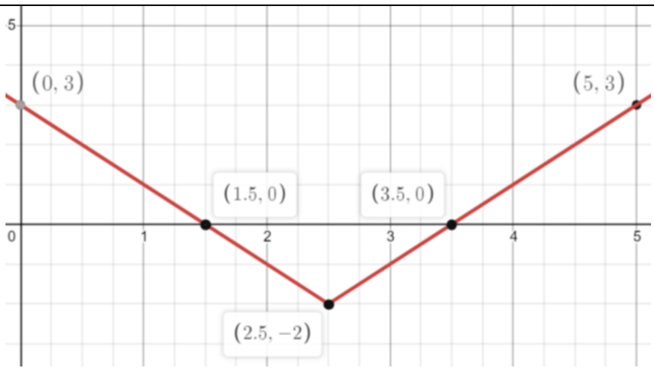
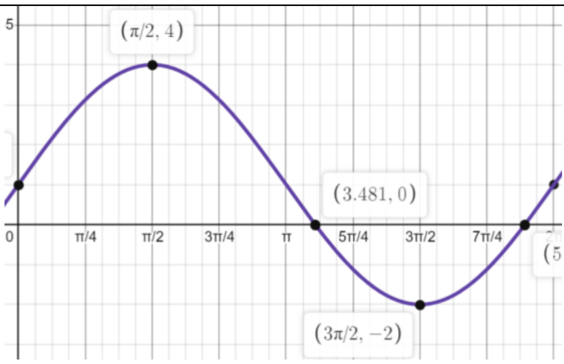
Paper F Sec 3 IP Solutions for students

Qn	Solutions
1(a) [4]	$\dfrac{(2x)^3-3^3}{2x-3}-5$ $=\dfrac{(2x-3)(4x^2+6x+9)}{2x-3}-5$ $=4x^2+6x+9-5$ $=4x^2+6x+4$
1(b) [2]	$9\lvert\sqrt{5}-2\rvert-4\lvert 2-\sqrt{5}\rvert=9\left(\sqrt{5}-2\right)-4\left(\sqrt{5}-2\right)$ $=5\left(\sqrt{5}-2\right)$ or $5\sqrt{5}-10$
2(a) [4]	$\sqrt{3-2x}-x=0$ $3-2x=x^2$ $x^2+2x-3=0$ $x=1$ or -3
2(b) [5]	To solve $2x^3-7x^2-2x+1=0$ Let $f(x)=2x^3-7x^2-2x+1$. $f(-\frac{1}{2})=-\frac{1}{4}-\frac{7}{4}+1+1=0$ $\Rightarrow\left(x+\frac{1}{2}\right)$ *is a factor of* $f(x)=2x^3-7x^2-2x+1$ $f(x)=\left(x+\frac{1}{2}\right)\left(2x^2-8x+2\right)$ or $\left(2x+1\right)\left(x^2-4x+1\right)$ $2x+1=0$ or $x^2-4x+1=0$ $\Rightarrow x=-\frac{1}{2}$ *or* $x=\dfrac{-(-4)\pm\sqrt{(-4)^2-4}}{2}\Rightarrow x=-\frac{1}{2}$ *or* $x=\dfrac{4\pm\sqrt{12}}{2}$ $\Rightarrow x=-\frac{1}{2}$ *or* $x=2\pm\sqrt{3}$
3(a) [3]	$5x-x^2-5+\text{x}-3>0\Rightarrow x^2-6x+8<0\Rightarrow\left(x-4\right)\left(x-2\right)<0\Rightarrow 2<x<4$
3(b) [4]	Equal roots $\Rightarrow$ Discriminant=0 $[-(c+2)]^2-4(4)(c-2)=0\Rightarrow c^2-12c+20=0$ $\Rightarrow(c-2)(c-10)=0\Rightarrow c=2$ or $c=10$
4 [3]	Let ax^3+x^2+bx be f(x). f(x) = $x(ax^2+x+b)$ $\text{f}(-1)=3\Rightarrow -a-b=2$ ....(1) $\text{f}(2)=6\Rightarrow b=1-4a$ ....(2) Sub (2) into (1), $a=1$, $b=-3$

	$\therefore \mathrm{f}(x) = x(x^2 + x - 3) \Rightarrow \mathrm{f}(3) = 27$
5(a) [4]	$\lg\left(x\sqrt{y}\right)^4 = 1.5 + \lg\frac{x}{y} \Rightarrow \lg\left(x\sqrt{y}\right)^4 - \lg\frac{x}{y} = 1.5$ $\Rightarrow \lg\frac{x^4y^2}{\frac{x}{y}} = 1.5 \Rightarrow\Rightarrow \lg x^3y^3 = 1.5 \Rightarrow 3\lg xy = 1.5$ $\Rightarrow \lg xy = 0.5 \Rightarrow 10^{0.5} = xy \Rightarrow y = \frac{\sqrt{10}}{x}$
5(b) [4]	$\log_2 7 - \log_2(11q - 2p) = 1 \Rightarrow \log_2\frac{7}{11q-2p} = 1 \Rightarrow 2 = \frac{7}{11q-2p} \Rightarrow$ $22q - 4p = 7$ ……….(1) $3^p = 9(27)^q \Rightarrow 3^p = 3^2(3^{3q}) \Rightarrow p = 2 + 3q$ ……….(2) Sub (2) into (1): $22q - 4(2+3q) = 7 \Rightarrow q = \frac{3}{2}$. Sub $q = \frac{3}{2}$ into (2): $\Rightarrow p = \frac{13}{2}$.
6 (a) [4]	$\mathrm{f}(x) = 2x^2 - 6x + 7 = 2(x^2 - 3x) + 7$ $= 2(x^2 - 3x + \left(\frac{3}{2}\right)^2 - \frac{9}{4}) + 7 = 2\left(x - \frac{3}{2}\right)^2 - \frac{9}{2} + 7$ $= 2\left(x - \frac{3}{2}\right)^2 - \frac{9}{2} + \frac{14}{2} = 2\left(x - \frac{3}{2}\right)^2 + \frac{5}{2}$ Turning point = $\left(\frac{3}{2}, \frac{5}{2}\right)$ $\mathrm{f}(0) = 7$ & $\mathrm{f}(5) = 27 \quad \Rightarrow \frac{5}{2} < \mathrm{f}(x) < 27$
6 (b) [3]	Let $\mathrm{g}(x) = \frac{x+1}{x-2}$ be y. $xy - 2y = x + 1$ $\Rightarrow xy - x = 2y + 1 \Rightarrow x(y-1) = 2y + 1$ $\Rightarrow x = \frac{2y+1}{y-1} \Rightarrow g^{-1}(x) = \frac{2x+1}{x-1}, x \neq 1$
7(i) [3]	$y = ax^2 + bx + c \Rightarrow c = -9$ $y = ax^2 + bx + c = a(x-1)(x-3) = a(x^2 - 4x + 3)$ Comparing constant term, $a = -3$ Comparing coefficients of x, $b = 12$
7(ii) [2]	When $x = 2$, y = -12+24-9 = 3. $\Rightarrow$ D (2,3)

8(i) [3]	
8(iia) [1]	$1.5 < x < 3.5$
8(iib) [1]	$1 \le x \le 2$ or $3 \le x \le 4$
9(i) [1]	$\begin{pmatrix} 500 & 350 & 400 \\ 700 & 500 & 850 \end{pmatrix}$
9(ii) [2]	$\begin{pmatrix} 500 & 350 & 400 \\ 700 & 500 & 850 \end{pmatrix}\begin{pmatrix} 6.8 \\ 3.6 \\ 1.4 \end{pmatrix} = \begin{pmatrix} 5220 \\ 7750 \end{pmatrix}$ 5220 kg of stainless steel, 7750 kg of aluminum.
9(iii) [2]	$\begin{pmatrix} 2.2 & 2.5 \end{pmatrix}\begin{pmatrix} 5220 \\ 7750 \end{pmatrix} = \begin{pmatrix} 30859 \end{pmatrix}$ $30,859
10(i) [2]	Gradient of BD = $\frac{1}{3}$
10(ii) [2]	Midpoint of BD = $\left(\frac{17}{2}, \frac{15}{2}\right)$, *Eqn AC:* $y = -3x + c$ Gradient of AC = -3 When $x = \frac{17}{2}, y = \frac{15}{2}, c = 33,$ $\Rightarrow$ Equation of AC *is* $y = -3x + 33$
10(iii) [3]	When $y = 0, 0 = -3x + 33 \Rightarrow x = 11 \Rightarrow A = (11, 0)$ Solve $y = 2x - 2$ *and* $y = -3x + 33$ *gives* $x = 7, y = 12$ $\Rightarrow C = (7, 12)$
10(iv) [2]	area of the kite, units^2 $= \frac{1}{2}\begin{vmatrix} 7 & 4 & 11 & 13 & 7 \\ 12 & 6 & 0 & 9 & 12 \end{vmatrix}$ $= \frac{1}{2}(42 + 99 + 156 - 48 - 66 - 63) = 60$ Check BD = $\sqrt{90}$ units, AC = $\sqrt{160}$ units
11(i) [2]	$(x-3)^2 + (y-4)^2 = 6^2$

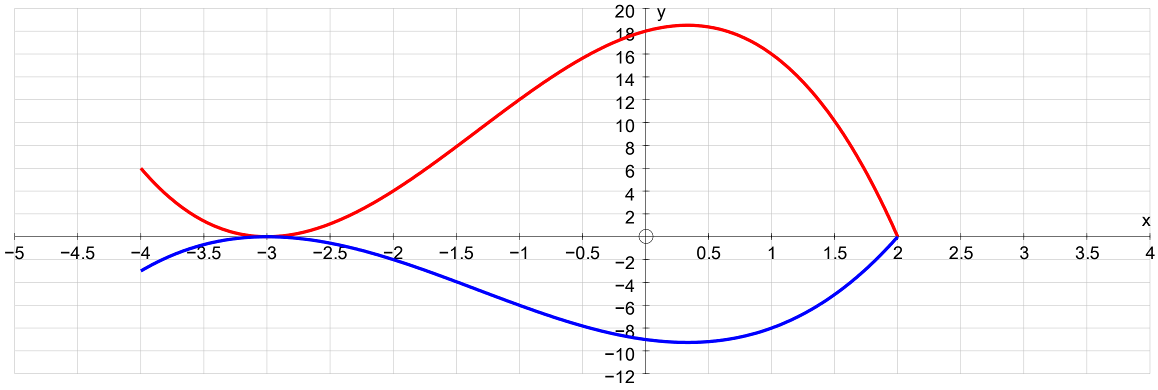
11(ii) [2]	When $x = 8$, $(8-3)^2 + (y-4)^2 = 6^2 \Rightarrow (y-4)^2 = 11$ $\Rightarrow y = 4 \pm \sqrt{11}$ $\tan^{-1}\text{BCA} = \dfrac{\sqrt{11}}{5} \Rightarrow \angle\text{BCD} = 2\tan^{-1}\left(\dfrac{\sqrt{11}}{5}\right) = 1.171\, rad$
11(iii) [3]	Area of sector , $\text{units}^2 = \dfrac{1}{2}6^2 \times 2\tan^{-1}\left(\dfrac{\sqrt{11}}{5}\right) = 21.08$ Area of shaded sector = $\pi(6^2) - 21.08 = 92.0\ \text{units}^2$ **or** Area of sector , $\text{units}^2 = \dfrac{1}{2}6^2 \times (2\pi - 2\tan^{-1}\left(\dfrac{\sqrt{11}}{5}\right)) = 92.0\,\text{units}^2$
12(ai)	
12(aii) [1]	range of y *is* $-2 < y < 4$
12(b) [3]	$\sec\left(z - 18^\circ\right) = -2 \Rightarrow \cos\left(z - 18^\circ\right) = -\dfrac{1}{2}$ $\Rightarrow z - 18^\circ = 120^\circ, 240^\circ \Rightarrow z = 138^\circ, 258^\circ$ Note: $-18^\circ < z - 18^\circ < 342^\circ$
13 (i)	$e^y = ax^2 + b \qquad Y = aX + b$
13 (ii)	On graph paper
13 (iiia)	$a \approx 3.99$ $b \approx 7$
13 (iiib)	When $x = 2.5$, $x^2 = 6.25 \quad \Rightarrow e^y \approx 32$ from the graph. $\Rightarrow y = \ln 32 \approx 3.47$
14 [6]	$AD^2 = 87^2 + 50^2 - 2(87)(50)\cos 55^\circ \Rightarrow AD = 71.3m$ $AC^2 = 26^2 + 84^2 - 2(26)(84)\cos 80^\circ \Rightarrow AC = 83.5m$ $104^2 = AD^2 + AC^2 - 2(\text{AD})(\text{AC})\cos\text{D}\hat{\text{A}}\text{C}$ $\cos\text{D}\hat{\text{A}}\text{C} = 0.10388 \Rightarrow \text{D}\hat{\text{A}}\text{C} = 84.0^\circ$ Area of the lake, m^2

	$\frac{1}{2}(87)(50)\sin 55^{\circ}+\frac{1}{2}(26)(84)\sin 80^{\circ}$ $=+\frac{1}{2}(\sqrt{5078.9\times 6973.5})\sin 84.0^{\circ}$ = 5816

15(i). Answer for (i) $y=-\frac{1}{2}\mathrm{f}(x)$

G1 - correct vertical stretch by factor $\frac{1}{2}$

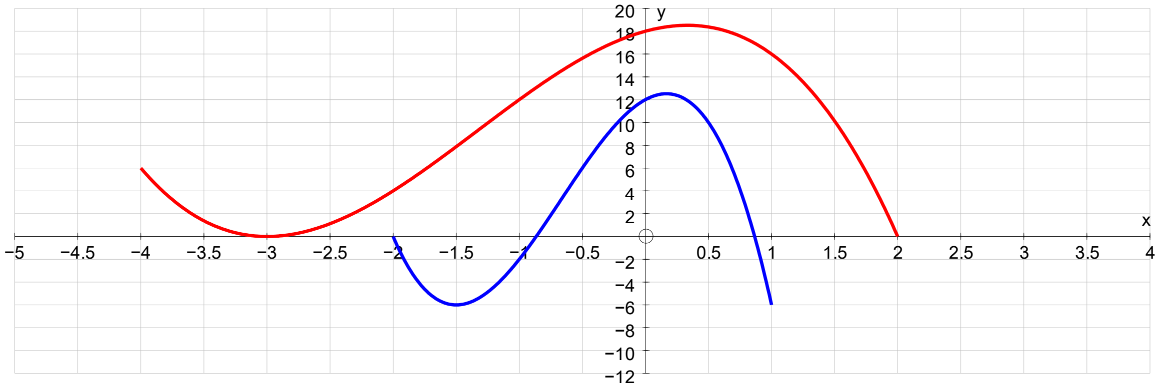
G1 - correct reflection about x-axis



15(ii)Answer for (ii) $y=\mathrm{f}(2x)-6$

G1 - correct horizontal stretch by factor $\frac{1}{2}$

G1 - correct vertical translation of 6 units down



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Q13) $y = \ln(ax^2+b)$

$e^y = ax^2+b$

Plot e^y against x^2

$X = x^2$	1	4	9	16	25
$Y = e^y$	11.02	23.34	42.95	70.11	106.70

$e^y = ax^2+b$

$Y = mX + C$

a = gradient

$= \frac{106.70 - 11.02}{24}$

ii) (a) $= 3.99$

b = Y-intercept / e^y-intercept

$= 7$

b) when $x = 2.5$

$x^2 = 6.25$

$e^y = 32$

$y = \ln 32$

$= 3.47$