6.4-0.2r)$ directly and save time in manual calculation.</td></tr>
</table>

11 The curve C_3 has parametric equations

$$x=1-p^2,\ y=p\left(1-p^2\right),\text{ where } p\geq 0.$$

(a) **(i)** The point P travels along the curve C_3 with its x-coordinate decreasing at a constant rate of 5 units per second. Find the rate of change of its y-coordinate when $x=\frac{1}{4}$. [4]

(ii) The distance between two points along a curve is the arc-length.

The arc-length between two points on C_3, where $p=\alpha$ and $p=\beta$, is given by the formula $\left|\int_{\alpha}^{\beta}\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}p}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}p}\right)^2}\,\mathrm{d}p\right|$.

Find the total length of the curve from the point $A(-3,-6)$ to $B(1,0)$. [3]

(b) Find the equation of the tangent to C_3 at $p=1$. Show algebraically that this tangent does not cut C_3 again. [5]

(c) Find the cartesian equation of C_3, expressing your answer in the form $y^2=\mathrm{f}(x)$. [2]

Qn 11 [14]	**Suggested Solutions**	**Comments [Compare with Tut 6b Q11]**
(a) (i)	$x=1-p^2,\ y=p\left(1-p^2\right)$, where $p>0$ $\frac{\mathrm{d}x}{\mathrm{d}p}=-2p$, $\frac{\mathrm{d}y}{\mathrm{d}p}=p(-2p)+1-p^2=-3p^2+1$ When $x=\frac{1}{4}$, $\frac{1}{4}=1-p^2 \Rightarrow p=\pm\frac{\sqrt{3}}{2}$ Since $p>0$, $p=\frac{\sqrt{3}}{2}$ Given $\frac{\mathrm{d}x}{\mathrm{d}t}=-5$ $\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}y}{\mathrm{d}p}\times\frac{\mathrm{d}p}{\mathrm{d}t}$ $\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}y}{\mathrm{d}p}\times\frac{\mathrm{d}p}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{\left(-3p^2+1\right)}{-2p}\times-5$	**Common mistakes**: 1. Misinterpreting the phrase **"x-coordinate decreasing at a constant rate"**, leading to using $\frac{\mathrm{d}x}{\mathrm{d}t}=5$ ✗ instead of the correct $\frac{\mathrm{d}x}{\mathrm{d}t}=-5$ ✓. 2. **Incorrect differentiation, such as writing** $\frac{\mathrm{d}x}{\mathrm{d}p}=1-2p$ ✗ instead of $\frac{\mathrm{d}x}{\mathrm{d}p}=-2p$ ✓ 3. **Omitting the chain rule**, by directly substituting (wrong)

Qn **11** **[14]**	**Suggested Solutions**	**Comments** **[Compare with Tut 6b Q11]**
	When $p=\frac{\sqrt{3}}{2}$, $\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{-3\left(\frac{3}{4}\right)+1}{2\left(\frac{\sqrt{3}}{2}\right)}\times 5$ $\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{5\left(-\frac{9}{4}+1\right)}{\sqrt{3}}=-\frac{25}{4\sqrt{3}}=-\frac{25\sqrt{3}}{12}$ Or its *y*-coordinate is **decreasing** at $\frac{25\sqrt{3}}{12}$ unit per second.	numbers instead of showing $\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}y}{\mathrm{d}p}\times\frac{\mathrm{d}p}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}t}$ 4. **Final answer not simplified**, e.g. leaving expressions like "$-\frac{25\sqrt{\frac{3}{4}}}{6}$", "$-\frac{25}{\sqrt{48}}$", etc instead of simplifying to $-\frac{25\sqrt{3}}{12}$ 5. Using **"*s*" instead of "*t*"** to represent time — acceptable but unconventional. 6. **Wrongly wrote** $y=px\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=p$ ✗since *p* is not a constant! *p* is the parameter for the parametric equation. A handful of students attempted to find the Cartesian equation of C_3 but **did not justify rejecting the negative root**. **One mark will be deducted** for **not providing a reason**.
(a) **(ii)**	At $A(-3,-6)$, $-3=1-p^2$ $p^2=4$ $p=\pm 2$ $p=2$ Since $p\geq 0$ At $B(1,0)$, $1=1-p^2$ $p=0$ Total length of the curve	**Common mistakes**: 1. Applying the given formula for the length of arc without **understanding *α* and *β* as specific *p* values** (and use -3 and 1 instead – no mark) 2. **Missing out the modulus sign** (not penalised) Students tend to give up and leave blank when encountering unfamiliar expressions like this. Students who made mistakes in their differentiation earlier in part **(a)(i)** will not be penalised for the same mistake in this part, so long

Qn 11 [14]	Suggested Solutions	Comments [Compare with Tut 6b Q11]
	$= \left\| \int_{\alpha}^{\beta} \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}p}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}p}\right)^2}\, \mathrm{d}p \right\|$ $= \left\| \int_0^2 \sqrt{(-2p)^2 + \left(-3p^2+1\right)^2}\, \mathrm{d}p \right\|$ =8.10 units	as their method in this part is correct.
(b)	**Method 1**: <u>Using Parametric Equations</u> $x = 1-p^2, y = p\left(1-p^2\right)$, where $p > 0$ $\frac{\mathrm{d}x}{\mathrm{d}p} = -2p$, $\frac{\mathrm{d}y}{\mathrm{d}p} = -3p^2+1$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}p} \times \frac{\mathrm{d}p}{\mathrm{d}x} = \frac{1-3p^2}{-2p}$ At $p = 1$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 1, x = 0, y = 0$ Equation of tangent: $y - 0 = 1(x-0)$ $y = x$ Since $y = x$ $1-p^2 = p\left(1-p^2\right)$ $p^3 - p^2 - p + 1 = 0$ $(p-1)\left(p^2-p-1\right) = 0$ $(p-1)^2(p+1) = 0$ Thus $p = -1$ (reject since $p > 0$) or $p = 1$ Therefore tangent only cuts the curve at $p = 1$ i.e $(0,0)$ and does not cut the curve again.	Many students did not **show clearly** how they **formed the equation of the tangent**. A handful wrongly used the gradient of normal instead (ie. $m_1 = -\frac{1}{m_2}$). Many students did not **put down the <u>correct reason</u> for concluding that the tangent only touches the curve once** – failing to **show <u>algebraically</u>.** "When $p = -1, x = 0$ and $y = 0$" is not the correct reason for showing – this is merely verifying. The actual reason is that $p \geq 0$ given by the question, implying the point corresponding to $p = -1$ is not even supposed to exist for this curve. Students who do not **write their conclusion** are not given the final mark.
	Method 2: <u>Using Cartesian Equation</u> $x = 1-p^2, y = p\left(1-p^2\right)$, where $p > 0$ $\frac{\mathrm{d}x}{\mathrm{d}p} = -2p$, $\frac{\mathrm{d}y}{\mathrm{d}p} = -3p^2+1$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}p} \times \frac{\mathrm{d}p}{\mathrm{d}x} = \frac{1-3p^2}{-2p}$ At $p = 1$,	Many students solve the equation $x^2 = x^2\left(1-x\right)$ by **merely cancelling the variable x** ✘away **from both sides** of the equation – this is a **<u>wrong way</u> of solving equations**!

Qn **11** **[14]**	**Suggested Solutions**	**Comments** **[Compare with Tut 6b Q11]**
	$\frac{\mathrm{d}y}{\mathrm{d}x}=1$, $x=0, y=0$ Equation of tangent: $y-0=1(x-0)$ $y=x$ If tangent to cut curve, sub $y=x$ into $y^2=x^2(1-x)$ $x^2=x^2(1-x)$ $x^3=0$ $x=0$ $y=0$ Therefore, tangent only cuts the curve at $(0,0)$ (or at $x=0$) and does not cut the curve again.	
(c)	$x=1-p^2, y=p(1-p^2)$ $y=xp$ $p=\frac{y}{x}$ Sub into $x=1-p^2$ $x=1-\left(\frac{y}{x}\right)^2$ $\left(\frac{y}{x}\right)^2=1-x$ $y^2=x^2(1-x)$	Most students get this part right. A small number of students made careless mistakes by writing the powers of *x* wrongly.