

Topical Revision Worksheet 1

Topic: Algebraic Manipulation, Surds and Indices (including Exponential Equations)

| Question 1 [2008 EOY, Q1]                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | marks |
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| <p>Express <math>\frac{1}{3-2\sqrt{2}}</math> in the form <math>a+b\sqrt{2}</math>, where <math>a</math> and <math>b</math> are real numbers.</p> <p><b>Solution:</b></p> $\frac{1}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}} = \frac{3+2\sqrt{2}}{9-8} = 3+2\sqrt{2}$                                                                                                                                                                                                                                                                                                                 | [2]   |
| Question 2 [2008 EOY, Q2]                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | marks |
| <p>Evaluate <math>25^{-1.5} \div 32^{0.4}</math> without a calculator, leaving your answer as a fraction in its simplest form.</p> <p><b>Solution:</b></p> $25^{-1.5} \div 32^{0.4} = 5^{-3} \div 2^2 = \frac{1}{500}$                                                                                                                                                                                                                                                                                                                                                                           | [2]   |
| Question 3 [2009 EOY, Q2]                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | marks |
| <p>Express <math>\frac{2\sqrt{5}-3}{2\sqrt{5}+4} + \frac{2}{\sqrt{3}}</math> in the form <math>a+b\sqrt{3}+c\sqrt{5}</math> where <math>a</math>, <math>b</math> and <math>c</math> are rational numbers.</p> <p><b>Solution:</b></p> $\begin{aligned} \frac{2\sqrt{5}-3}{2\sqrt{5}+4} + \frac{2}{\sqrt{3}} &= \left( \frac{2\sqrt{5}-3}{2\sqrt{5}+4} \times \frac{2\sqrt{5}-4}{2\sqrt{5}-4} \right) + \left( \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \right) \\ &= \frac{32-14\sqrt{5}}{4} + \frac{2\sqrt{3}}{3} \\ &= 8 + \frac{2}{3}\sqrt{3} - \frac{7}{2}\sqrt{5} \end{aligned}$ | [3]   |
| Question 4 [2010 EOY, Q1]                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | marks |
| <p>(a) Simplify <math>(\sqrt[3]{3} + \sqrt[3]{2})(\sqrt[3]{9} - \sqrt[3]{6} + \sqrt[3]{4})</math>.</p> <p><b>Solution:</b></p> $\begin{aligned} (\sqrt[3]{3} + \sqrt[3]{2})(\sqrt[3]{9} - \sqrt[3]{6} + \sqrt[3]{4}) &= \sqrt[3]{27} - \sqrt[3]{18} + \sqrt[3]{12} + \sqrt[3]{18} - \sqrt[3]{12} + \sqrt[3]{8} \quad [\text{M1}] \\ &= \sqrt[3]{27} + \sqrt[3]{8} = 5 \quad [\text{A1}] \end{aligned}$                                                                                                                                                                                           | [2]   |

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| <p>(b) A triangle has an area of <math>(5\sqrt{6} + 18)</math> units<sup>2</sup> and the length of its base is <math>(2\sqrt{3} - 3)</math> unit. Find its height in the form <math>(a\sqrt{2} + b\sqrt{3} + c\sqrt{6} + d)</math> units, where <math>a, b, c</math> and <math>d</math> are integers.</p> <p><b><u>Solution:</u></b></p> $\begin{aligned} \text{Height} &= \frac{5\sqrt{6} + 18}{2\sqrt{3} - 3} \times 2 \text{ units} \\ &= \frac{(10\sqrt{6} + 36)(2\sqrt{3} + 3)}{(2\sqrt{3} - 3)(2\sqrt{3} + 3)} \text{ units} \\ &= \frac{20\sqrt{18} + 30\sqrt{6} + 72\sqrt{3} + 108}{12 - 9} \text{ units} \\ &= \frac{60\sqrt{2} + 30\sqrt{6} + 72\sqrt{3} + 108}{3} \text{ units} \\ &= 20\sqrt{2} + 24\sqrt{3} + 10\sqrt{6} + 36 \text{ units} \end{aligned}$ | <p>[3]</p> |
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| Question 5 [2010 EOY, Q2]                                                                                                                                                                                                                                                                                                                                                                                                                                  | marks      |
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| <p>Solve for <math>x</math> when <math>\frac{1}{2}\sqrt{2x-3} = \sqrt{\frac{3x+5}{2}}</math>.</p> <p><b><u>Solution:</u></b></p> $\begin{aligned} \frac{1}{2}\sqrt{2x-3} &= \sqrt{\frac{3x+5}{2}} \Rightarrow \frac{2x-3}{4} = \frac{3x+5}{2} \\ \Rightarrow 4x - 6 &= 12x + 20 \\ \Rightarrow x &= -\frac{13}{4} \end{aligned}$ <p>However, <math>x = -\frac{13}{4}</math> is rejected as <math>\sqrt{2x-3}</math> will be undefined. So no solution.</p> | <p>[3]</p> |

| Question 6 [2010 EOY, Q3]                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | marks |
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| <p>(a) Make <math>h</math> the subject of the formula <math>\frac{g}{2r} = \frac{1}{y} \sqrt{\frac{h^2 - 1}{2 - h^2}}</math>.</p> <p><b>Solution:</b></p> $\frac{g}{2r} = \frac{1}{y} \sqrt{\frac{h^2 - 1}{2 - h^2}} \Rightarrow \frac{g^2}{4r^2} = \frac{1}{y^2} \left( \frac{h^2 - 1}{2 - h^2} \right)$ $\Rightarrow g^2 y^2 (2 - h^2) = 4r^2 (h^2 - 1)$ $\Rightarrow 4r^2 h^2 + h^2 g^2 y^2 = 2g^2 y^2 + 4r^2$ $\Rightarrow h^2 = \frac{2g^2 y^2 + 4r^2}{4r^2 + g^2 y^2}$ $\Rightarrow h = \pm \sqrt{\frac{2g^2 y^2 + 4r^2}{4r^2 + g^2 y^2}}$                                                                                                                             | [2]   |
| <p>(b) Simplify <math>\frac{x^2 - 3x + 2}{2x^2 - x - 3} \div \frac{2x^2 + 7x + 6}{8x^2 - 18} \times \frac{x^2 + x - 2}{x^2 - 2x + 1}</math>.</p> <p><b>Solution:</b></p> $\frac{x^2 - 3x + 2}{2x^2 - x - 3} \div \frac{2x^2 + 7x + 6}{8x^2 - 18} \times \frac{x^2 + x - 2}{x^2 - 2x + 1}$ $= \frac{(x-1)(x-2)}{(2x-3)(x+1)} \times \frac{2(2x-3)(2x+3)}{(2x+3)(x+2)} \times \frac{(x+2)(x-1)}{(x-1)^2}$ $= \frac{2(x-2)}{x+1} \quad \text{or} \quad \frac{2x-4}{x+1}$                                                                                                                                                                                                        | [3]   |
| Question 7 [2011 EOY, Q1]                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | marks |
| <p>Without using a calculator, simplify <math>\frac{6}{1-\sqrt{3}} + \frac{9}{\sqrt{3}}</math>.</p> <p><b>Solution:</b></p> $\left( \frac{6}{1-\sqrt{3}} + \frac{9}{\sqrt{3}} \right) = \frac{6(\sqrt{3}) + 9(1-\sqrt{3})}{\sqrt{3}(1-\sqrt{3})}$ $= \frac{9-3\sqrt{3}}{\sqrt{3}-3} \cdot \left[ \frac{\sqrt{3}+3}{\sqrt{3}+3} \right]$ $= \frac{9\sqrt{3} + 27 - 9 - 9\sqrt{3}}{3-9}$ $= \frac{18}{-6} = -3$ <p style="text-align: center;">OR</p> $\left( \frac{6}{1-\sqrt{3}} + \frac{9}{\sqrt{3}} \right) = \frac{6(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})} + \frac{9\sqrt{3}}{(\sqrt{3})^2}$ $= -3(1+\sqrt{3}) + 3\sqrt{3} \quad M1$ $= -3 - 3\sqrt{3} + 3\sqrt{3} = -3$ | [3]   |

| Question 8 [2011 EOY, Q3]                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | marks |
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| <p>(i) Show that <math>(2x + y)(4x^2 - 2xy + y^2) = 8x^3 + y^3</math>.</p> <p><b><u>Solution:</u></b></p> $\begin{aligned} & (2x + y)(4x^2 - 2xy + y^2) \\ &= 8x^3 - 4x^2y + 2xy^2 + 4yx^2 - 2xy^2 + y^3 \\ &= 8x^3 + y^3 \text{ (shown)} \end{aligned}$                                                                                                                                                                                                                                                      | [1]   |
| <p>(ii) Hence simplify <math>\frac{8x^3 + y^3}{x^2 - xy - 6y^2} \times \left( \frac{x^2 - 3xy}{2x + y} \right) \div \sqrt{\frac{16x^4}{x^2 + 4xy + 4y^2}}</math>.</p> <p><b><u>Solution:</u></b></p> $\begin{aligned} & \frac{(2x)^3 + y^3}{(x + 2y)(x - 3y)} \cdot \frac{x(x - 3y)}{(2x + y)} \cdot \left[ \frac{(x + 2y)}{4x^2} \right] \\ &= \frac{(2x + y)(4x^2 - 2xy + y^2)}{(x + 2y)(x - 3y)} \cdot \frac{x(x - 3y)}{2x + y} \cdot \frac{x + 2y}{4x^2} \\ &= \frac{4x^2 - 2xy + y^2}{4x} \end{aligned}$ | [4]   |
| Question 9 [2012 EOY, Q2]                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | marks |
| <p>Without using a calculator, simplify <math>\frac{3}{(2 - \sqrt{5})^2} - \frac{2}{(2 + \sqrt{5})^2}</math>.</p> <p><b><u>Solution:</u></b></p> $\begin{aligned} & \frac{3}{(2 - \sqrt{5})^2} - \frac{2}{(2 + \sqrt{5})^2} \\ &= \frac{3(2 + \sqrt{5})^2 - 2(2 - \sqrt{5})^2}{[(2 - \sqrt{5})(2 + \sqrt{5})]^2} \\ &= \frac{3(9 + 4\sqrt{5}) - 2(9 - 4\sqrt{5})}{(1)} \\ &= 9 + 20\sqrt{5} \end{aligned}$                                                                                                    | [4]   |

| Question 10 [2012 EOY, Q10 a, b]                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | marks |
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| <p>(a) Factorise <math>81a^4 - 27a^2b^2 - 4b^4</math> completely</p> <p><b>Solution:</b></p> $81a^4 - 27a^2b^2 - 4b^4 = (9a^2 + b^2)(9a^2 - 4b^2)$ $= (9a^2 + b^2)(3a + 2b)(3a - 2b)$                                                                                                                                                                                                                                                                                                                                                                                                   | [2]   |
| <p>(b) Given that <math>2x = 3\sqrt{px^2 + q}</math>, express <math>x</math> in terms of <math>p</math> and <math>q</math>.</p> <p><b>Solution:</b></p> $2x = 3\sqrt{px^2 + q}$ $4x^2 = 9px^2 + 9q$ $x^2(4 - 9p) = 9q$ $x^2 = \frac{9q}{4 - 9p}$ $x = \pm \sqrt{\frac{9q}{4 - 9p}}$                                                                                                                                                                                                                                                                                                     | [2]   |
| Question 11 [2013 EOY, Q1 a, b]                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | marks |
| <p>(a) Find the product of <math>(x^{\frac{1}{3}} - y^{\frac{1}{3}})(x^{\frac{1}{3}} + y^{\frac{1}{3}})</math> and <math>(x^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}} + y^{\frac{2}{3}})(x^{\frac{2}{3}} + x^{\frac{1}{3}}y^{\frac{1}{3}} + y^{\frac{2}{3}})</math> and simplify the answer.</p> <p><b>Solution:</b></p> $(x^{\frac{1}{3}} - y^{\frac{1}{3}})(x^{\frac{1}{3}} + y^{\frac{1}{3}}) \times (x^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}} + y^{\frac{2}{3}})(x^{\frac{2}{3}} + x^{\frac{1}{3}}y^{\frac{1}{3}} + y^{\frac{2}{3}})$ $= (x - y)(x + y)$ $= x^2 - y^2$ | [2]   |
| <p>(b) If <math>y = \frac{\sqrt{x+a} - \sqrt{x-a}}{\sqrt{x+a} + \sqrt{x-a}}</math>, simplify <math>y + \frac{1}{y}</math>.</p> <p><b>Solution:</b></p> $\frac{\sqrt{x+a} - \sqrt{x-a}}{\sqrt{x+a} + \sqrt{x-a}} + \frac{\sqrt{x+a} + \sqrt{x-a}}{\sqrt{x+a} - \sqrt{x-a}}$ $= \frac{(x+a) - 2\sqrt{x+a}\sqrt{x-a} + (x-a) + (x+a) + 2\sqrt{x+a}\sqrt{x-a} + (x-a)}{(x+a) - (x-a)}$ $= \frac{2x + 2a + 2x - 2a}{2a}$ $= \frac{2x}{a}$                                                                                                                                                    | [3]   |

| Question 12 [2013 EOY, Q6]                                                                                                                                                                                                                                                                                                                  | marks |
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| <p>(a) Simplify <math>\frac{a-b}{(b+c)(a+c)} + \frac{b-c}{(a+c)(a+b)} + \frac{c-a}{(a+b)(b+c)}</math>.</p> <p><b>Solution:</b></p> $\frac{(a-b)(a+b) + (b-c)(b+c) + (c-a)(c+a)}{(a+b)(b+c)(a+c)}$ $= \frac{a^2 - b^2 + b^2 - c^2 + c^2 - a^2}{(a+b)(b+c)(a+c)}$ $= \frac{0}{(a+b)(b+c)(a+c)} = 0$                                           | [3]   |
| <p>(b) Simplify <math>\frac{2x^2 - 6x + 4}{x^2 - 6x + 5} \times \frac{x^2 - x - 20}{3x^2 + 6x - 24}</math>.</p> <p><b>Solution:</b></p> $\frac{2(x^2 - 3x + 2)}{(x-1)(x-5)} \times \frac{(x-5)(x+4)}{3(x^2 + 2x - 8)}$ $= \frac{2(x-1)(x-2)}{(x-1)(x-5)} \times \frac{(x-5)(x+4)}{3(x-2)(x+4)}$ $= \frac{2}{3}$                             | [3]   |
| Question 13 [2014 EOY, Q1]                                                                                                                                                                                                                                                                                                                  | marks |
| <p>(a) Factorise completely <math>(x-3)^3 + (x-1)^3</math>.</p> <p><b>Solution:</b></p> $(x-3)^3 + (x-1)^3$ $= (x-3+x-1)[(x-3)^2 - (x-3)(x-1) + (x-1)^2]$ $= (2x-4)(x^2 - 6x + 9 - x^2 + 4x - 3 + x^2 - 2x + 1)$ $= 2(x-2)(x^2 - 4x + 7)$                                                                                                   | [3]   |
| <p>(b) Simplify <math>\frac{x - \frac{x-1}{2}}{2 - \frac{x-1}{x}}</math>.</p> <p><b>Solution:</b></p> $\frac{2x - x + 1}{2} \div \frac{2x - x + 1}{x}$ $= \frac{x}{2} \quad \text{or} \quad \frac{x - \frac{x-1}{2}}{2 - \frac{x-1}{x}} = \frac{2x^2 - x(x-1)}{4x - 2(x-1)} = \frac{x^2 + x}{2x + 2} = \frac{x(x+1)}{2(x+1)} = \frac{x}{2}$ | [3]   |

| Question 14 [2014 EOY, Q2]                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | marks |
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| <p>The area of a rectangle <math>ABCD</math> is given by <math>(1+3\sqrt{2})\text{ m}^2</math>, and the length of side <math>AB</math> is <math>(2+\sqrt{8})\text{ m}</math>. Find, without using a calculator, the length of side <math>BC</math> in the form of <math>(a+b\sqrt{2})\text{ m}</math>, where <math>a</math> and <math>b</math> are real numbers.</p> <p><b><u>Solution:</u></b><br/> Length of the side <math>BC</math></p> $= \frac{1+3\sqrt{2}}{2+\sqrt{8}}$ $= \frac{(1+3\sqrt{2})(2-2\sqrt{2})}{(2+\sqrt{8})(2-2\sqrt{2})}$ $= \frac{2-2\sqrt{2}+6\sqrt{2}-12}{-4}$ $= \frac{-10+4\sqrt{2}}{-4}$ $= \left(\frac{5}{2}-\sqrt{2}\right)\text{ m}$ | [4]   |
| Question 15 [2015 EOY, Q3]                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | marks |
| <p>(a) Make <math>p</math> the subject of the formula if <math>s = \frac{4(2p^2+1)}{u^2}</math>.</p> <p><b><u>Solution:</u></b></p> $s = \frac{4(2p^2+1)}{u^2}$ $\Rightarrow su^2 = 8p^2 + 4$ $\Rightarrow p^2 = \frac{su^2 - 4}{8}$ $\Rightarrow p = \pm \sqrt{\frac{su^2 - 4}{8}}$                                                                                                                                                                                                                                                                                                                                                                                | [3]   |

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| <p>(b) Simplify into a single fraction <math>\frac{x^3 + 8y^3}{x^2 - 4y^2} - \frac{16xy^2 - 4x^3}{(4y - 2x)^2}</math>.</p> <p><b><u>Solution:</u></b></p> $\begin{aligned} & \frac{x^3 + 8y^3}{x^2 - 4y^2} - \frac{16xy^2 - 4x^3}{(4y - 2x)^2} \\ &= \frac{(x + 2y)(x^2 - 2xy + 4y^2)}{(x + 2y)(x - 2y)} - \frac{4x(2y - x)(2y + x)}{4(2y - x)^2} \\ &= \frac{x^2 - 2xy + 4y^2}{x - 2y} - \frac{2xy + x^2}{2y - x} \\ &= \frac{2x^2 + 4y^2}{x - 2y} \end{aligned}$                                                                            | <p>[4]</p>   |
| <p>Question 16 [2015 EOY, Q4]</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | <p>marks</p> |
| <p>(a) Given that <math>x^2 - 4x = y^2 - 4y</math> and that <math>x \neq y</math>, find the value of <math>\sqrt[3]{x + y}</math>.</p> <p><b><u>Solution:</u></b></p> $x^2 - 4x = y^2 - 4y \Rightarrow (x - y)(x + y) = 4(x - y)$ <p>Since <math>x \neq y</math>, <math>x - y \neq 0 \Rightarrow x + y = 4</math></p> <p>Hence <math>\sqrt[3]{x + y} = \sqrt[3]{4} = 6</math></p>                                                                                                                                                             | <p>[3]</p>   |
| <p>(b) Using a suitable substitution, or otherwise, solve for <math>x</math> if <math>2(27^x) = 45(3^x) + \frac{243}{3^x}</math>.</p> <p><b><u>Solution:</u></b></p> $2(27^x) = 45(3^x) + \frac{243}{3^x} \Rightarrow 2(81^x) = 45(9^x) + 243$ <p>Let <math>y = 9^x</math>, equation becomes</p> $2y^2 = 45y + 243 \Rightarrow 2y^2 - 45y - 243 = 0$ <p>Solving, <math>y = 27</math> (<math>y = -\frac{9}{2}</math> is rejected since <math>9^x</math> cannot be negative)</p> <p>Hence <math>9^x = 27 \Rightarrow x = \frac{3}{2}</math></p> | <p>[4]</p>   |