

Name: _____ () Class: _____



華僑中學

HWA CHONG INSTITUTION

Practice Paper B

INTEGRATED MATHEMATICS

Level : Secondary Three

Duration : 2 hours 30 minutes

Do not open this booklet until you are told to do so.

INSTRUCTIONS TO CANDIDATES

Write your name, class and index number on all the work you hand in.

Answer **all** questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 100.

You are expected to use an electronic calculator to evaluate explicit numerical expressions. **The use of a graphing calculator is not permitted.**

You are reminded of the need for clear presentation in your answers. Omission of essential working will result in loss of marks.

This question paper consists 6 printed pages, including this page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 (a) Factorise completely $(x-3)^3 + (x-1)^3$. [3]

(b) Simplify $\frac{x - \frac{x-1}{2}}{2 - \frac{x-1}{x}}$. [3]

2 The area of a rectangle $ABCD$ is given by $(1+3\sqrt{2})\text{m}^2$, and the length of side AB is $(2+\sqrt{8})\text{m}$. Find, without using a calculator, the length of side BC in the form of $(a+b\sqrt{2})\text{m}$, where a and b are real numbers. [3]

3 Solve the equations

(a) $\log_{(x-1)}(3x^2 - 7x - 2) = 2$, [3]

(b) $\ln|\sqrt{x} - 2| + \frac{1}{2}\ln x = 0$, where $x > 0$. [4]

4 Two outlets of a local bakery sold the following types of cakes on a particular day.

	Pandan cake	Chocolate cake
Outlet A	100	120
Outlet B	70	85

The costs of baking a pandan cake and a chocolate cake are \$5 and \$6 respectively. In both outlets, each pandan cake was sold at \$ x , and each chocolate cake was sold at \$ y .

It is given that $\mathbf{P} = \begin{pmatrix} 100 & 120 \\ 70 & 85 \end{pmatrix}$, $\mathbf{Q} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$ and $\mathbf{R} = \begin{pmatrix} x \\ y \end{pmatrix}$.

(i) Write down an expression connecting $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{R}$, such that it represents the profit earned by each outlet on that particular day. [1]

(ii) If the profits earned by Outlet A and Outlet B on that particular day were \$1710 and \$1205 respectively, find the selling price for each type of cake using the inverse matrix method. [4]

5 Find the range of values of m for which $y = mx^2 - (m+1)x + m$ is always positive for all real values of x . [4]

- 6 It is given that the equation $x^2 + 2k + 1 = (k + 2)x$ has two real roots α and β .
- (i) Find the range of values of k . [3]
- (ii) If $\alpha^2 + \beta^2 = 11$, determine the value(s) of k . [3]
- 7 (a) Solve the equation $2x^3 = 5x^2 - x - 2$. [4]
- (b) Find the remainder when $(x - 3)^{200} - (x - 2)^{100}$ is divided by $x^2 - 5x + 6$. [4]
- 8 S is the sum of two parts. One part varies directly as the square of x and the other part varies inversely as y . When $x = 2$ and $y = 3$, $S = 1$. When $x = 4$ and $y = 6$, $S = 7\frac{1}{2}$. Express S in terms of x and y . [3]
- 9 (a) Solve the trigonometric equation $6 - 4\cos^2\left(\frac{x}{2} + 30^\circ\right) = 9\sin\left(\frac{x}{2} + 30^\circ\right)$ for $0^\circ \leq x \leq 720^\circ$. [4]
- (b) Prove that $2\sin^2 x - \frac{\tan x - \cot x}{\tan x + \cot x} = 1$. [3]
- 10 Answer the whole of this question on a sheet of graph paper.

The table shows experimental values of two variables x and y . It is known that x and y are related by an equation $y = ae^{-bx}$, and that one of these values of y given in the table is subject to an abnormally large error.

x	1.5	2.5	3.5	4.5	5.0	6.0
y	9.9	6.0	3.6	2.2	1.7	1.4

- (i) Draw a straight line graph of $\ln y$ against x , using a scale of 2 cm to 1 unit on the x -axis and 1 cm to 0.2 units on the $\ln y$ -axis. [3]
- (ii) Use your graph to estimate the value of a and of b . [3]
- (iii) Identify the abnormal reading of y and estimate its correct value. [2]

- 11** A function f is defined, for $x \neq 0$, by $f : x \mapsto \frac{a}{x} + 1$, where a is a real constant.

Given that $2f(-1) + f^{-1}(2) = 4$, calculate the value of a . [4]

- 12** The function f is given by $f : x \mapsto 2 - 3\cos 2x$ for the domain $0^\circ \leq x \leq 180^\circ$. Sketch the graph $y = f(x)$.

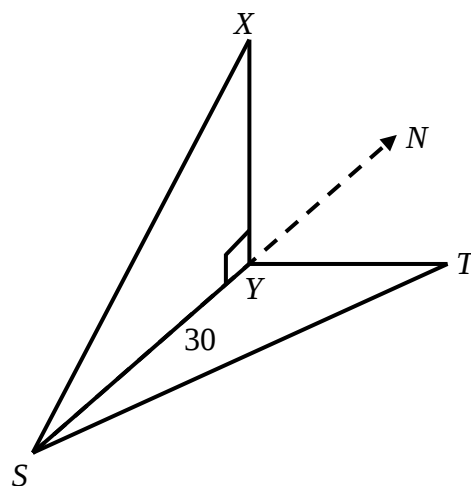
Hence, state the range of the function for its given domain. [4]

- 13 (a)** Sketch the graphs of $y = 10^x$ and $y = \left| \frac{1}{2}(2x-1)(x-2) \right|$ on the same axes, showing clearly the x -intercepts, y -intercepts and turning points, if any.

Hence, state the range of values of x such that $10^x < \left| \frac{1}{2}(2x-1)(x-2) \right|$. [5]

- (b)** Describe the sequence of transformations to obtain a graph of $y = 2\ln(x-1)$ from the graph of $y = e^x + 1$. [2]

- 14** Three points, S , Y and T are on level ground. Y is the foot of a vertical flagpole XY , and S is 30 m due south of Y .

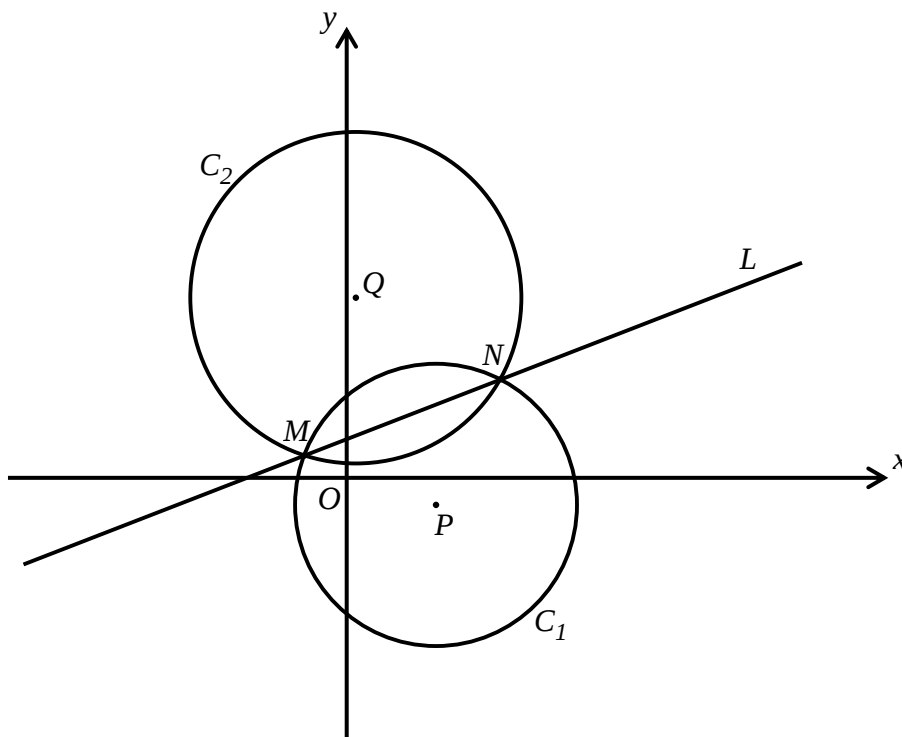


- (i)** The angle of elevation of X from S is 25° . Calculate the height of the flagpole. [2]
- (ii)** The bearing of T from S and the bearing of T from Y are 042° and 075° respectively, calculate the distance ST . [3]
- (iii)** A man is standing at the midpoint of ST . Calculate the angle of elevation of X from the man. [4]

- 15 Given that θ is an angle that lies in the 4th quadrant, simplify the expression

$$\frac{1}{\cos \theta \sqrt{1 + \tan^2 \theta}} + \frac{2 \cot \theta}{\sqrt{\frac{1}{\sin^2 \theta} - 1}}. \quad [3]$$

- 16 The diagram shows two intersecting circles C_1 and C_2 with centres P and Q respectively. A straight line L passes through M and N , which are the points of intersections of the two circles.



- (i) Given that the equation of the circle C_1 is $x^2 + y^2 - 6x + 4y - 21 = 0$, find the coordinates of P and the radius of C_1 . [3]
- (ii) It is also given that the equation of the line L is $4y - x = 6$, find the coordinates of M and of N . [4]
- (iii) Find the equation of the perpendicular bisector of MN . [3]
- (iv) Given that the radius of circle C_2 is $\frac{1}{2}\sqrt{221}$ units, find the coordinates of Q . [4]
- (v) Calculate the area of the quadrilateral $PMQN$. [2]

End of Paper