SUMMARY: Equations and Inequalities

Graphs of Polynomials

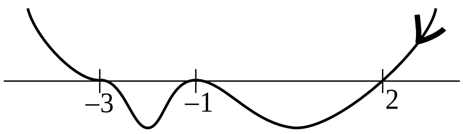
Make sure the coefficient of the highest power is positive so that graph always start from the top right-hand corner.

Factors $(x-a)^n$

- n is odd: cuts the x-axis
- n is even: Turning point at x-axis

E.G. $(x+1)^2(2-x)(3+x)^3 \le 0$

$\Rightarrow (x+1)^2(x-2)(3+x)^3 \ge 0$



$\Rightarrow x \ge 2,\ x = -1 \text{ or } x \le -3$

Other Methods

- number line
- substitute values using GC

Rational Functions

- Do not cross multiply! Make one side zero.
- Always factorise! If there are no roots, that means the expression is always positive or negative. Use discriminant or complete square to justify.
- For exact roots of quadratic equations, apply the formula if necessary, i.e. $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$.
- Do not include the x-values that makes the denominator 0.

E.G. $\dfrac{12}{x-3} \ge x+1$

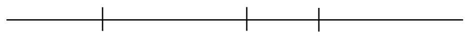
$\dfrac{12}{x-3} - (x+1) \ge 0$ $\left\{\begin{array}{l}\text{bring all the terms to one side}\\ \text{make one side zero}\end{array}\right.$

$\Rightarrow \dfrac{12-(x+1)(x-3)}{x-3} \ge 0$

$\Rightarrow \dfrac{12-x^2+2x+3}{x-3} \ge 0$

$\Rightarrow \dfrac{x^2-2x-15}{x-3} \le 0$ $\left\{\begin{array}{l}\text{make coefficient of}\\ \text{highest power +ve}\end{array}\right.$

$\Rightarrow (x-5)(x+3)(x-3) \le 0,\ x \ne 3$ $\left\{\begin{array}{l}\text{multiply by}\\ (x-3)^2\end{array}\right.$



$\Rightarrow x \le -3 \text{ or } 3 < x \le 5$

E.G. $\dfrac{x^2+2x+5}{x^2+3x-28} < 0$

$x^2+2x+5 = (x+1)^2+4 > 0$ for all x

$\therefore \dfrac{1}{x^2+3x-28} < 0$ $\left\{\begin{array}{l}x^2+2x+5 \text{ can be divided}\\ \text{without affecting sign}\end{array}\right.$

$\Rightarrow \dfrac{1}{(x+7)(x-4)} < 0 \Rightarrow -7 < x < 4$

Modulus Function

$$|\Delta| = \begin{cases} \Delta, & \Delta \ge 0 \\ -\Delta, & \Delta < 0 \end{cases}$$

Two very useful inequalities

$|x| < a \Leftrightarrow -a < x < a$

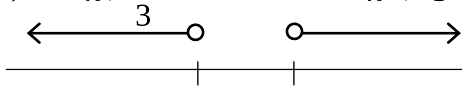
$|x| > a \Leftrightarrow x < -a$ or $x > a$

E.G. $|2x+1| < x-2$

$-(x-2) < 2x+1 < x-2$

$\Rightarrow -x+2 < 2x+1$ AND $2x+1 < x-2$

$\Rightarrow \quad x > \dfrac{1}{3} \qquad x < -3$



no intersection, therefore no solution.

Squaring both sides can only be used when both sides are always positive. Bring to one side and factorise after squaring both sides!

E.G. $|2x+3|<|3x+1|$

Square both sides

$(2x+3)^2<(3x+1)^2$

$\Rightarrow (3x+1)^2-(2x+3)^2>0$ $\left\{\begin{array}{l}\text{factorise using}\\ a^2-b^2=(a+b)(a-b)\end{array}\right.$

$\Rightarrow (3x+1+2x+3)(3x+1-2x-3)>0$

$\Rightarrow (5x+4)(x-2)>0$

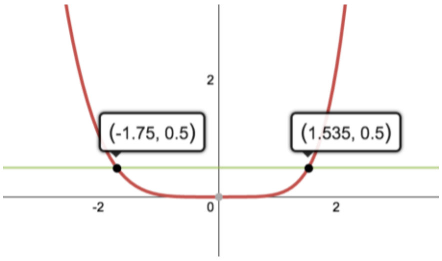
$\Rightarrow x>2$ or $x<-\frac{4}{5}$

Solve Inequalities using GC

E.G. Find the values of x for which xe^x is within 0.5 of $x+x^2+\frac{1}{2}x^3$.

Find x s.t. $\left|xe^x-(x+x^2+\frac{1}{2}x^3)\right|<0.5$

Sketch $y=\left|xe^x-(x+x^2+\frac{1}{2}x^3)\right|$ and $y=0.5$.



$\therefore -1.75<x<1.535$

Equations

Polynomial Equations

Solve using GC app PlySmlt2

Solving Equations Using GC

E.G. Find the points of intersections of the curves $y=\ln(x-2)$ and $y=\sqrt{x+2}$

Plot $y=\ln(x-2)-\sqrt{x+2}$ and find the intersection with the x-axis using "zero"

System of Linear Equations

Form linear equations and solve using GC app PlySmlt2

- Equations with more unknowns than equations

E.G. At a funfair, Royston spent \$76 on 20 food items consisting of cup corn, nachos and churros to share with his friends. The cost of each portion of cup corn, nachos and churros are \$3, \$4 and \$5 respectively. Given that he bought at least 5 portions of each food item, how many portions of each food item did he buy?

Solution: Let the number of portions of cup corn, nachos and churros be x, y and z respectively.

$$3x+4y+5z=76$$
$$x+y+z=20$$

From GC,

$$x=4+z\ \geq 5 \Rightarrow\ z\geq 1$$
$$y=16-2z\geq 5\ \Rightarrow\ z\leq 5.5$$
$$z=z\geq 5$$

Since x, y and z must be at least 5, we deduce that $z=5$, $x=9$, $y=6$