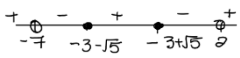
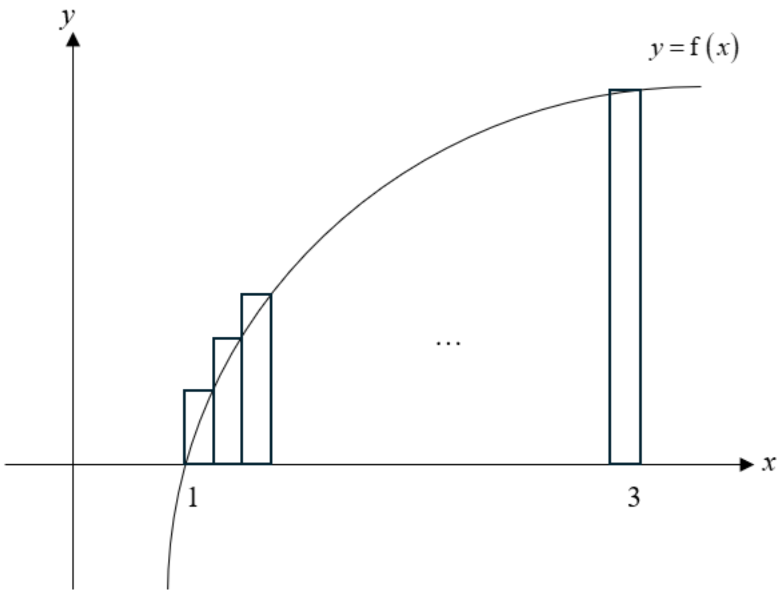
Q1	Suggested Answers
(a)	**<u>Method 1:</u>** From GC, set required is $\{x \in \mathbb{R} : 2 \le x \le 8\}$.
	<u>Method 2:</u> $3\lvert x-3\rvert \le \lvert 1-2x\rvert$ $9(x-3)^2 \le (1-2x)^2$ $9x^2 - 54x + 81 \le 1 - 4x + 4x^2$ $5x^2 - 50x + 80 \le 0$ $x^2 - 10x + 16 \le 0$ $(x-2)(x-8) \le 0$ The set required is $\{x \in \mathbb{R} : 2 \le x \le 8\}$.
(b)	$\dfrac{x+18}{x^2+5x-14} \ge -1$ $\dfrac{x+18+(x^2+5x-14)}{(x+7)(x-2)} \ge 0 \qquad x \ne -7 \text{ or } 2$ $\dfrac{x^2+6x+4}{(x+7)(x-2)} \ge 0$ $\dfrac{(x+3)^2-5}{(x+7)(x-2)} \ge 0$ $\dfrac{(x+3-\sqrt{5})(x+3+\sqrt{5})}{(x+7)(x-2)} \ge 0$  $\therefore x < -7 \text{ or } -3-\sqrt{5} \le x \le -3+\sqrt{5} \text{ or } x > 2$

Q2	Suggested Answers
(a)	$-\mathrm{i}x^3 + 5\mathrm{i}x^2 + ax + b = 0$ Replacing $a = \lambda\mathrm{i}$ and $b = \mu\mathrm{i}$ where $\lambda, \mu \in \mathbb{R}$: $-\mathrm{i}x^3 + 5\mathrm{i}x^2 + \lambda\mathrm{i}x + \mu\mathrm{i} = 0$ $x^3 - 5x^2 - \lambda x - \mu = 0$ Since the coefficients of the equation are all real, the complex roots occur in conjugate pair.
(b)	$\mathrm{i}x^3 + 5\mathrm{i}x^2 + a^* x + b = 0$ Since a is purely imaginary, $a^* = -a$: $-\mathrm{i}(-x)^3 + 5\mathrm{i}(-x)^2 + a(-x) + b = 0$ $-x = 2-\mathrm{i},\ 2+\mathrm{i} \text{ and } 1$ $\therefore x = -2+\mathrm{i},\ -2-\mathrm{i} \text{ and } -1$

Q3	Suggested Answers
(a)	 The width of each of the n rectangles is $\frac{2}{n}$. The height of the first rectangle is $f\left(1+\frac{2}{n}\right)$, second rectangle is $f\left(1+\frac{4}{n}\right)$, …and the n rectangle is $f\left(1+\frac{2n}{n}\right)$. The area of the n rectangles is $\frac{2}{n}\left\{f\left(1+\frac{2}{n}\right)+f\left(1+\frac{4}{n}\right)+...+f\left(1+\frac{2n}{n}\right)\right\}$. When $n\to\infty$, the area of the n rectangles tends towards the area under the curve of $y=f(x)$ from $x=1$ to $x=3$ which is $\int_1^3 f(x)\ dx$.
(b)	$\lim_{n\to\infty}\sum_{k=1}^{n}\ln\left(1+\frac{2k}{n}\right)\frac{2}{n}$ $=\int_1^3 \ln x\ dx$ $=\left[x\ln x\right]_1^3-\int_1^3\frac{1}{x}(x)\ dx$ $=\left[x\ln x\right]_1^3-\int_1^3 1\ dx$ $=\left[x\ln x\right]_1^3-\left[x\right]_1^3$ $=3\ln 3-(3-1)$ $=3\ln 3-2$

Q4	Suggested Answers
(a)	Differentiating wrt x : $3x^2 + x\frac{\mathrm{d}y}{\mathrm{d}x} + y + 6y^2\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-y-3x^2}{x+6y^2}$
(b)	If tangent is parallel to y-axis, $x+6y^2=0$, ie, $x=-6y^2$ Point of contact of tangent with C is given by $\left(-6y^2\right)^3 + y\left(-6y^2\right) + 2y^3 = k$ ie, $-216y^6 - 6y^3 + 2y^3 = k \Rightarrow 216y^6 + 4y^3 + k = 0$ (shown) For y to be real, discriminant ≥ 0, ie, $4^2 - 4(216)(k) \geq 0$ $4 \geq 216k \Rightarrow k \leq \frac{4}{216}$, ie, $k \leq \frac{1}{54}$
(c)	When $x=-6$, $-6y^2=-6 \Rightarrow y=\pm 1$ Therefore, $k=-220$ or -212

Q5	**Suggested Answers**
(a)	Let θ be the acute angle between p_1 and p_2. Then $\cos\theta = \dfrac{\begin{pmatrix}4\\1\\1\end{pmatrix}\bullet\begin{pmatrix}4\\3\\-1\end{pmatrix}}{\left\|\begin{pmatrix}4\\1\\1\end{pmatrix}\right\|\left\|\begin{pmatrix}4\\3\\-1\end{pmatrix}\right\|} = \dfrac{18}{\sqrt{18}\cdot\sqrt{26}} = \dfrac{3}{\sqrt{13}}$. Therefore $\sin\theta = \dfrac{2}{\sqrt{13}}$.
(b)	Since $4(1)+0+4=8$ and $4(1)+3(0)-4=0$, the point $A(1,0,4)$ lies on p_1 and p_2. $\begin{pmatrix}4\\1\\1\end{pmatrix}\times\begin{pmatrix}4\\3\\-1\end{pmatrix}=\begin{pmatrix}-4\\8\\8\end{pmatrix}=4\begin{pmatrix}-1\\2\\2\end{pmatrix}$. The vector equation of the line of intersection of p_1 and p_2 is $\mathbf{r}=\begin{pmatrix}1\\0\\4\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\2\end{pmatrix},\ \lambda\in\mathbb{R}$.

(c)

$$\overrightarrow{AB}\cdot\begin{pmatrix}-1\\2\\2\end{pmatrix}=\begin{pmatrix}0\\3\\-3\end{pmatrix}\cdot\begin{pmatrix}-1\\2\\2\end{pmatrix}=6-6=0$$

So AB is perpendicular to l.

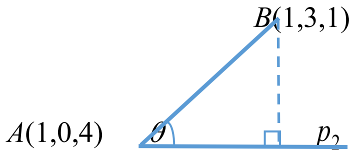
$$AB=\left|\begin{pmatrix}0\\3\\-3\end{pmatrix}\right|=\sqrt{3^2+(-3)^2}=3\sqrt{2}.$$

By **(a)**, perpendicular distance from B to p_2

$=AB\sin\theta$

$=3\sqrt{2}\left(\dfrac{2}{\sqrt{13}}\right)$

$=\dfrac{12}{\sqrt{26}}$



(d)

Since p_3 is perpendicular to both p_1 and p_2, the normal vectors of p_1 and p_2 are both parallel to p_3.

From **(b)**, we infer that a normal vector of p_3 is $\begin{pmatrix}-1\\2\\2\end{pmatrix}$.

Equation of p_3 is therefore

$$\mathbf{r}\cdot\begin{pmatrix}-1\\2\\2\end{pmatrix}=\begin{pmatrix}1\\3\\1\end{pmatrix}\cdot\begin{pmatrix}-1\\2\\2\end{pmatrix}=-1+6+2=7$$

which in cartesian form is $-x+2y+2z=7$.

Q6	Suggested Answers

(a)(i)

If two scores are equal (Event C), suppose x and x, sum of two scores will be $2x$ which is even (which is thus not odd). A and C cannot occur concurrently.

So Event A and C are mutually exclusive

(a)(ii)

A and B are independent.

$\text{P(A)} = \text{P(1 odd and 1 even)} = 2(\frac{1}{2}(\frac{1}{2})) = \frac{1}{2}$

P(B) = P(at least 1 of scores >4)
= P(1st die =5,6) +P(2$^{\text{nd}}$ die = 5,6) – P(1st and 2$^{\text{nd}}$ die =5 or 6)

$= \frac{1}{3} + \frac{1}{3} - (\frac{1}{3})^2 = \frac{5}{9}$

$\text{P}(A \cap B) = 2\text{P}(5,2 \text{ or } 4 \text{ or } 6) + 2\text{P}(6,1 \text{ or} 3) = \frac{10}{36} = \frac{5}{18}$

Alternatively, by using table of outcomes:

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$\text{P(B)} = \frac{20}{36} = \frac{5}{9}$

$\text{P}(A \cap B) = \frac{10}{36} = \frac{5}{18}$ (Look for number of)

$\text{P}(A)\text{P}(B) = \frac{1}{2}(\frac{5}{9}) = \frac{5}{18} = \text{P}(A \cap B)$

Thus A and B are independent.

(b)

$\text{P}(C \mid B)$

$= \frac{\text{P}(C \cap B)}{P(B)}$

$= \frac{\text{P}(5,5) + \text{P}(6,6)}{\text{P}(B)}$

$= \frac{\frac{2}{36}}{\frac{5}{9}}$

$= \frac{1}{10}$

Q7	Suggested Answers
(a)	s =0,1,2,3,4,5 ; f = 5,4,3,2,1,0 d = 5,3,1 D can take values 1, 3 or 5
(b)	$S \sim \mathrm{B}\left(5, \frac{1}{3}\right)$ $\mathrm{P}(D=1)$ $= \mathrm{P}(S=3 \text{ or } S=2)$ $= \frac{40}{81}$
(c)	<table><tr><td>d</td><td>1</td><td>3</td><td>5</td></tr><tr><td>P(D=d)</td><td>40/81</td><td>10/27</td><td>11/81</td></tr></table> E(D^2) = $\sum d^2 \mathrm{P}\left(D=d\right)$ = 65/9.
(d)	$S \sim \mathrm{B}\left(5, \frac{1}{3}\right)$ $\mathrm{E}(S) = 5\times\frac{1}{3} = \frac{5}{3}$, $\mathrm{Var}(S) = 5\times\frac{1}{3}\times\frac{2}{3} = \frac{10}{9}$ $\mathrm{E}\left(S^2\right) = \mathrm{Var}(S) + (\mathrm{E}(S))^2 = \frac{10}{9} + (\frac{5}{3})^2 = \frac{35}{9}$
(e)	$D^2 = \lvert S-(5-S)\rvert^2 = 4S^2 - 20S + 25$ $\mathrm{E}(D^2) = \mathrm{E}(4S^2 - 20S + 25)$ $= 4\mathrm{E}(S^2) - 20\mathrm{E}(S) + 25$ $= 4(\frac{35}{9}) - 20\left(\frac{5}{3}\right) + 25$ $= \frac{65}{9}$ (verified)

<table>
<tr><th>Q8</th><th>Suggested Answers</th></tr>
<tr><td>(a)</td><td>

$$\bar{x} = \frac{\sum x}{n} = \frac{80.1}{45} = 1.78$$

$$s^2 = \frac{1}{n-1}\sum\left(x-\bar{x}\right)^2$$

$$= \frac{1}{44}(24.29)$$

$$= 0.55205 \text{ (5sf)}$$

$$= 0.552 \text{ (3sf)}$$

</td></tr>
<tr><td>(b)</td><td>

Let μ be the population mean mass of adult trout.

To test $\mathrm{H_0} : \mu = w$

$\mathrm{H_1} : \mu \neq w$

at 10% level of significance

Under H_0, since $n = 45$ is large, by the Central Limit Theorem,

$$\bar{X} \sim \mathrm{N}\left(w, \frac{s^2}{45}\right) \text{ approximately}$$

$$Z = \frac{\bar{X} - w}{s/\sqrt{45}} \sim \mathrm{N}(0,1) \text{ approximately}$$

Reject $\mathrm{H_0}$ if $z_{\text{calc}} \leq -1.6448$ or $z_{\text{calc}} \geq 1.6448$ (5 s.f.)

$$z_{\text{calc}} = \frac{1.78 - w}{\frac{s}{\sqrt{45}}}$$

Since $\mathrm{H_0}$ is not rejected,

$$-1.6448 < z_{\text{calc}} < 1.6448$$

$$-1.6448 < \frac{1.78 - w}{\frac{s}{\sqrt{45}}} < 1.6448$$

$$-1.6448 < \frac{1.78 - w}{\frac{\sqrt{0.55205}}{\sqrt{45}}} < 1.6448$$

$$-0.18218 < 1.78 - w < 0.18218$$

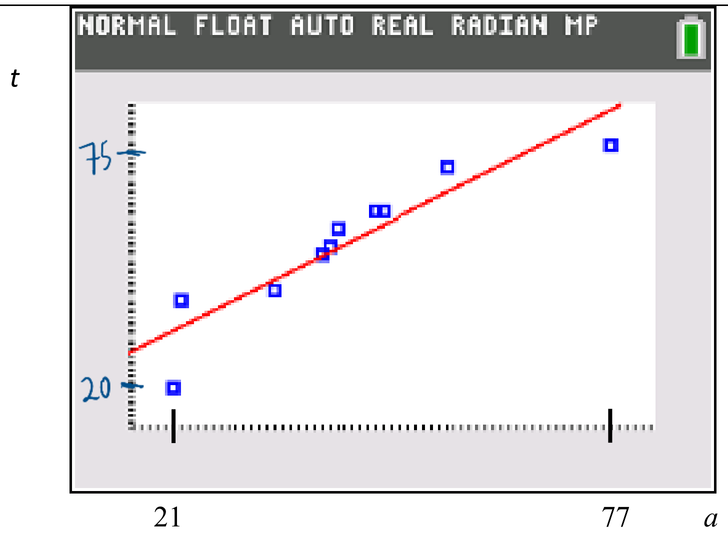
$$1.5978 < w < 1.9622$$

$$1.60 < w < 1.96$$

$$\{w \in \mathbb{R} : 1.60 < w < 1.96\}$$

</td></tr>
<tr><td>(c)</td><td>Since the <u>sample size (n=45) is large</u>, we can approximate the <u>sample mean mass</u> of adult trout with a normal distribution using Central limit theorem.</td></tr>
</table>

Q9	Suggested Answers
(a)	The probability that a plate being faulty is constant. The event that a plate is faulty is independent of the other plates (being faulty).
(b)	Let X be the number of faulty plates out of 100 plates. $X \sim \mathrm{B}(100, p)$ $\mathrm{P}(X=3)=\binom{100}{3}p^3(1-p)^{100-3}$ $=161700p^3(1-p)^{97}$.
(c)	Since $X=3$ is the mode $\mathrm{P}(X=3)>\mathrm{P}(X=2)$ $161700p^3(1-p)^{97}>4950p^2(1-p)^{98}$ $\frac{98}{3}p>1-p$ $\frac{101}{3}p>1$ $p>\frac{3}{101}$ and $\mathrm{P}(X=3)>\mathrm{P}(X=4)$ $161700p^3(1-p)^{97}>3921225p^4(1-p)^{96}$ $\frac{97}{4}p<1-p$ $\frac{101}{4}p<1$ $p<\frac{4}{101}$ Hence $\frac{3}{101}<p<\frac{4}{101}$
(d)	P(no faulty items)+P(1 faulty plate) +P(1 faulty bowl) = 0.88 $(0.99)^2(1-q)^2+2(0.99)(0.01)(1-q)^2+(0.99)^2(2q)(1-q)=0.88$ $0.9999(1-q)^2+(1.9602)q(1-q)=0.88$ Using GC, since $0<q<1$, $q=0.33333\approx 0.333$

Q10	Suggested Answers
(a)	By GC, $\bar{a} = 42.8$ and $\bar{t} = \dfrac{485+k}{10}$. Since the point $(\bar{a}, \bar{t})$ lies on the line $t = 0.8957914a + 14.16013$, $\dfrac{485+k}{10} = 0.8957914(42.8) + 14.16013$ $k = 40$
(b)	
(c)	Time spent by a 60-year old $0.8957914(60) + 14.16013$ $= 67.9$ ≈ 68 minutes.
(d)	By GC, the sum of residuals of the least squares regression line of t on a $= 352.1937458$ Since the sum of residuals of the least squares regression line of t on a is the least amongst all the lines $t = ma + c$ which approximate the same set of data, $S = \sum_{i=1}^{10} \left[t_i - (ma_i + c) \right]^2 \geq 353$.

NORMAL FLOAT DEC REAL RADIAN MP

L1	L2	L3	L4	L5
21	20	32.972	168.27	------
56	70	64.324	32.212	
42	56	51.783	17.78	
77	75	83.136	66.196	
22	40	33.868	37.607	
34	42	44.617	6.8489	
40	50	49.992	6.7E-5	
41	52	50.888	1.2375	
48	60	57.158	8.0763	
47	60	56.262	13.97	
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L4(1)=168.2662824964

NORMAL FLOAT DEC REAL RADIAN MP

1-Var Stats
x̄=35.21937458
Σx=352.1937458
Σx²=35772.25499
Sx=50.95554874
σx=48.34067803
n=10
minX=6.7469796E-5
↓Q1=6.8488858

	In the GC screen on the left, L4 gives the values of $\left[t_i-\left(ma_i+c\right)\right]^2$. On the right, sum of squares of residuals for the least squares regression line of t on a is $\sum x = 352.1937458$.
(e)	Model (A): $t=p+\frac{q}{a}=p+q\left(\frac{1}{a}\right)$ so we consider the regression line of t on $\frac{1}{a}$. Model (B): $t=pe^{qa} \Rightarrow \ln t = qa+\ln p$ so we consider the regression line of $\ln t$ on a. Model | Product moment correlation coefficient, r A | -0.929 B | 0.833 Since the $\lvert r\rvert_{\text{Model A}}=0.929>\lvert r\rvert_{\text{Model B}}=0.833$, Model A is more appropriate as its magnitude is closer to 1 than Model B.

Q11	Suggested Answers
(a)	Let X be the mass of a randomly chosen apple. $X \sim \mathrm{N}(\mu, \sigma^2)$ $\mathrm{P}(X > 120) = 0.15$ $\mathrm{P}(Z > \frac{120-\mu}{\sigma}) = 0.15$ $\frac{120-\mu}{\sigma} = 1.0364$ $120 - \mu = 1.0364\sigma$..........(1) $\mathrm{P}(X < 80) = 0.10$ $\mathrm{P}(Z < \frac{80-\mu}{\sigma}) = 0.1$ $\frac{80-\mu}{\sigma} = -1.2816$ $80 - \mu = -1.2816\sigma$............(2) Using GC, $\mu = 102.12 \approx 102, \quad \sigma = 17.256 \approx 17.3$ (shown)
(b)	$X_1 - X_2 \sim \mathrm{N}(0,\ 17.256^2 + 17.256^2)$ i.e. $X_1 - X_2 \sim \mathrm{N}(0,\ 595.54)$ $\mathrm{P}(\|X_1 - X_2\| \geq 20)$ $= \mathrm{P}(X_1 - X_2 \geq 20) + \mathrm{P}(X_1 - X_2 \leq -20)$ $= 2 \times \mathrm{P}(X_1 - X_2 \geq 20)$ $= 0.41247 \approx 0.412$
(c)	$X \sim \mathrm{N}(102.12,\ 17.256^2)$ $\mathrm{P}(110 < X < 140) = 0.30988$ Expected number of premium apples = 20(0.30988) = 6.1976 ≈ 6.20
(d)	**<u>Method 1:</u>** Let T be the total mass of 20 apples in a box $T = X_1 + X_2 + ... + X_{20}$ $\overline{T} = \frac{X_1 + X_2 + ... + X_{20}}{20} \sim \mathrm{N}\left(102.12,\ \frac{17.256^2}{20}\right)$ i.e. $\overline{T} \sim \mathrm{N}\ (102.12,\ 14.888)$ $\mathrm{P}(\overline{T} \leq 105) = 0.77229 \approx 0.772$
	<u>Method 2:</u> $T = X_1 + X_2 + ... + X_{20} \sim \mathrm{N}(20 \times 102.12,\ 20 \times 17.256^2)$ $T = X_1 + X_2 + ... + X_{20} \sim \mathrm{N}\ (2042.4,\ 5955.4)$ $\mathrm{P}(\mathrm{T} \leq 105 \times 20) = 0.77228 \ \approx 0.772$
(e)	**<u>Method 1:</u>** $C = 0.0029(T_1 + T_2 + T_3)$ $C \sim \mathrm{N}\left(0.0029(3 \times 20 \times 102.12),\ 0.0029^2(3 \times 20 \times 17.256^2)\right)$ i.e. $C \sim \mathrm{N}(17.769,\ 0.15025)$ $\mathrm{P}(C > 18) = 0.27560 \approx 0.276$
	<u>Method 2:</u> $T_1 + T_2 + T_3 \sim \mathrm{N}\left(3 \times 20 \times 102.12,\ 3 \times 20 \times 17.256^2\right)$ i.e. $T_1 + T_2 + T_3 \sim \mathrm{N}(6127.2,\ 17866.17)$ $\mathrm{P}\left(T_1 + T_2 + T_3 > \frac{18 \times 1000}{2.9}\right) = 0.27551 \approx 0.276$