Temasek Junior College (IP)

2025 Year 4 Mathematics

AM

Unit 16 – Graphing with Graphing Calculator (GC)[1]

Lesson 3: More Graphing, Graphical Solutions and Equation Solving on the GC

Learning Objectives By the end of this lesson you should be able to: 1) Sketch the graphs of different functions with the aid of a graphing calculator (GC). 2) Solve equations using a GC. 3) Solve simple problems involving graphs using the GC.

You may be asked to use a GC to aid you in graphing a variety of functions, both seen and unseen. As it is not possible to explore every type of function, you should learn to be competent in the use of the GC, such that you are ready to plot any function when required.

In **Lessons 1** and **2**, we have attempted to plot some of the functions learned previously (polynomial, logarithmic, exponential, reciprocal, root *etc*) on the GC. In this lesson, we shall look at a few more.

3.1 Solving Inequalities using GC

Sometimes we may be given questions that we are unable to solve easily using algebraic methods learnt so far. In this case, GC is able to help us visualize the problems and obtain the solutions.

Example 1

(i) With the aid of a graphing calculator, sketch the graph of $y = x^3 + 2x^2 - 5x - 5$ and $y = 0.5x^2 + x + 1$, labeling coordinate of intersects and axial intercepts.

(ii) Hence find the values of x that satisfies

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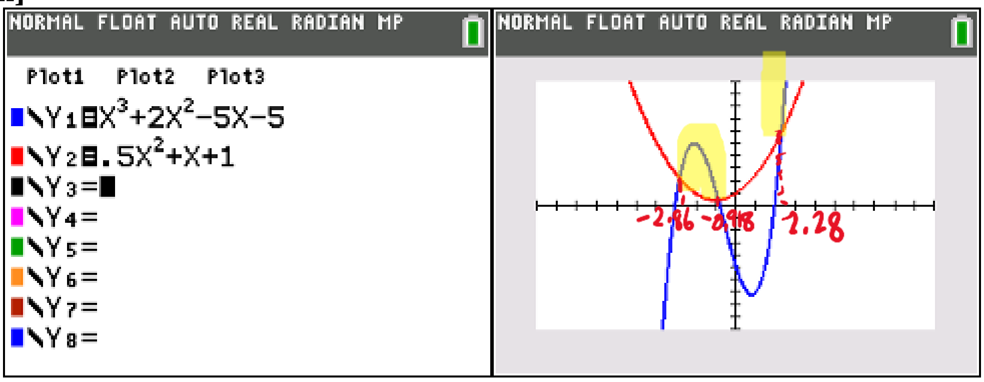
(a) $x^3 + 2x^2 - 5x - 5 = 0.5x^2 + x + 1$,

(b) $x^3 + 2x^2 - 5x - 5 > 0.5x^2 + x + 1$.

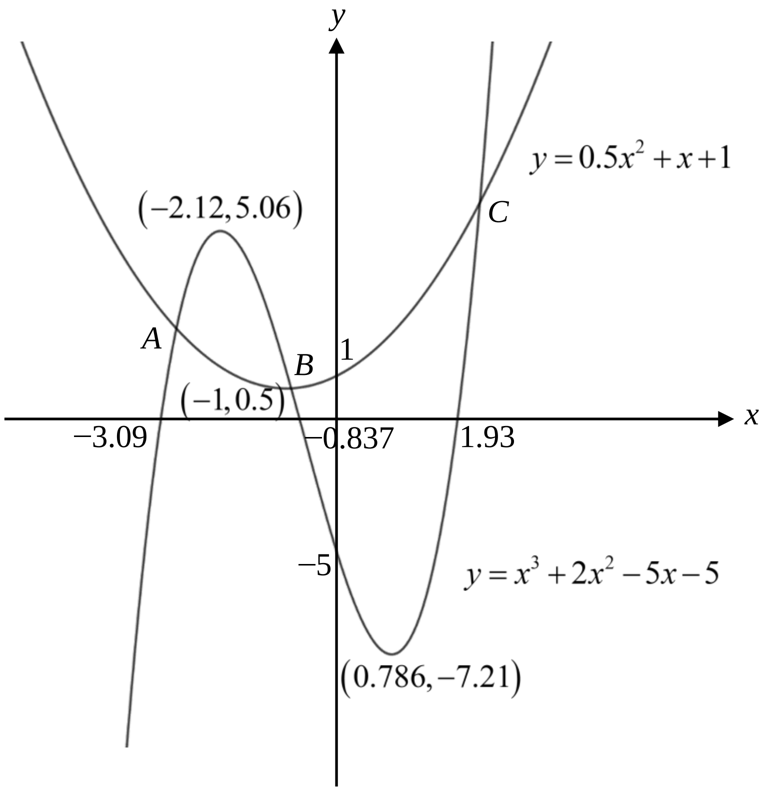
above

[Solution]



[1] *This set of notes is to be used in conjunction with the TI-84 Plus CE graphing calculator. Operation of other brands and models of graphing calculators may differ*

(i)



The coordinates of the intersection points A, B and C are $(-2.86, 2.24)$, $(-0.918, 0.503)$ and $(2.28, 5.89)$ respectively.

(ii) **(a)** The values of x that satisfy $x^3 + 2x^2 - 5x - 5 = 0.5x^2 + x + 1$ are the x-coordinates of the intersection points between the 2 graphs, $x = -2.86$, $x = -0.918$ or $x = 2.28$.

(b) The range of values of x that satisfy $x^3 + 2x^2 - 5x - 5 > 0.5x^2 + x + 1$ are the range of values of x when the y-coordinate of the cubic graph is greater than the y-coordinate of the quadratic graph.

$\therefore -2.86 < x < -0.918$ or $x > 2.28$.

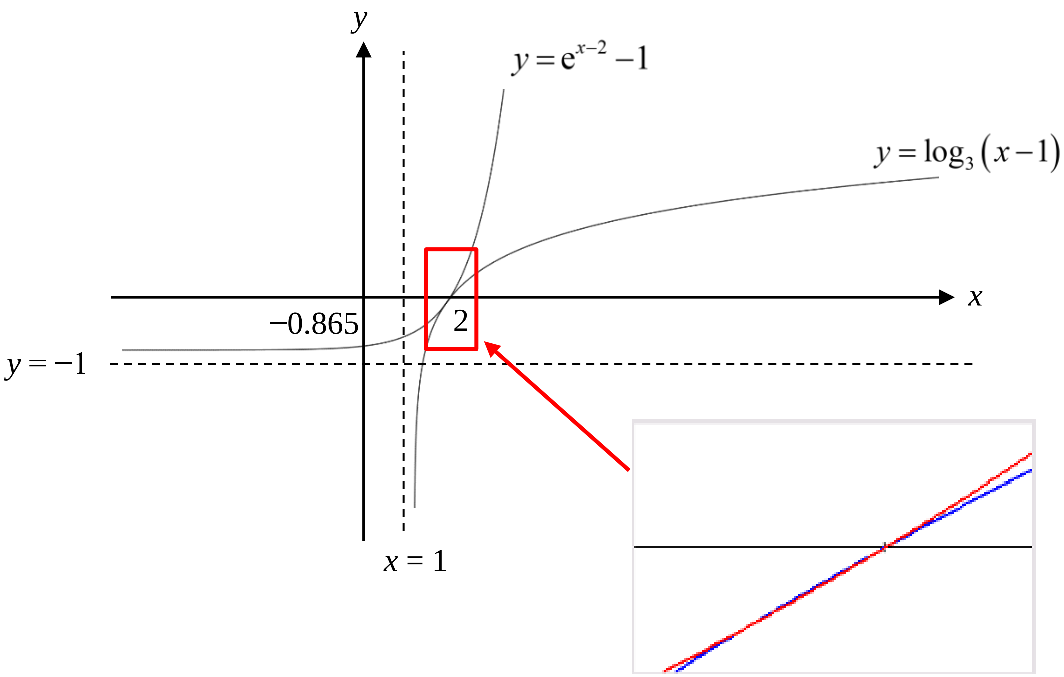
Exercise 1

With the aid of a graphing calculator,

(a) sketch the graph of $y = e^{x-2} - 1$ and $y = \log_3(x-1)$.

(b) solve $e^{x-2} - 1 = \log_3(x-1)$.

[Solution]



By zooming in closer to the region highlighted above, we see that there are two points of intersection instead of just one point of intersection. The coordinates of the intersection points between the graphs of $y = e^{x-2} - 1$ and $y = e^{x-2} - 1$ are $(1.91, -0.0884)$ and $(2, 0)$.

$\therefore$ The solutions for $e^{x-2} - 1 = \log_3(x-1)$ are $x = 1.91$ or $x = 2$.

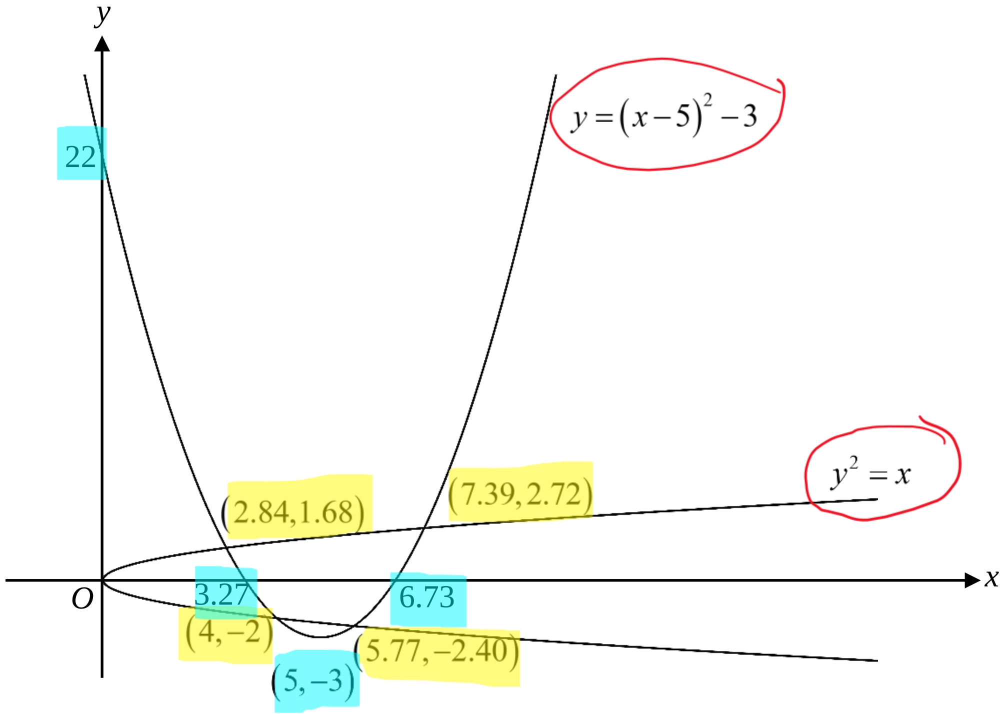
Note: this question was meant to highlight the importance of checking if there is more than one intersection point, especially when the GC does not give you a clear picture in standard view.

Exercise 2

With the aid of a graphing calculator, solve the pair simultaneous equations

$$\begin{cases} y^2 = x \\ y = (x-5)^2 - 3 \end{cases} \Rightarrow y = \pm\sqrt{x}$$

[Solution]

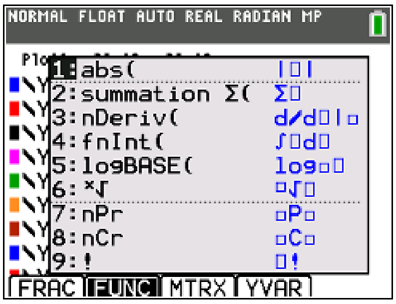


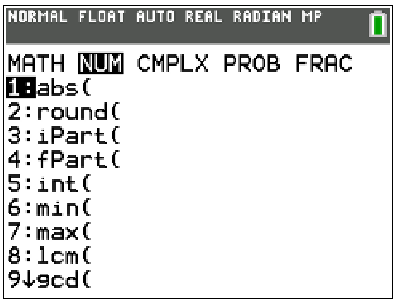
Using GC, the solutions are $x = 2.84$ or $x = 4$ or $x = 5.77$ or $x = 7.39$.

3.2 Modulus Functions

On the GC, the modulus is denoted as **abs(** and has the same symbolic form $|\ |$ as the modulus notation when written.

It can be accessed using ALPHA WINDOW 1 or MATH ▶ 1 (the 1st option in the NUM sub-menu). To make entries outside the modulus notation, press the ◀ or ▶ buttons until the cursor is outside the modulus notation.





<u>Exercise 3</u>

(i) With the aid of a graphing calculator, sketch the graph of $y=|x-2|$ and $y=|2x+1|$ labeling coordinate of intersects and axial intercepts.

(ii) Hence find the values of *x* that satisfies $|x-2|=|2x+1|$.

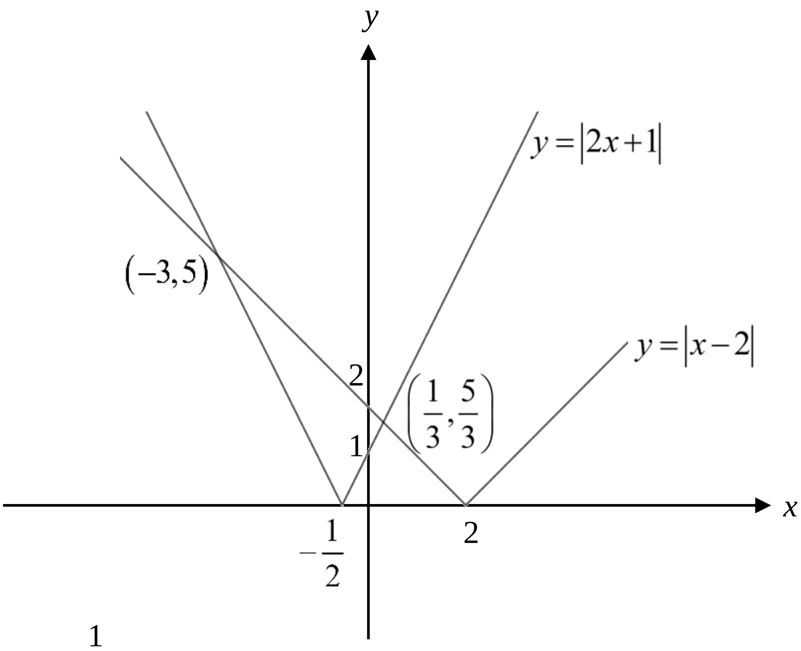
(iii) Using an algebraic method, find the values of *x* that satisfies $(x-2)^2=(2x+1)^2$. What do you notice about your answers in **(iii)** and in **(ii)**?

(iv) With the aid of a graphing calculator, explain if the solution to $|x-2|=|2x+1|$ satisfy the equation $|x-2|=2x+1$.

(v) Using your answers in **(ii),** find the values of *x* that satisfies $|x-2|\geq|2x+1|$.

[Solution]

(i)



(ii) $x=-3$ or $x=\frac{1}{3}$

(iii) $(x-2)^2=(2x+1)^2$

$x^2-4x+4=4x^2+4x+1$

$3x^2+8x-3=0$

$(3x-1)(x+3)=0$

$\therefore x=\frac{1}{3}$ or $x=-3$

The solutions for part **(ii)** and **(iii)** are the same. Letting $(x-2)^2=(2x+1)^2$ is a possible method to solve the equation $|x-2|=|2x+1|$.

(iv) The graphs of $y=|x-2|$ and $y=2x+1$ only intersect at $\left(\frac{1}{3},\frac{5}{3}\right)$, and not at $x=-3$. Therefore, not all solutions for $|x-2|=|2x+1|$ will satisfy the equation $|x-2|=2x+1$. Therefore, we must check additionally if the condition $2x+1\geq 0$ i.e. $x\geq-\frac{1}{2}$ is satisfied.

(v) From graph, $-3\leq x\leq\frac{1}{3}$.

3.3 Trigonometric Functions

Recall the following:

- Graphs of the sine, cosine and tangent functions are periodic.

- Graphs of the sine and cosine functions have maximum and minimum points while graphs of the tangent function have asymptotes.

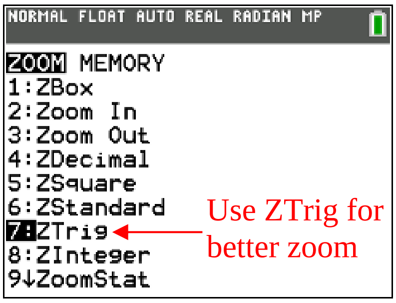
- Graphs of sine and tangent functions exhibit rotational symmetry while graphs of the cosine function exhibit line symmetry.

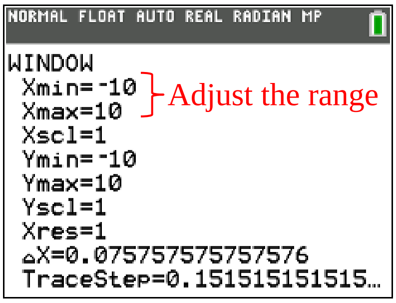
The sine, cosine and tangent functions can be accessed via the SIN, COS and TAN buttons.

In most circumstances, you will need to graph trigonometric functions over a range of values. To ensure a good display on the GC, the following steps can be taken in sequential order:

- Use ZOOM 7, ZTrig, to obtain a better zoom of the function.

- Use WINDOW to adjust Xmin and Xmax to the given range.





<u>IMPORTANT!</u>

- Always check that the GC is in the correct angle measurement (degree mode or radian mode) before graphing a trigonometric function

- When required to graph **BOTH** trigonometric and non-trigonometric functions on the GC, ensure that your GC is **SET TO RADIAN MODE**.

Example 5 (Self-Read)

With the aid of a graphing calculator, sketch the graph of $y = 3\sin(2x)$ for $-\pi \le x \le \pi$, stating the turning points and intersections with the axes. State

(i) the maximum and minimum values of *y*, and

(ii) the period of the function.

By adding a suitable line,

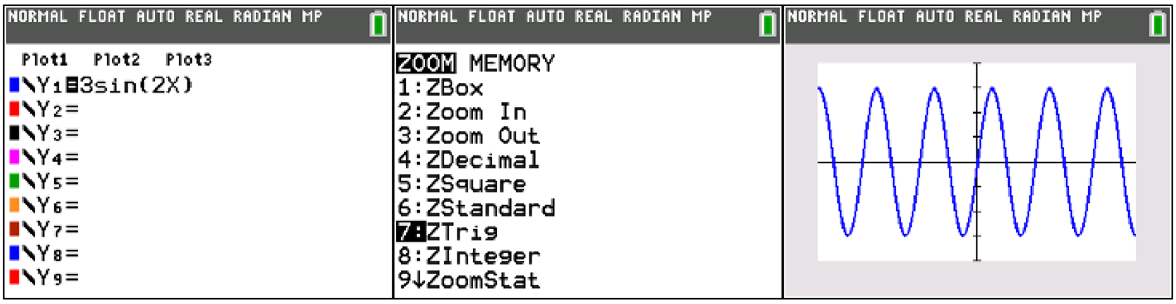
(iii) deduce the number of solutions to $3\sin(2x) - 1 + 2x = 0$ for $-\pi \le x \le \pi$, and

(iv) solve the equation $3\sin(2x) - 1 + 2x = 0$ for $-\pi \le x \le \pi$, giving your answer to 3 significant figures.

[Solution]

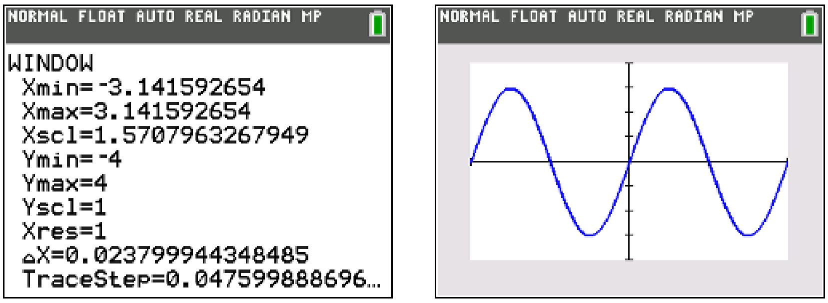
- Enter the function $y = 3\sin(2x)$ on the graphing interface.

- Press $\boxed{\text{ZOOM}}$ $\boxed{7}$ to set the display to ZTrig. The graph will be displayed automatically.



- Adjust the display using $\boxed{\text{WINDOW}}$ to change Xmin to $-\pi$ and Xmax to π.

- Press $\boxed{\text{GRAPH}}$ to display the graph with the adjusted settings.



To obtain Xmin and Xmax above, use $\boxed{(-)}$ $\boxed{\text{2ND}}$ $\boxed{\wedge}$ to get $-\pi$ and $\boxed{\text{2ND}}$ $\boxed{\wedge}$ to get π.

(i) To obtain the maximum and minimum values, consider $-1 \le \sin(2x) \le 1$.

$\Rightarrow -3 \le 3\sin(2x) \le 3$

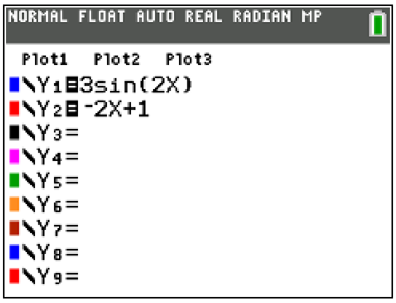
Hence the maximum value of y is 3 and the minimum value of y is -3.

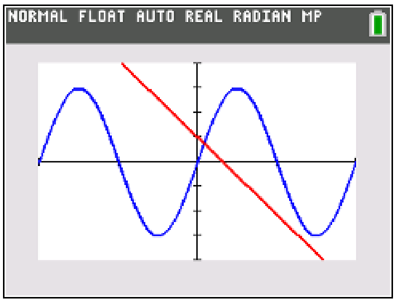
There is no need to use the GC to locate the maximum and minimum values in this case since the range of values of y can be worked out quickly (as long as there is a sound understanding of the sine function in this case).

(ii) Period $=\dfrac{2\pi}{2}=\pi$

(iii) $3\sin(2x)-1+2x=0 \;\Rightarrow 3\sin(2x)=-2x+1$

Hence the line $y=-2x+1$ needs to be added.



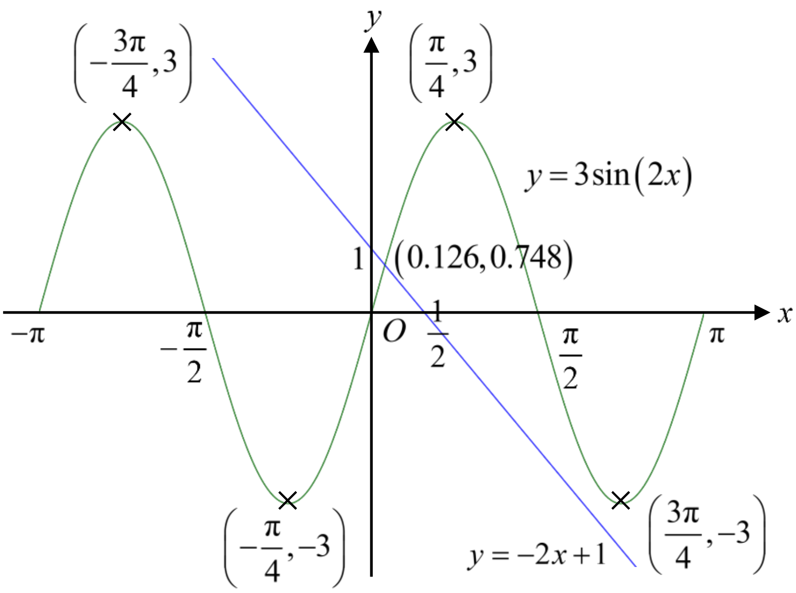


From GC, there is 1 point of intersection between the 2 graphs. Hence there is 1 solution to the equation $3\sin(2x)-1+2x=0$, for $-\pi\le x\le\pi$.

(iv) **Question:** Can you determine the solution using your GC?

The solution to the equation $3\sin(2x)-1+2x=0$, for $-\pi\le x\le\pi$ is $x=0.126$ (3 s.f.).

Sketch the actual graphs, labelling all the asymptotes, intercepts and intersections.



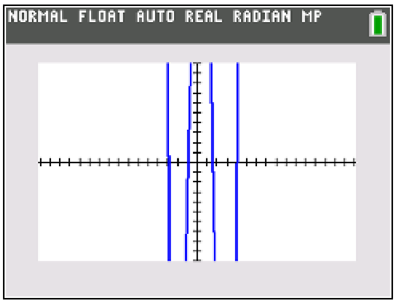
3.4 Customising Window Settings (Self-Read)

Example 6

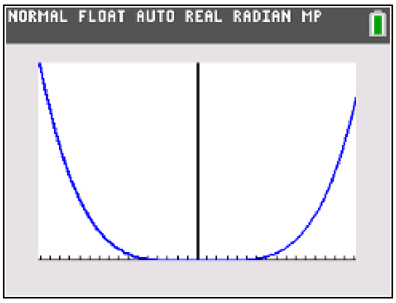
With the aid of a graphing calculator, sketch the graph of $y = 2x^4 - 3x^3 - 26x^2 + 15x + 36$ and find all the turning points and intersections with the axes.

[Solution]

Entering the function on the graphing interface and graphing it on ZStandard ($\boxed{\text{ZOOM}}$ $\boxed{6}$) setting, we obtain the following:



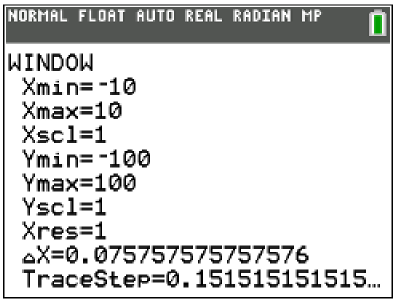
Using ZoomFit, we obtain the following:

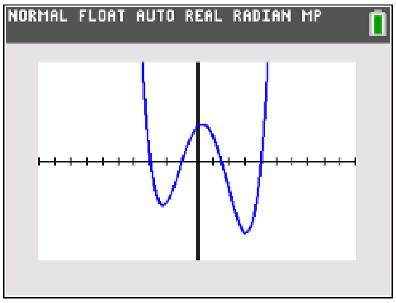


** You may try other* $\boxed{\text{ZOOM}}$ *settings but will unlikely be able to obtain a satisfactory display.*

One of the limitations of the GC is that you might not be able to view the important characteristics of a graph using the $\boxed{\text{ZOOM}}$ settings provided. In this case, you will need to make use of your knowledge of the function and customize the $\boxed{\text{WINDOW}}$ settings instead. This will involve adjusting Xmin, Xmax, Ymin and Ymax.

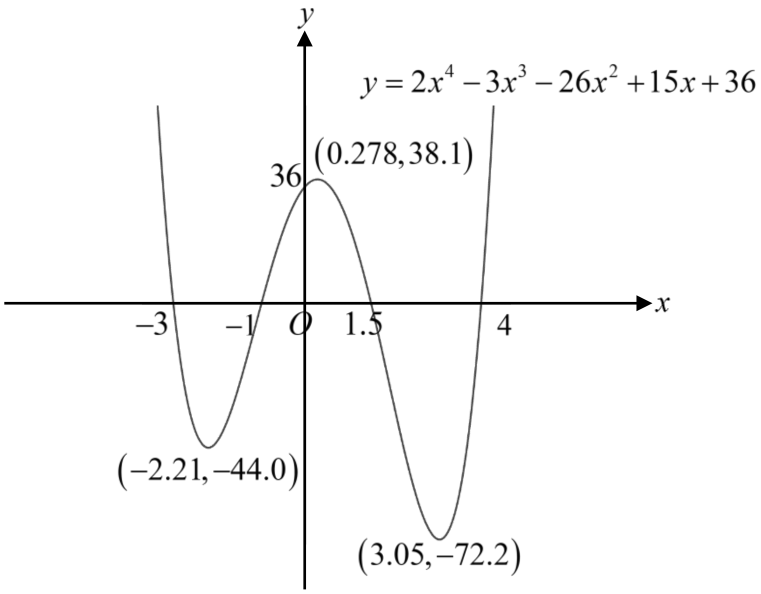
From our understanding of polynomial functions of order 4, we know that the graph of $y = 2x^4 - 3x^3 - 26x^2 + 15x + 36$ will cut the x-axis at most 4 times and have 3 turning points. Adjusting Ymin and Ymax on the $\boxed{\text{WINDOW}}$ settings, we are able to obtain the following:





** You may need to undergo a trial and error process to obtain a reasonable display.*

Use your GC to find the intercepts and turning points. Then sketch the graph.



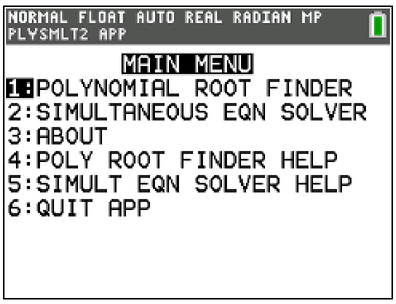
3.5 Equation Solving (Self-Read)

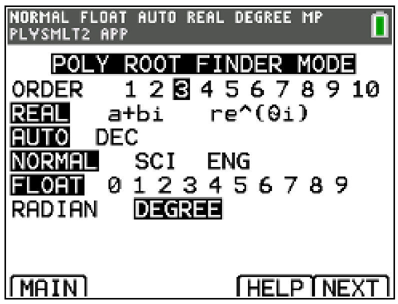
The PlySmlt2 application under APPS can be used to solve polynomial equations and simultaneous polynomial equatons.

<u>Example 7</u>
Find all the roots of the equation $x^3+2x^2-3x-1=0$.

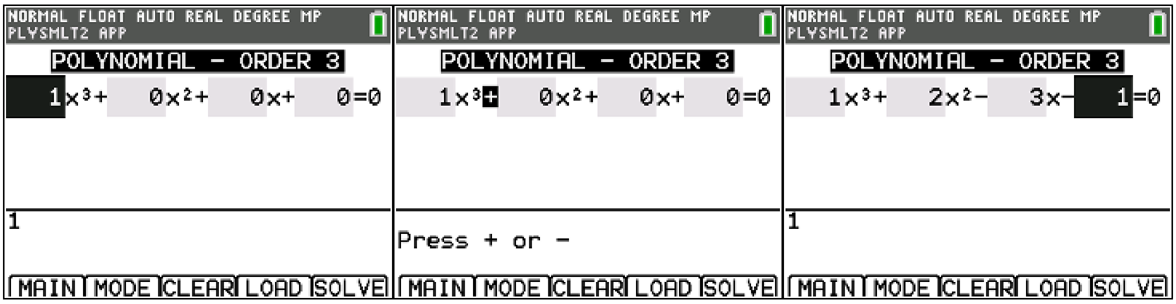
[Solution]

- Under APPS, select PlySmlt2.
- In the main menu, select 1: Polynomial Root Finder
- Since this is a cubic equation, ensure that the order is 3 (highest power of *x* is 3).
- Leave the remaining settings as shown below.



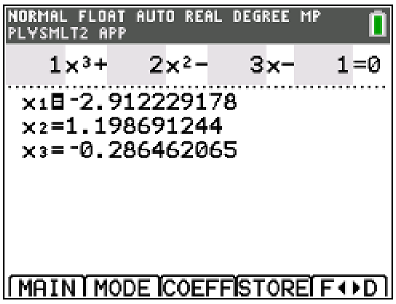


- Press GRAPH to proceed. This allows the NEXT option to be selected.
- Enter the coefficient of x^3 and press ENTER
- You will be given the option to select the sign ‘+’ or ‘–’. Select as appropriate and press ENTER.
- Repeat the process of entering the coefficients and selecting the signs until the entire polynomial equation is entered. (Remember to press ENTER one last time after entering the constant term, before proceeding to SOLVE.)



These interfaces are applicable for linear, quadratic and cubic equations only. Higher order polynomials have a different interface. See **Example 7**.

- Press GRAPH to proceed. This allows the SOLVE option to be selected.



- x_1, x_2 and x_3 gives the roots of the equation.

Hence, the roots of the equation $x^3+2x^2-3x-1=0$ are $x=-2.91$ or $x=1.20$ or $x=-0.286$ (to 3 s.f.).

3.6 Some Final Tips

<u>Final tips on graphing on the graphic calculator</u>

1) The GC has a default window of $-10 \le x \le 10$ and $-10 \le y \le 10$. This means that sometimes a graph will not be fully shown on the default screen, or parts of a graph will be completely missing.

2) It is very important to know all the characteristics of basic graphs when using the GC. Knowledge of such graphs will allow you to detect when a part of the graph is missing (use the various Zoom and Window options to see the whole graph), when a typo has occurred in the keying in of the equation (the shape of the graph is not what you know it should be) and when some asymptotes are missing (knowledge of the particular graph is needed to add the asymptote manually).

3) Check how the graph behaves when x is very large and very small (either use table of values or trace or analyse graph equation). Where asymptotes exist, use dotted lines to draw and label the equation of the asymptotes on the diagram. Also make sure that the shapes of the graphs near asymptotes accurately show the asymptote behavior of graph, e.g. the graph of $y = \ln x$ needs to be extended to show its asymptote behavior near $x=0$. DO NOT bend the curve away from asymptotes.

4) It is important that you read the questions carefully to decide whether the use of the GC is sufficient. If an algebraic or analytical method is required by the question, then complete working must be presented clearly.

5) Do explore and try out the various functions of the GC on your own. The more familiar you are with it, the more efficient you will be in using it and the less likely you will make a mistake when using its functions.

Final tips on sketching graphs

1) Each sketch graph should be at least one quarter of a page in size. Use a pencil to sketch the graph and a ruler to ensure the axes are straight. Label the *x*- and *y*-axes.

2) Label on the diagram (and not elsewhere): all intercept(s), equation(s) of graph(s), asymptote(s) (including equations of asymptotes), max/min point(s).

3) When it is not required to give exact values, leave values correct to 3 significant figures.

4) Where the range of *x* is given, sketch the graph within the given range and label the coordinates of the end-points.

5) Where the graph should be symmetrical (e.g. quadratic graphs, reciprocal graphs etc), your sketch must show the symmetry as well.

Homework 16B

1 Sketch the graph of $y = \dfrac{\ln x}{x}$ for $x > 0$ with the aid of a GC. Label **all** the intersections with the axes, the maximum and minimum points, and the equations of the asymptotes (if any).

2 Solve the following simultaneous equations using a graphical method.

$$\begin{cases} y = \cos x \\ y = |x| \end{cases}$$

Sketch the graphs on the same set of axes and label **all** the intersections with the axes, the maximum and minimum points, and the equations of the asymptotes (if any).

3 **(i)** Sketch the graph of $y = \dfrac{1}{5^x}$.

(ii) In order to solve the equation $x = \log_5\left(\dfrac{1}{4-3x}\right)$, a suitable straight line has to be drawn on the same set of axes as the graph of $y = \dfrac{1}{5^x}$. Find the equation of the straight line and with the aid of a GC, find the solutions to the equation $x = \log_5\left(\dfrac{1}{4-3x}\right)$.