

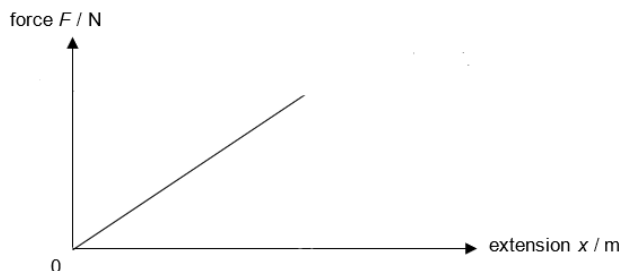


FORCES AND MOMENTS TUTORIAL SOLUTIONS

Self-Check Questions

- S1 Hooke's Law states that the extension (or compression) of a material is directly proportional to the force required to extend (or compress) it, provided the limit of proportionality is not exceeded.

S2



- S3 The moment of a force about a pivot is the product of the force and the perpendicular distance from the pivot to the line of action of the force.

The torque of a couple is the turning effect of a couple which is the product of one of the forces and the perpendicular distance between their lines of action.

- S4 There must be **no resultant force** acting on the object.
There must be **no resultant torque** on the object about any point.

Self-Practice Questions

- P1 Using Hooke's law,

$$F = kx$$

$$10 = k(70 \times 10^{-3} - 50 \times 10^{-3})$$

$$k = 500 \text{ N m}^{-1}$$

Therefore when tension is 20N,

$$20 = 500x$$

$$x = 0.040 \text{ m}$$

Thus, the length of the spring is $0.040 + 0.050 = 0.090 \text{ m}$

- P2 Resolving the forces to be perpendicular to the ruler $= 2.0 \sin 50^\circ$
Torque of the couple $= (2.0 \sin 50^\circ)(\text{distance between the 2 forces})$
 $= 2.0 \sin 50^\circ(0.30) = 0.46 \text{ N m}$

P3

C

The three non-parallel forces of an object in equilibrium need to intersect at a single point

P4

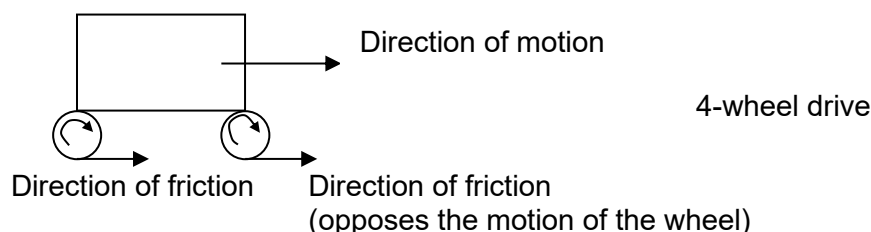
Taking moments about X,

$$(\text{weight of load})(0.8) + (180)(1.5) = (220)(3)$$

$$\text{Weight of load} = 490 \text{ N}$$

Discussion Questions

- 1 (a) Friction is a force which resists motion and is dissipative in nature. Energy is lost as heat and/or sound due to friction. The amount of friction acting on an object, placed on a surface, depends on the normal contact force exerted by the surface on the object. As the wheels rotate clockwise, the tyres exert a leftward frictional force on the road surface. By Newton's third law of motion, the road surface will exert an equal but opposite rightward frictional force on each of the rotating tyres.



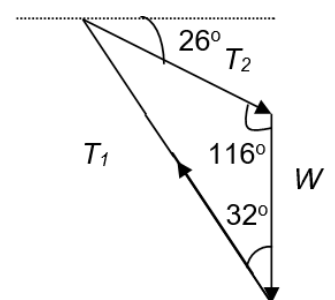
- (b) Friction also enables us to hold a pen, to open the cover of a bottle, to mount one's spectacles in front of the eyes, to fix a nail in the wood, to walk, etc.
- (c) The motion of a free falling object is produced without friction. The motion is produced by the gravitational attraction on the object. If the object is falling in a more viscous fluid, the object experiences a viscous force, which opposes its motion.

2 (a) $F = kx \rightarrow (0.140)(9.81) = k(0.108 - 0.080)$
 $\rightarrow k = 49.1 \text{ N m}^{-1}$

(b) (i) $k = \frac{mg}{L_2 - L_1}$
 $\frac{\Delta k}{k} = \frac{\Delta m}{m} + \frac{\Delta(L_2 - L_1)}{(L_2 - L_1)} = 0.01 + \frac{0.002}{0.028} = 0.081 = 8.1\%$

(ii) $\frac{\Delta k}{k} = 0.081 \rightarrow \Delta k = (0.081)(49.1) = 4 \text{ N m}^{-1}$
 $\rightarrow k = (49 \pm 4) \text{ N m}^{-1}$

- 3 Cable car is stationary. i.e. in equilibrium,
 $\rightarrow W, T_1, T_2$ should form a closed Δ .
 [refer to vector triangle in (a)]
 Δ is isosceles with (magnitude of) $T_2 =$ (magnitude of) W
 $\sum F_y = 0$
 $W + T_2 \sin 26^\circ = T_1 \cos 32^\circ$
 $W + W \sin 26^\circ = T_1 \cos 32^\circ$
 $T_1 = 1.36 \times 10^5 \text{ N}$

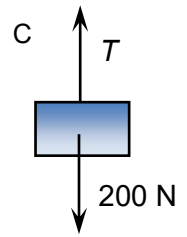
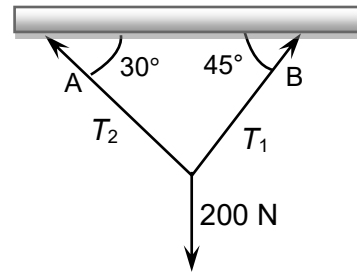


- 4 (a) $T = 200 \text{ N}$ (considering the mass only)
 Considering forces at the knot,
 Horizontally, $T_1 \cos 45^\circ = T_2 \cos 30^\circ$
 Vertically, $T_1 \sin 45^\circ + T_2 \sin 30^\circ = 200$

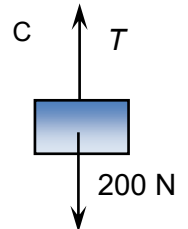
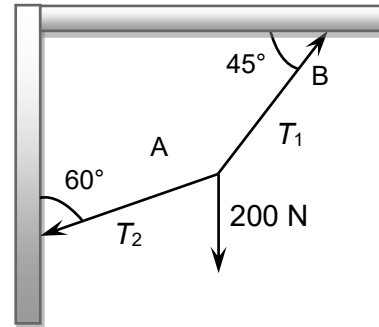
Solving the above eqn,

$$T_1 = 179 \text{ N}$$

$$T_2 = 146 \text{ N}$$



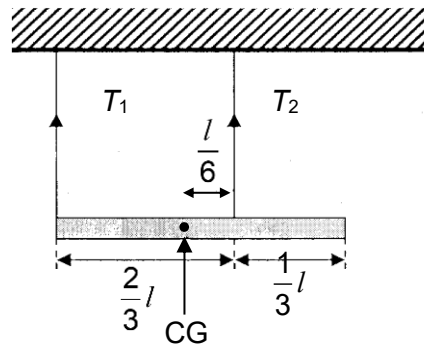
- (b) $T = 200 \text{ N}$ (considering the mass only)
 Considering forces at the knot,
 Horizontally, $T_1 \cos 45^\circ = T_2 \sin 60^\circ$
 Vertically, $T_1 \sin 45^\circ = T_2 \cos 60^\circ + 200$
 Solving the above eqn,
 $T_1 = 669 \text{ N}$
 $T_2 = 546 \text{ N}$



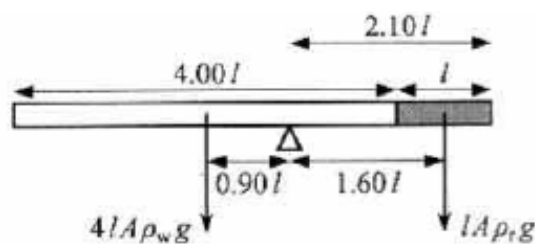
- 5 Taking moments about CG,

$$T_1 \left(\frac{3l}{6} \right) = T_2 \left(\frac{l}{6} \right)$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{1}{3} = 0.33$$



6



$$\text{weight} = mg = V\rho g = \ell A \rho g$$

$$(4\ell A \rho_w g)(0.9\ell) = (\ell A \rho_t g)(1.6\ell) \rightarrow \frac{\rho_t}{\rho_w} = 2.25$$

- 7 (a) (i) Force Y must not have a horizontal component.
Therefore force Y must act vertically upwards, which means $\theta = 90^\circ$.
Force Y must balance the weight of object S .
Magnitude of force Y is 60.0 N .

- (ii) Horizontal component of $X = 200 \cos 30^\circ$
Vertical component of $X = 200 \sin 30^\circ$

Since object S is in translational equilibrium,

$$Y_{\text{horizontal}} = 200 \cos 30^\circ = 173.2 \text{ N}$$

$$Y_{\text{vertical}} + 200 \sin 30^\circ = 60$$

$$Y_{\text{vertical}} = -40 \text{ N} \text{ (i.e. } Y_{\text{vertical}} \text{ is } 40 \text{ N downwards)}$$

$$\text{magnitude of } Y = \sqrt{40^2 + 173.2^2} = 178 \text{ N}$$

$$\theta = \tan^{-1}\left(-\frac{40}{173.2}\right) = -13.0^\circ \text{ (i.e. } 13.0^\circ \text{ below horizontal)}$$

- (b) Force X has a horizontal component pointing towards the right. In order to achieve translational equilibrium of object S , this horizontal force has to be balanced by a leftward component in force Y .

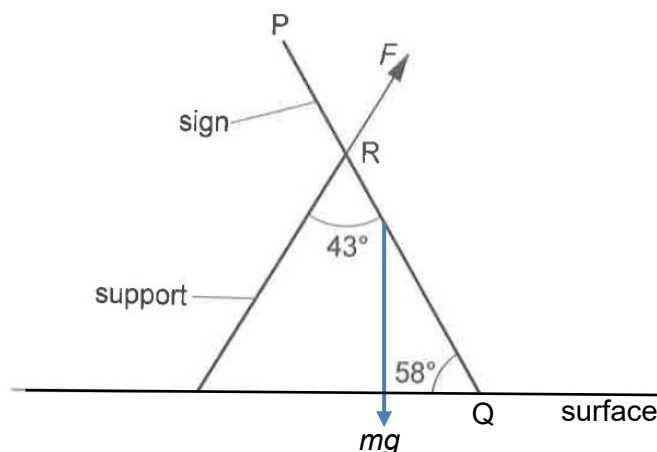
If rope B is parallel to the weight of S , there will not be a leftward horizontal force available and thus equilibrium will not be achieved.

- 8 (a) Let length $PQ = 6x$

Taking moments about Q ,

$$mg(3x \cos 58^\circ) = F \sin 43^\circ(4x)$$

$$\begin{aligned} F &= \frac{mg \cos 58^\circ(3)}{4 \sin 43^\circ} \\ &= \frac{2.3(9.81) \cos 58^\circ(3)}{4 \sin 43^\circ} \\ &= 13.1 \text{ N} \end{aligned}$$



- (b) In order to balance the horizontal component of F , there must be a horizontal contact force at Q acting on the sign by the surface, which is the friction.

So force on sign at Q is not vertical.

- 9 Using cosine rule, $\theta = 29^\circ$.
Taking moments about A ,
 $Tx = mgy$
 $\Rightarrow T(0.80 \sin \theta) = 10.0g(0.40) \sin 2\theta$
 $\Rightarrow T = 85.8 \text{ N}$

For vertical equilibrium,

$$f_r + T \cos \theta = 10.0g$$

$$\Rightarrow f_r = 23.0 \text{ N}$$

