Topical Revision Paper 7

Term 2 Unit 7: Graph Transformations

Question 1:

(i) Sketch the graphs of $y = \ln x$ and $y = e^x$ **on the same axes**. [2]

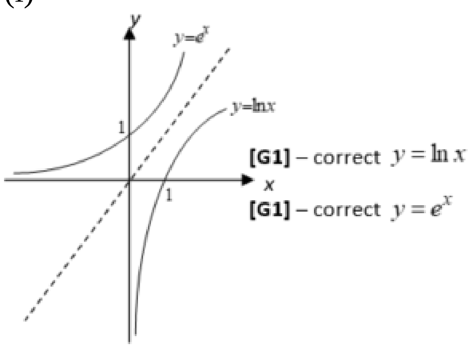
(ii) Describe using geometrical transformations how you would obtain the graphs of

(a) $y = \ln x^2$ from $y = \ln x$ [1]

(b) $y = 2 + e^{-x}$ from $y = e^x$ [2]

Solution:

(i)



(ii) (a)

$y = \ln x^2 \Rightarrow y = 2\ln x$.

Stretch graph of $y = \ln x$ by a scale factor of 2 parallel to the *y*-axis

(iii) (b)

Step 1: Reflect the graph of $y = e^x$ about *y*-axis to obtain $y = e^{-x}$.

Step 2: Translate the graph of $y = e^{-x}$ in the positive y-direction by 2 units to obtain $y = 2 + e^{-x}$.

Question 2:

A graph with equation $y = -x^3 + 3$ undergoes the following two successive transformations.

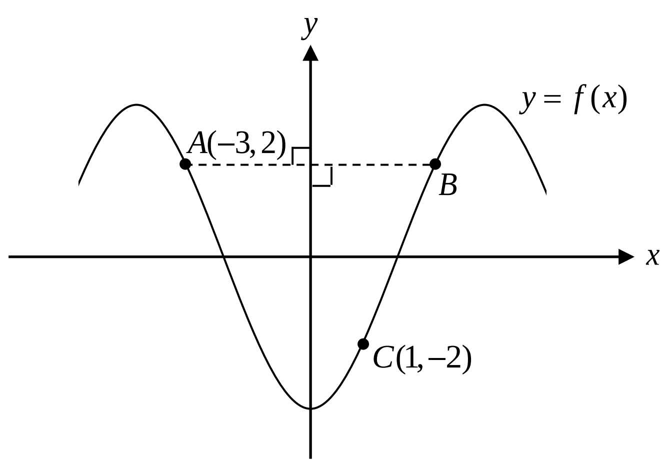
I A translation of 3 units in the positive y direction,

II A stretch parallel to the y -axis with a scale factor of 2 .

Find the equation of the resultant graph $y = \mathrm{g}(x)$. [2]

Solution:

I :
$$y = -x^3 + 3 + 3$$
$$y = -x^3 + 6$$

II : $y = 2(-x^3 + 6)$ $y = -2x^3 + 12$ $\therefore g(x) = -2x^3 + 12$
<u>Question 3:</u> Given that $f(x) = \ln x$ and $g(x) = 2\ln(3+x)$, describe the transformation of the graph of $f(x)$ to the graph of $g(x)$ in two successive steps. [2]
<u>Solution:</u> **<u>Step 1:</u>** Translate the graph of $y = \ln x$ in the negative x-direction by 3 units to obtain $y = \ln(x+3)$ **<u>Step 2:</u>** Stretch the graph of $y = \ln(x+3)$ by a scale factor of 2 parallel to y-axis to obtain $y = 2\ln(x+3)$.
<u>Question 4:</u> Given that the graph of $y = 5e^{-x}$ is drawn, find the equation of the straight line to be drawn to obtain the solution to the equation $(x-2)e^x = 5$. [2]
<u>Solution:</u> $(x-2)e^x = 5$ $x - 2 = \frac{5}{e^x}$ $y = x - 2$
<u>Question 5:</u> The diagram below shows partially the graph of a function *f* which is symmetrical about the *y*-axis. The points *A*, *B* and *C* lie on the graph.  **(a)** Write down the new coordinates of *C* under the transformation $y = 3f(x)$. **(b)** Sketch the graph of $y = f(x+2)$, and label clearly the new coordinates of *A*, *B* and *C*.

Solution:

(a) $C_{new}:(1,-6)$

(b)

$A:(-3,2)$

$A_{new}:(-5,2)$

$B:(3,2)$

$B_{new}:(1,2)$

$C:(1,-2)$

$C_{new}:(-1,2)$