華僑中學

Hwa Chong Institution
Secondary 3 Examination 2021

CANDIDATE NAME					
CLASS				REGISTER NUMBER	

Higher Mathematics 1 hour

Additional materials:

Writing papers

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work that you hand in.
Answer **4 out of 7** questions.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 40.

This document consists of **4** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Mensuration

Arc Length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

2. TRIGONOMETRY

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Answer 4 out of 7 questions

1 **(a)** Expand and simplify $x^3+y^3-(x+y)^3+3xy(x+y)$. [2]

(b) **(i)** Explain why the minimum value of x^2-4x+6 is 2. [2]

(ii) Considering **(i)**, show that the range of values of x satisfying

$$\frac{1}{x+1}\geq\frac{x^2-4x+6}{6}$$ is ~~$x<-1$~~ or $-1<x\leq 0$ or $1\leq x\leq 2$. [3]

(iii) Hence or otherwise, solve the inequality $\frac{1}{\ln x+1}-1\geq\frac{(\ln x)^2-4\ln x}{6}$. [3]

2 A circle passes through points (–1, 1), (3, 1) and (3, –1).

(i) Write down the coordinates of the centre. [2]

(ii) Find the equation of the circle. [2]

(iii) Find the range of values of k if point $(k+1,k-1)$ lies inside the circle. [3]

(iv) Find the minimum distance from point (3, 4) to a point on the circle. [3]

3 **(a)** Factorise $x^6+4x^2y^4$ completely. [4]

(b) A function f is defined as $\mathrm{f}:x\mapsto(x-1)^2-5, x\leq 0$.

(i) Find f^{-1}, stating the domain clearly. [3]

(ii) By considering the line of symmetry of the graph $y=\mathrm{f}(x)$ and $y=\mathrm{f}^{-1}(x)$, solve the equation $\mathrm{f}(x)=\mathrm{f}^{-1}(x)$. [3]

4 **(a)** Solve the equation $\log_9 x-2\log_x 27=2$. [5]

(b) A parabola with the equation $(y-1)^2=4x-4$ has the directrix $x=k$ and the focus point $F(a, b)$.

(i) State the value of k. [1]

(ii) Find the coordinates of the point on the parabola such that the distance from the point to the focus point F is 5 units. [4]

[Turn Over

5 **(a)** Solve the equation $\left(\sqrt{2+\sqrt{3}}\right)^{x} = 2-\sqrt{3}$. [3]

(b) The function f is defined by $\mathrm{f}: x \mapsto \mathrm{e}^{-x} - 2$.

(i) Explain why $\mathrm{f}(|x|) = 0$ has no real solution. [2]

(ii) Sketch $y = \mathrm{f}(|x|)$, indicating clearly the x-intercepts, the y-intercepts and the asymptotes, if any. [4]

(iii) Hence, state the number of solutions for the equation $2\mathrm{f}(|x|) + 3 = 0$. [1]

6 **(a)** A polynomial $\mathrm{f}(x) = ax^5 - bx^3 + x^2 + cx + 4$, where a, b and c are constants, has a remainder of 10 when it is divided by $(x+1)$. Show that the polynomial has a factor $(x-1)$. [4]

(b) In triangle ABC, $\angle A = 60°$, $AB = 4$ cm and $BC = 6$ cm.

(i) State the number of such possible triangles. Justify your answer. [2]

(ii) **Without using a calculator,** find the possible area(s) of triangle ABC, leaving your answer(s) in the form $\left(a\sqrt{2} + b\sqrt{3}\right)$ cm where a and b are integers. [4]

7 **(a)** If $\mathrm{f}(x) = \dfrac{\lg x}{\lg\left(2022x - x^2\right)}$, show that $\mathrm{f}(x) + \mathrm{f}(2022 - x) = 1$. Hence find the sum $\mathrm{f}(1) + \mathrm{f}(2) + \mathrm{f}(3) + \cdots + \mathrm{f}(2021)$. [4]

(b) It is given that $\mathbf{A} = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.5 \end{pmatrix}$ and $\mathbf{B}$ is a matrix such that $\mathbf{AB} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

(i) Calculate matrix $\mathbf{B}$. [3]

(ii) Find $\mathbf{B}^2$ and $\mathbf{B}^3$. [2]

(iii) Hence, write down a formula for $\mathbf{B}^n$ where n is a positive integer. [1]

End of Paper

華僑中學 HWA CHONG INSTITUTION

Name Wu An Hao Subject Math Date 11th Oct

Q	1	2	3	4	5			
M	4	9	8	4	3			

C.T Group/Class 3I1 Serial No 27

Nothing is to be written on this margin

25/40

1. (a) $x^3+y^3-(x+y)^3+3xy(x+y)$

$= x^3+y^3+3x^2y+3xy^2-(x+y)^3$ ✓

$= (x+y)^3-(x+y)^3$

$= ⓵$ ✗ 1

(b)(i) y-value of Turning point $= \frac{4\times1\times6-4^2}{4}$

$= 2$ ✓

Since the minimum range of x^2-4x+6 is 2, the minimum value is 2. ✓ 2

(ii) $\frac{1}{x+1} \geq \frac{x^2-4x+6}{6}$

$\frac{6}{x+1} \geq x^2-4x+6$

$\frac{6}{x+1} \cdot \frac{1}{x^2-4x+6} \geq 0$

$(x+1)(x^2-4x+6) \leq 6$

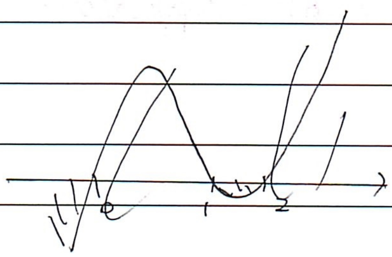
$x^3-4x^2+6x+x^2-4x+6 \leq 6$

$x^3-3x^2+2x \leq 0$

$(x-2)(x^2-x) \leq 0$

$(x-2)(x-1)x \leq 0$

$\therefore x \leq 0$ or $1 \leq x \leq 2$



Page

(ii) $\frac{6}{x+1} \geq x^2-4x+6$

Case 1:

$x+1>0$

$x>-1$

$6 \geq (x+1)(x^2-4x+6)$

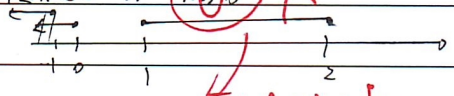
$x^3-4x^2+6x+x^2-4x+6 \leq 6$

$x^3-3x^2+2x \leq 0$

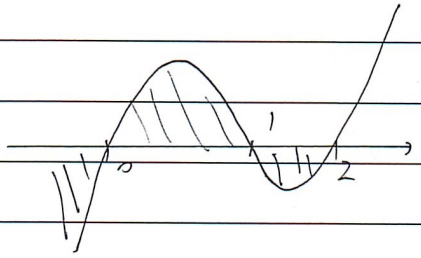
$(x-2)(x^2-x) \leq 0$

$(x-2)(x-1)x \leq 0$

$1 \leq x \leq 2$ or $x \leq 0$



$\therefore 1 \leq x \leq 2$



What happened?

Case 2:

$x+1<0$

$x<-1$

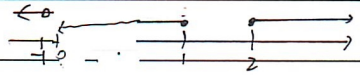
$6 \leq (x+1)(x^2-4x+6)$

$x^3-3x^2+2x \geq 0$

$(x-2)(x-1)x \geq 0$

doesn't exist.

$0 \leq x \leq 1$ or $x \geq 2$



$\therefore$

2. (i) Midpoint of $(-1,1)$ and $(3,-1)$: $\left(\frac{-1+3}{2}, \frac{1-1}{2}\right)$ ✓

$= (1,0)$ ✓

Center: $(1,0)$ 2

(ii) Distance between $(1,0)$ and $(3,1)$ $= \sqrt{(3-1)^2 + (1-0)^2}$ units ✓

$= \sqrt{5}$ units ✓

Eqn of circle: $(x-1)^2 + y^2 = 5$ ✓ 2

(iii) Distance between k and center of circle $= \sqrt{(k+1-1)^2 + (k-1-0)^2}$ ✓ units

$= \sqrt{k^2 + k^2}$ ✓ ✗

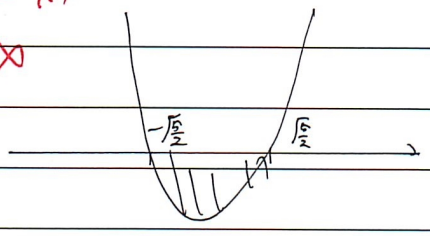
$= \sqrt{2k^2}$ ✗

$\sqrt{2k^2} < \sqrt{5}$ m2

$2k^2 < 5$

$k^2 < \frac{5}{2}$

$\therefore -\sqrt{\frac{5}{2}} < k < \sqrt{\frac{5}{2}}$ ✓



(iv) Distance between $(3,4)$ & center of circle: $\sqrt{(3-1)^2 + (4-0)^2}$ units ✓

$= \sqrt{20}$ units ✓

Minimum distance $= \sqrt{20} - \sqrt{5}$ units ✓ 3

$= 2.24$ units (3sf) ✓

華僑中學 HWA CHONG INSTITUTION

Name Wu Run Hao | Subject Math | Date 11th Oct
C.T Group/Class 3E1 | Serial No 27

Q								
M								

Nothing is to be written on this margin

3 (a) $x^6 + 4x^2y^4$

$= x^2(x^4 + 4y^4)$

$= x^2\{(x^2+2y^2)^2 - 4x^2y^2\}$

$= x^2\{(x^2+2y^2)^2 - (2xy)^2\}$

$= x^2(x^2+2y^2-2xy)(x^2+2y^2+2xy)$

4

(b)(i) $f(x) = (x-1)^2 - 5$

Let $f(x)$ be y.

$\therefore y = (x-1)^2 - 5$

$y+5 = (x-1)^2$

$x-1 = -\sqrt{y+5}$, ($\sqrt{y+5}$ reject as $x \leq 0$)

$x = -\sqrt{y+5} + 1$

$f(y) = -\sqrt{y+5} + 1$

$f^{-1}(x) = -\sqrt{x+5} + 1$

$R_f = D_{f^{-1}}$,

Domain of f^{-1} is $x \leq -4$

$\therefore f^{-1}: x \mapsto -\sqrt{x+5}, x \leq -4$ +1

1

(ii) $f(x) = f^{-1}(x) = x$

$f(x) = x$

$\therefore (x-1)^2 - 5 = x, x < 0$

$x^2 - 2x + 1 - 5 = x$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = -1$ (or 4 reject)

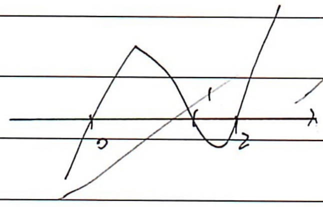
3

(ii) $\frac{(x^2-4x+6)(x+1)}{6} \leq 1$ or $\frac{(x^2-4x+6)(x+1)}{6} \geq 1$

$x^3 - \ldots \leq 0$ or $x^3-3x+2x \geq 0$

$(x-2)(x-1)x \leq 0$ or $(x-2)(x-1)x \geq 0$

$\therefore 1 \leq x \leq 2$ or $x \leq 0$ or $0 \leq x \leq 1$ or $x \geq 2$



4.(a) $\log_9 x - 2\log_x 27 = 2$

$\frac{\log_x x}{\log_x 9} - 3\log_x 9 = 2$

Let $\log_x 9$ be y, ($y>0$)

$\frac{1}{y} - 3y = 2$

$1-3y^2 = 2y$

$3y^2+2y-1=0$

$y = \frac{1}{3}$ or -1 (rejected)

$\log_x 9 = \frac{1}{3}$

$x^{\frac{1}{3}} = 9$ $\log_x$

$x = 729$

4

SINGAPORE