

1. (a) Factorise the following expression completely. [2]

$$8x^3 - 27y^3$$

- (b) Express the following as a single fraction in its simplest form.

$$\frac{2}{x-3} - \frac{x+1}{x^2-4x+3} - \frac{x}{1-x} \quad [4]$$

2. Use the substitution  $u = \sqrt{x-5}$  to solve the equation

$$3\sqrt{x-5} + \frac{8}{\sqrt{x-5}} = 14. \quad [4]$$

3. Solve the equation  $x + 3\sqrt{x+2} = 2$ . [4]

4. (a) It is given that  $(2 - \sqrt{5})^2 - \frac{20}{3 + \sqrt{5}} = p + q\sqrt{5}$ , where  $p$  and  $q$  are integers.

Find the value of  $p$  and of  $q$ . [4]

- (b) Find the values of  $x$  and  $y$  which satisfy the equations

$$4^{x+y} \times 2^{-3y} = 1,$$

$$\frac{81^x}{3^{3x+2y}} = \frac{1}{27}. \quad [4]$$

- (c) Given that  $y = x^2 + (w+6)x + \frac{3}{4}w + 18$ , where  $w$  is an integer. Find the value(s) of  $w$  such that  $x$ -axis is a tangent to the graph of  $y$ . [4]

[Turn Over]

5. Solve the following equations.

(a)  $\log_4(2-x) - 1 = \log_{16} x + \log_4\left(\frac{1}{4}\right)$  [5]

(b)  $5^{2x-3} = 2e^{x+2}$  [4]

6. A glass of liquid substance is left to cool in a room. Its temperature  $T^\circ\text{C}$  after  $t$  minutes, is given by  $T = 25 + T_0 e^{-at}$ , where  $T_0$  and  $a$  are constants. The initial temperature of the liquid substance is  $85^\circ\text{C}$  and it drops to  $70^\circ\text{C}$  after 10 minutes.

(i) Show that  $T_0 = 60$  and  $a = 0.0288$ , correct to 3 significant figures. [3]

(ii) Find the temperature of the liquid substance an hour from the beginning, giving your answer to one decimal place. [2]

(iii) After a very long time, estimate the temperature of the liquid substance. Explain your answer. [2]

7. The equation of a curve is  $y = -x^2 - 6x + 3$ . Find the range of values of the constant  $m$  for which the line  $y = mx + 4$  intersects the curve at 2 distinct points. [4]

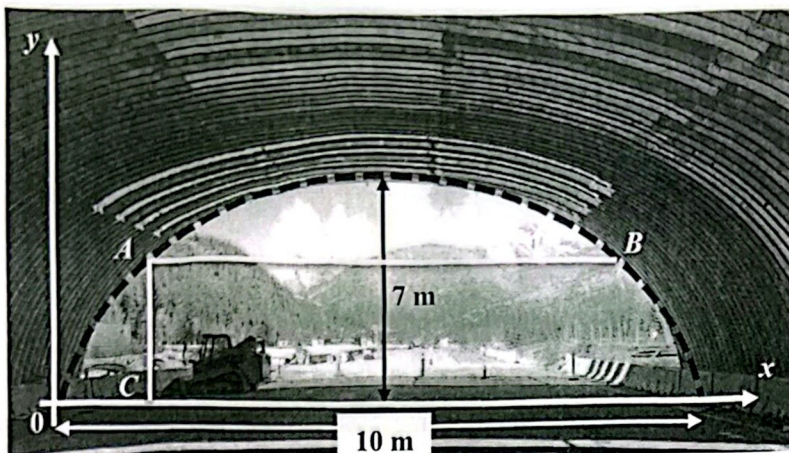
8. Given a cubic polynomial  $f(x) = 10x^3 + x^2 - 5x + 1$ ,

(i) show that  $2x - 1$  is a factor of  $f(x)$ . [1]

(ii) Hence, solve the equation  $f(x) = 0$ , leaving your answers in exact form. [5]

9. A polynomial  $P(x)$  leaves a remainder of 4 when divided by  $x - 3$  and a remainder of  $-1$  when divided by  $x + 2$ . Find the remainder when  $P(x)$  is divided by  $(x - 3)(x + 2)$ . [4]

10. An arched underpass has the shape of a parabola as shown below.



In the diagram,  $x$  m is the horizontal distance from left end of the arch on the ground and  $y$  m is the height of the arch above the ground. A one-way road passing under the arch is 10 m wide and the maximum height of the arch is 7 m.

The width of the arch at a point is defined as the horizontal distance from that point to the other side of the arch. One such width is illustrated in the figure above,  $AB$ .

The height of the arch at a point is defined as the vertical distance from that point to the level ground. One such height is illustrated in the figure above,  $AC$ .

- (i) Find a quadratic function in the form  $y = a(x-h)^2 + k$  to represent the arch, where  $a$ ,  $h$  and  $k$  are constants. [4]
- (ii) A point on the arch is 2.5 m vertically from the ground. What is the width of the arch at this point? [3]
- (iii) Find the height of the arch when its width is 4 m. [2]
- (iv) Decide whether it is possible for a truck that is 4 m wide and 6 m tall to navigate through the underpass. Explain your answer. [2]
11. The graph of a cubic polynomial function  $y = f(x)$  crosses the  $x$ -axis at  $2$ ,  $\sqrt{3}$  and  $-\sqrt{3}$  and crosses the  $y$ -axis at 30. Find an expression for the cubic polynomial  $f(x)$ , express it in the general form,  $f(x) = ax^3 + bx^2 + cx + d$ , where  $a$ ,  $b$ ,  $c$  and  $d$  are integers. [4]

[Turn Over]

12. Answer the whole of this question on a sheet of Graph Paper.

The table shows experimental values of two variables,  $x$  and  $y$ , which are connected by an equation of the form  $e^y = ax^b$ , where  $a$  and  $b$  are constants.

|     |      |      |      |      |      |
|-----|------|------|------|------|------|
| $x$ | 2    | 3    | 5    | 7    | 10   |
| $y$ | 2.28 | 2.94 | 3.69 | 4.30 | 4.88 |

- (i) Plot  $y$  against  $\ln x$  for the given data and draw a straight line graph. [3]
- (ii) Use your graph to estimate the value of  $a$  and of  $b$ . [4]
- (iii) By drawing a suitable straight line on the same axes, or otherwise, estimate the value of  $x$  when  $e^y = x^5$ . [2]

End of Paper