1 Evaluate each of the following.

(a) $\begin{pmatrix} 3 & 4 \\ 2 & -1 \end{pmatrix} - \begin{pmatrix} -2 & 6 \\ 1 & 5 \end{pmatrix}$ **[1]**

$= \begin{pmatrix} 5 & -2 \\ 1 & -6 \end{pmatrix}$

(b) $\begin{pmatrix} -2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ **[1]**

$= (-5)$

(c) $\begin{pmatrix} 3 \\ 2 \end{pmatrix} \begin{pmatrix} -2 & 1 \end{pmatrix}$ **[1]**

$= \begin{pmatrix} -6 & 3 \\ -4 & 2 \end{pmatrix}$

3

2 Given matrices $\mathbf{T} = \begin{pmatrix} 0 & 1 \\ 4 & -1 \end{pmatrix}$, $\mathbf{U} = \begin{pmatrix} 4 & a \\ -3 & 1 \end{pmatrix}$, $\mathbf{V} = \begin{pmatrix} -1 & 2 \\ b & 1 \end{pmatrix}$, find

(a) the value of a and of b such that $2\mathbf{U} + 3\mathbf{V} = \mathbf{V}^2$, **[4]**

$2U+3V=V^2$

$2\begin{pmatrix} 4 & a \\ -3 & 1 \end{pmatrix}+3\begin{pmatrix} -1 & 2 \\ b & 1 \end{pmatrix}=\begin{pmatrix} -1 & 2 \\ b & 1 \end{pmatrix}\begin{pmatrix} -1 & 2 \\ b & 1 \end{pmatrix}$

$\begin{pmatrix} 8 & 2a \\ -6 & 2 \end{pmatrix}+\begin{pmatrix} -3 & 6 \\ 3b & 3 \end{pmatrix}=\begin{pmatrix} 1+2b & 0 \\ 0 & 2b+1 \end{pmatrix}$

$\begin{pmatrix} 5 & 2a+6 \\ 3b-6 & 5 \end{pmatrix}=\begin{pmatrix} 1+2b & 0 \\ 0 & 2b+1 \end{pmatrix}$ ✓

$2a+6=0$ $\quad$ $3b-6=0$

$2a=-6$ $\quad$ $3b=6$

$a=-3$ ✓ $\quad$ $b=2$ ✓

$\therefore a=-3, b=2$

$2\begin{pmatrix} 4 & -3 \\ -3 & 1 \end{pmatrix}+3\begin{pmatrix} -1 & 2 \\ 2 & 1 \end{pmatrix}=\begin{pmatrix} -1 & 2 \\ 2 & 1 \end{pmatrix}$

$\begin{pmatrix} 8 & -6 \\ -6 & 2 \end{pmatrix}+\begin{pmatrix} -3 & 6 \\ 6 & 3 \end{pmatrix}=\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$

$\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}=\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$

4

(b) matrix $\mathbf{R}$ if $\mathbf{TR} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$. **[3]**

let R be $\begin{pmatrix} x \\ y \end{pmatrix}$

$TR=\begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$\begin{pmatrix} 0 & 1 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ ✓

$\begin{pmatrix} y \\ 4x-y \end{pmatrix}=\begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$\begin{cases} y=2 \text{ --(1)} \\ 4x-y=2 \text{ --(2)} \end{cases}$ ✓

Sub (1) into (2)

$4x-2=2$

$4x=4$

$x=1$

$\therefore x=1$ ✓, $y=2$ ✓

$R=\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ A0

$\begin{pmatrix} 0 & 1 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$=\begin{pmatrix} 2 \\ 2 \end{pmatrix}$

2

$\frac{34}{40}$ $\quad$ $\frac{35}{40}$ chng

6

3 An Active Ageing Centre runs morning and afternoon activity sessions for 5 days each week. Each senior attends either the morning session or the afternoon session every day.

The seniors are grouped into three activity groups – Wellness, Fitness and Arts. The table shows the number of seniors in each group in week 1.

	Wellness	Fitness	Arts
Morning	16	10	15
Afternoon	10	15	20

(a) Represent the data in the table by using a 2×3 matrix **H**. **[1]**

$H = \begin{pmatrix} 16 & 10 & 15 \\ 10 & 15 & 20 \end{pmatrix}$ ✓

(b) Each senior is charged a fee for each session.

The session fee is \$5 for Wellness, \$8 for Fitness and \$6 for Arts.

Represent the fees in a 3×1 column matrix **F**. **[1]**

$F = \begin{pmatrix} 5 \\ 8 \\ 6 \end{pmatrix}$ ✓

(c) Given that matrix **M** = **HF**, evaluate **M**. **[2]**

$M = HF$

$= \begin{pmatrix} 16 & 10 & 15 \\ 10 & 15 & 20 \end{pmatrix} \begin{pmatrix} 5 \\ 8 \\ 6 \end{pmatrix}$

$\cancel{\begin{pmatrix} 245 \\ 290 \end{pmatrix}}$

$= \begin{pmatrix} 250 \\ 290 \end{pmatrix}$ ✓ 4

(d) State what the elements of matrix **M** represent. **[1]**

~~245~~ 250 represents the total amount of money in dollars earned from the three activity groups in the morning session. 290 represents the total amount of money earned in dollars from the three ~~ag~~ activity groups in the ~~ar~~ afternoon session.

1

(e) Using matrix multiplication and matrix **M**, calculate the total amount of fees collected by the centre in week 1. **[2]** 1

5 days a week

~~$\begin{pmatrix} 5 & 5 \end{pmatrix}\begin{pmatrix} 245 \\ 290 \end{pmatrix}$~~

~~$= \$2675$~~

$\begin{pmatrix} 5 & 5 \end{pmatrix}\begin{pmatrix} 250 \\ 290 \end{pmatrix}$

$= \$2700$ ✗

~~1440~~ M1

A0

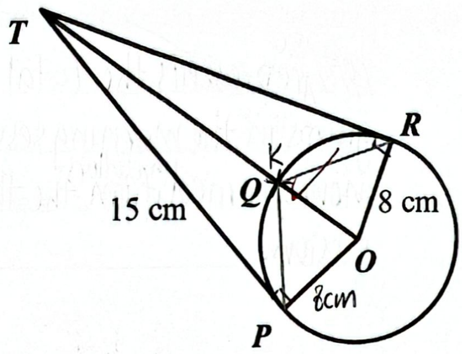
missing ( )

$\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 250 \\ 290 \end{pmatrix}$

$= \$540$

②

4 In the diagram, the points P, Q and R lie on the circle with centre O. TP and TR are tangents to the circle at P and R respectively. OQT is a straight line. The radius of the circle is 8 cm and $PT = 15$ cm.



(a) State the length of TR. **[1]**

$TR = TP$ (tangents from ext. pt. are equal)
$= 15cm$

(b) Explain briefly why $\angle ORT$ is 90°. **[1]**

The ~~radius~~ tangent TR is perpendicular to the radius OR
(tangent ⊥ radius)

(c) Find the length of QT. **[2]**

$QT = TO - QO$
$QO = 8cm$ (radius)
By Pythagoras Theorem
$TO^2 = TP^2 + OP^2$
$= 15^2 + 8^2$
$= 289$

$TO = \sqrt{289}$
$= 17$
$QT = 17cm - 8cm$
$= 9cm$

(d) Point K lies on the circle so that triangle RKT is congruent to triangle PKT.
On the diagram, mark with a cross, ONE possible position for the point K. **[1]**

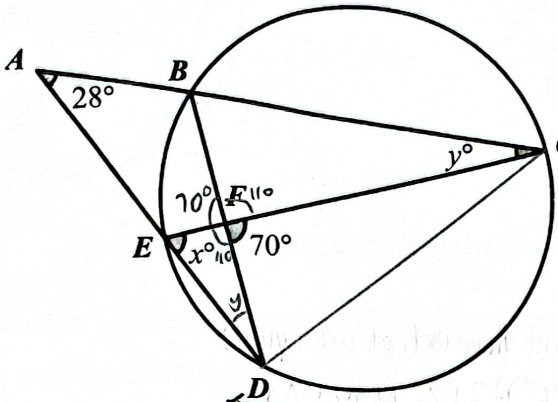
Point K could be on Q

Heck K can be any point on TO.

3

5 In the diagram, points B, C, D and E are points on the circumference of the circle. AED, ABC, BFD and EFC are straight lines.

Given that angle $EAB = 28°$, angle $DFC = 70°$, angle $FED = x°$ and angle $FCB = y°$, find the value of x and of y, stating your reasons clearly. **[4]**



$y+28+(180-x)=180$
$x+y+110=180$

$\angle ADB = \angle ACE$ ($\angle$s in the same segment)
$\angle AEC = 180° - x°$
$y° + 28° + (180° - x°) = 180°$ ($\angle$ sum of Δ)
$x° + y° + 110° = 180°$ ($\angle$ sum of Δ)

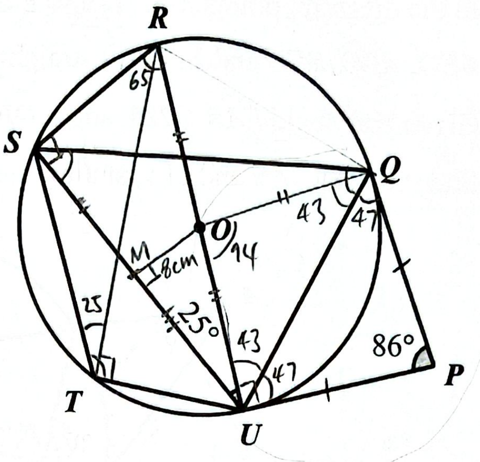
$\begin{cases} y+28+(180-x)=180 \quad (1) \\ x+y+110=180 \quad (2) \end{cases}$

From (2),
$y = 70 - x$ — (3)
Sub (3) into (1)
$(70-x)+28+(180-x)=180$
$70-x+28+180-x=180$
$-2x=-98$
$x=49$
Sub $x=49$ into (3)
$y=70-49$
$=21$

$x=49, y=21$

4

6 Q, R, S, T and U are points on the circle with centre O. PQ and PU are tangents to the circle, and ROU is a straight line.
It is given that angle $QPU = 86°$ and angle $SUO = 25°$.



(a) Find each of the following, stating your reasons clearly.

(i) $\angle QUP$, **[2]**

QP=QU (tangent from ext. pt. are equal)
$\angle QUP = (180° - 86°) \div 2$ ($\angle$s of isos Δ)
$= 47°$

2

(ii) $\angle QSU$, **[2]** any other way?

$\angle QSU = \frac{1}{2}\angle QOU$ ($\angle$ at centre = 2 $\angle$ at circumference)
~~reflex $\angle QOU = 2\angle QPU$ ($\angle$ at centre = 2 $\angle$ at circumference)~~
~~$=172°$~~
$\angle OUP = 90°$ (tangent $\perp$ radius)
$\angle OUQ = 90° - 47°$
$= 43°$

$\angle QOU = 180° - 43° - 43°$ ($\angle$ sum of Δ)
$= 94°$
$\angle QSU = 94° \div 2$
$= 47°$

2

Alt Angle Thm $\angle QUP = \angle QSU$
$= 47°$

(iii) $\angle SQU$, **[3]**

$\angle SQU = 180° - \angle QSU - \angle SUQ$ ($\angle$ sum of Δ)
$= 180° - 47° - (25° + 43°)$
$= 65°$

(iv) $\angle STU$ **[2]**

$\angle STR = \angle SUR$ ($\angle$s in the same segment)
$= 25°$

$\angle RTU = 90°$ (rt $\angle$ in semicircle)
$\angle STU = 90° + 25°$
$= 115°$

9

6 **(b)** Given that M is the mid-point of SU and $OM = 8$ cm, find the radius of the circle. **[2]**

$\angle OMU = 90°$ (equal bisector of chords)

OU = radius

$\sin 25° = \frac{8}{OU}$

$OU = \frac{8}{\sin 25°}$

$= 18.9$ cm (3sf)

2

(c) Jane claims that a circle of diameter QU will pass through the point O. Do you agree with Jane? Explain your reasoning. **[3]**

If circle w diameter QU pass through O, PQOU will be a cyclic quadrilateral.

$\angle OQP + \angle OUP$ should be 180° → $\angle OQP + \angle OUP = 90° + 90°$ (tangent ⊥ radius) $= 180°$

~~$\angle OQP + \angle OUP = (43°+47°)+(47°+47°) = 180°$~~

$\angle QOU = 94°$, $\angle QPU = 86°$ total 180°

~~Yes,~~ I agree with Jane. If a circle with diameter QU passes through O, PQOU will be a cyclic quadrilateral, $\angle OQP + \angle OUP$ should be 180° (opp ∠s of cyclic quad) and $\angle QOU + \angle QPU$ should be 180° (opp ∠s of cyclic quad). $\angle OQP + \angle OUP = 90° + 90°$ (tangent ⊥ radius) and is equal to 180°. $\angle QOU + \angle QPU = 94° + 86°$ and is equal to 180°. Thus a circle with diameter QU will pass through point O.

0

PQOU yes is a cyclic quad but QU is not the diameter. Check out why.

2

END OF PAPER

Did you have fun marking? Have a great day ahead!

almost missed this.