Unit 14: Electric Field

LEARNING OUTCOMES

Candidates should be able to:

(a) recall and use Coulomb's law in the form $F = \frac{Q_1 Q_2}{4\pi\varepsilon_o r^2}$ for the electric force between two point charges in free space or air.

(b) recall and use $E = \frac{Q}{4\pi\varepsilon_o r^2}$ for the field strength due to a point charge, in free space or air, to solve problems.

(c) define electric potential at a point as the work done per unit charge by an external force in bringing a small positive test charge from infinity to that point.

(d) use the equation $V = \frac{Q}{4\pi\varepsilon_o r}$ for the electric potential in the field due to a point charge, in free space or air.

(e) show an understanding that the electric potential energy of a system of two point charges is $U_E = \frac{1}{4\pi\varepsilon_o}\frac{Q_1 Q_2}{r}$.

(f) recall that electric field strength at a point is equal to the negative potential gradient at that point and use this to solve problems.

(g) calculate the field strength of the uniform electric field between charged parallel plates in terms of the potential difference and plate separation.

(h) calculate the forces on a charge in a uniform electric field.

(i) describe the effect of a uniform electric field on the motion of a charged particle.

(j) define capacitance as the ratio of the charge stored to the potential difference and use $C = \frac{Q}{V}$ to solve problems.

(k) recall that the electric potential energy stored in a capacitor is given by the area under the graph of potential difference against charge stored, and use this and the equations $U = \frac{1}{2}QV$, $U = \frac{1}{2}\frac{Q^2}{C}$ and $U = \frac{1}{2}CV^2$ to solve problems.

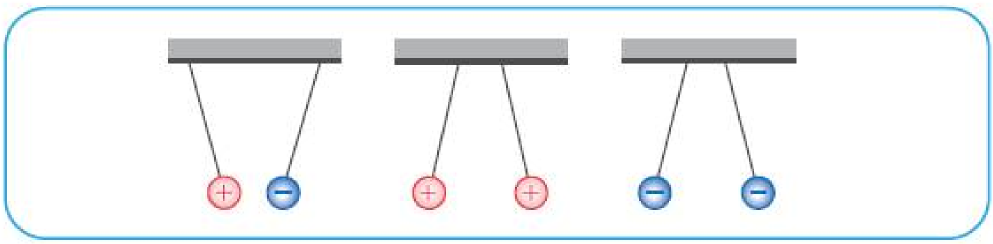
1. 2 TYPES OF ELECTRIC CHARGES

There are 2 types of electric charge – positive charge and negative charge.

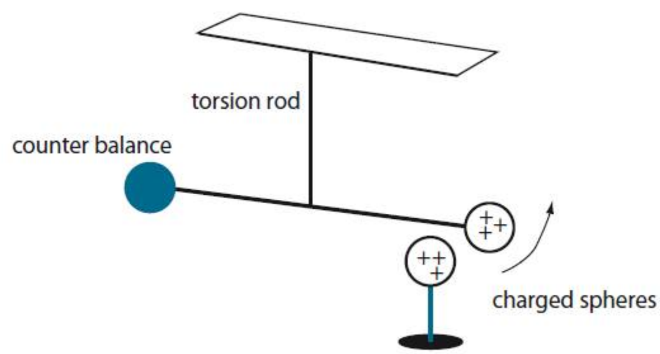
Using a simple model, we can consider matter to be made up of three types of particles: electrons (which have negative charge), protons (positive) and neutrons (neutral). An uncharged object has equal numbers of protons and electrons, net charge is zero.

When one material is rubbed against another, there is friction between them, and electrons may be rubbed off one material onto the other. The material that has gained electrons is now negatively charged, and the other material is positively charged.

It was observed that like charges repel and unlike charges attract each other.



This shows that there exists an electric force of repulsion or attraction between two charges. The French physicist, Charles Coulomb (1738-1806), using a torsion balance of his own invention shown in the figure below, confirmed the existence of an inverse square law of the electric force.



A Coulomb torsion balance

On the basis of his experiments, he concluded that
1. the force F between two point charges Q_1 and Q_2 was directly proportional to the product of the two point charges.
$$F \propto Q_1 \times Q_2$$

2. the force between the two point charges was inversely proportional to the square of the distance between them r^2.
$$F \propto 1/r^2$$

Combining the two equations give the Coulomb's law of electric force described in the next section.

2. COULOMB'S LAW

Coulomb's Law states that the electric force between two point charges is directly proportional to the product of their charges and inversely proportional to the square of their separation.

i.e.

$$F \propto \frac{Q_1 Q_2}{r^2}$$

For two charges Q_1 and Q_2 placed in vacuum (free space), we write Coulomb's Law as

$$\boxed{F = \frac{1}{4\pi\varepsilon_o}\frac{Q_1 Q_2}{r^2}}$$

where $\frac{1}{4\pi\varepsilon_o}$ is the constant of proportionality and ε_o is the *permittivity of free space.*

The quantities ε_o = 8.85 x 10^{-12} C^2 N^{-1} m^{-2} (or F m^{-1}) and $\frac{1}{4\pi\varepsilon_o}$ = 8.99 x 10^9 m F^{-1}.

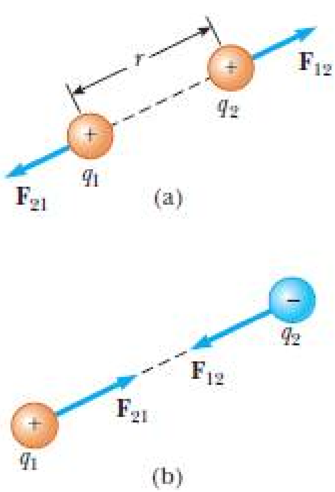
Points to note:

1. If the charges are of the same sign, the force is repulsive and if the charges are of opposite sign, the force is attractive.

2. The electric forces which two charges exert on each other constitute an action-reaction pair.

3. The permittivity of air at s.t.p. is approximately equal to ε_o.

4. The smallest unit of charge is the charge on an electron or proton which has an absolute value of e = 1.60 x 10^{-19} C. Since electric charge Q is quantized, we can write $Q = Ne$, where N is an integer.

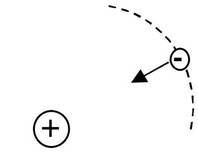


Two point charges separated by a distance r exert a force on each other given by Coulomb's law. The force on q_1 is equal in magnitude and opposite in direction to the force on q_2. (a) When the charges are of the same sign, the force is repulsive. (b) When the charges are of opposite sign, the force is attractive.

Example 1 Forces in a Hydrogen Atom

The electron and proton of a hydrogen atom are separated by a distance of 5.3 x 10^{-11} m.

Find the magnitudes of the
(a) electric force between them,
(b) gravitational force between them
(c) ratio of the electric force to the gravitational force .



Solution

(a) Use Coulomb's Law to find electric force.

$$F_E = \frac{1}{4\pi\varepsilon_o}\frac{q_e q_p}{r^2} = \frac{1}{4\pi x 8.85x10^{-12}}\frac{(-1.60x10^{-19})(1.60x10^{-19})}{(5.3x10^{-11})^2} = -8.2\ x10^{-8}\ N$$

Note that the negative sign shows that it is an attractive force.

(b) Use Newton's law of gravitation to find the gravitational force.

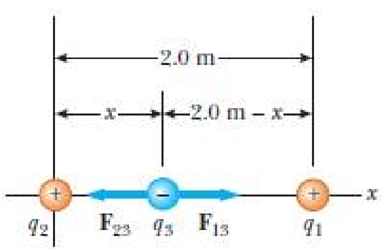
$$F_G = G\,\frac{m_e m_p}{r^2} = 6.67\ x\ 10^{-11}\ \frac{(9.1x10^{-31})(1.67x10^{-27})}{(5.3x10^{-11})^2} = 3.6\ x10^{-47}\ N$$

(c) The ratio of the 2 forces: $\frac{F_E}{F_G} = 2.27\ x\ 10^{39}$ indicates that the electric force is a much stronger force and so the gravitational force between the charged constituents of the atom is considered negligible when compared with the electric force between them.

Example 2

Three charges lie along the x-axis as shown in the figure. The positive charge q_1 is 15 μC at a distance x = 2.0 m from the positive charge q_2 = 6.0 μC.

Where must a *negative* charge q_3 be placed on the *x*-axis so that the resultant electric force acting on q_3 is zero?



Solution

Let F_{13} and F_{23} be the attractive electric forces exerted by q_1 *and* q_2 *on* q_3 respectively.
Their magnitudes are

$$F_{13} = \frac{1}{4\pi\varepsilon_o}\frac{15x10^{-6}q_3}{(2.0-x)^2}$$

$$F_{23} = \frac{1}{4\pi\varepsilon_o}\frac{6x10^{-6}q_3}{x^2}$$

Let the two forces cancel each other, $F_{13} = F_{23}$

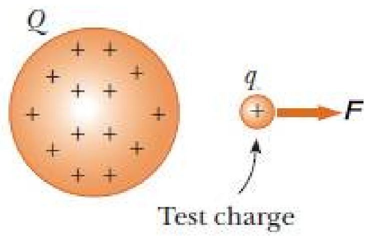
$$\frac{1}{4\pi\varepsilon_o}\frac{15x10^{-6}q_3}{(2.0-x)^2} = \frac{1}{4\pi\varepsilon_o}\frac{6x10^{-6}q_3}{x^2}$$

$$6(2 - x)^2 = 15x^2$$

Solving gives x = 0.77 m

3. The Electric Field

A charge Q sets up an electric field around it. This field is *invisible* but it can be detected by placing a small test charge q nearby. If this small charge q experiences a force even though they are not in contact, it provides evidence that charge Q sets up an electric field or `*field of force*' around it.



An electric field E is said to exist at a region in space if a small charge *q* placed in it experiences an electric force.

The electric field strength at a point is defined as the electric force per unit *positive* charge placed at that point.

That is,

$$\text{electric field strength} = \frac{\text{force acting on positive test charge}}{\text{magnitude of test charge}}$$

$$\boxed{E = \frac{F}{q}}$$

Unit of E: **N C^{-1} or V m^{-1}**

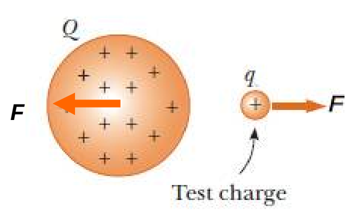
<u>Note:</u>

1. The electric field strength E is a vector quantity.

2. The direction of the electric field strength at any point is the same as the direction of the force experienced by a **positive** test charge placed at that point.

<u>Example 3</u> Electric Field strength

A small charge q is placed in the electric field of a large charge Q. Both charges experience a force F.



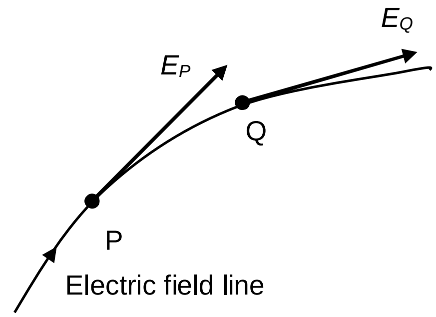
What is the electric field strength of the charge Q at the position of the charge q?

A $\frac{F}{Qq}$ **B** $\frac{F}{Q}$ **C** Fq **D** $\frac{F}{q}$

4. Electric field lines (or lines of force)

Electric field lines are useful for **visualizing** the electric field in any region of space. They are *imaginary lines along which a small positive test charge would move.*

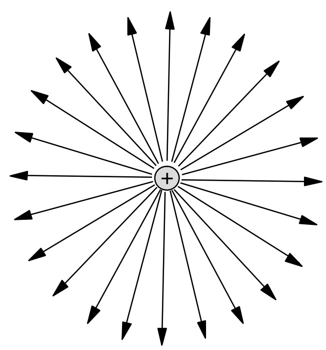
The tangent to a field line at a point gives the direction of electric field strength ***E*** at that point.



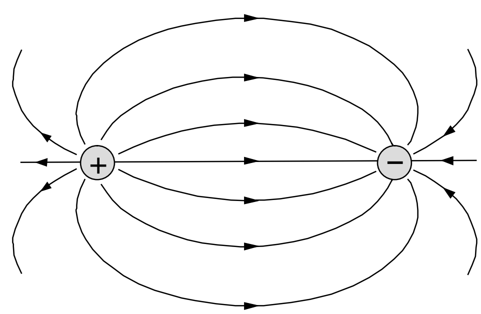
4.1 Properties of electric field lines

1. they start on a positive charge and end on a negative charge
2. they meet on the surface of charged objects at right angles
3. they never cross over one another
4. their density is an indication of the strength of the electric field
5. the electric field is uniform between two oppositely charged parallel conducting plates.

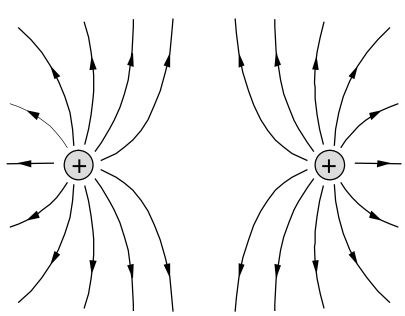
4.2 Examples of electric field lines for different arrangements of charges



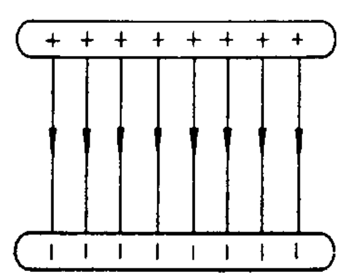
isolated positive point charge



two equal charges, one positive and one negative

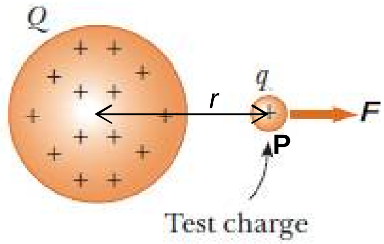


two equal positive charges



two plane parallel plates with equal and opposite charges

4.3 <u>Electric Field due to a point charge *Q*</u>

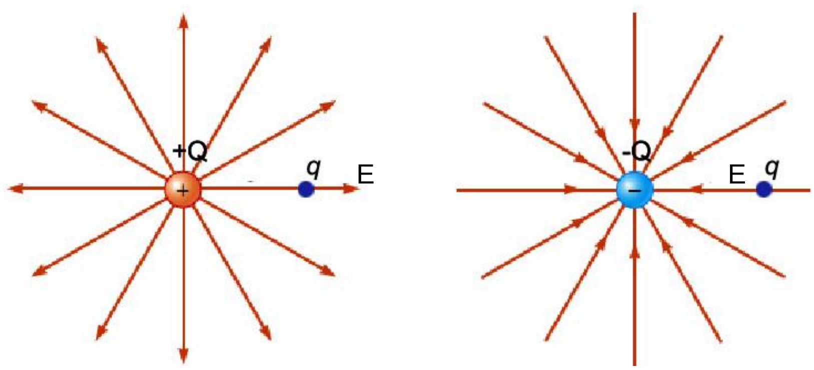


To find the electric field at a point **P**, distance *r* from a point charge *Q* in free space or air, imagine a test charge *q* placed at **P**. From Coulomb's law, force on test charge is

$$F = \frac{1}{4\pi\varepsilon_o}\frac{Qq}{r^2}$$

The *electric field strength* at **P** is $E = \frac{F}{q} \Rightarrow \boxed{E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}}$

The direction of the electric field is the direction of the force that acts on a positive test charge q placed in the field.

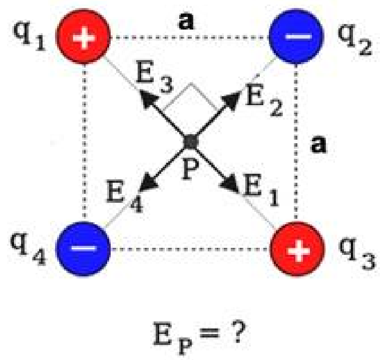


Thus its direction is radially outward for a positive point charge and radially inward for a negative point charge.

Note: If 2 or more charges are present, the total electric field due to a group of charges equals the vector sum of the electric fields of the individual charges.

<u>Example 4</u>

Four point charges of same magnitude q but different signs are situated at the corners of a square with sides of length *a*, as shown in the figure. What is the resultant electric field strength at the centre P?



<u>Solution</u>
The electric field strengths due to the four charges are E_1, E_2, E_3 and E_4 respectively.

Their directions at P are drawn as shown.

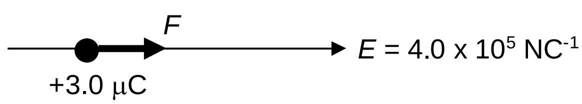
Their magnitudes are equal, given by

$$|E| = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

As E_1 cancels out E_3 and E_2 cancels out E_4, the resultant electric field strength $E_p = 0$

<u>Example 5</u>

A test charge of 3.0 μC is at a point *P* where the electric field due to other charges is directed to the right and has a magnitude of 4 x 10^6 NC^{-1}.

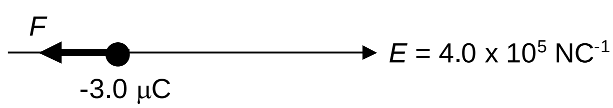


(a) What is the magnitude and direction of the electric force *F* acting on it?
(b) If the test charge is -3 μC, what is the direction of the electric force?

<u>Solution</u>
(a) Using the equation $F = qE$, since the test charge q is positive, the direction of the electric force *F* is in the same direction as the electric field *E*. The magnitude is

$$F = qE = 3.0 \times 10^{-6} \times (4.0 \times 10^5) = 1.2 \text{ N}$$

(b) The direction of the electric force will be *opposite* in direction to the electric field.



5. Electric Potential and Electric Potential Energy

The electric potential at a point is defined as the work done per unit positive charge by an external agent in bringing a small test charge from infinity to that point.

Electric Potential $\boxed{V = \dfrac{W}{q}}$

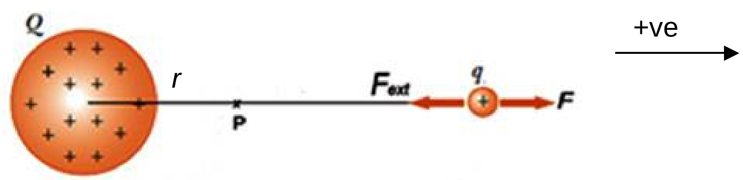
Unit of V: **J C^{-1}** or **volt, V**. It is a scalar quantity.

The electric potential energy at a point is defined as the work done by an external agent in bringing a small test charge from infinity to that point.

Electric Potential Energy $\boxed{U = W = qV}$

Unit of U: **J.** It is a scalar quantity.

5.1 Derivation of Electric Potential V due to a point charge *Q*



Consider a positive test charge *q* moved by an external agent at constant speed from *infinity* to a point P, distance *r* away from charge +*Q*. The force due to the external agent, $\boldsymbol{F_{ext}}$, is equal in magnitude but opposite in direction to the electric force of repulsion, ***F***, due to the field.

$$F_{ext} = -F = -\frac{Qq}{4\pi\varepsilon_o r^2}$$

Therefore, the external work done in moving +*q* from infinity to a point P, distance *r* from Q is

$$W = \int_{\infty}^{r} F_{ext}\,dr = \int_{\infty}^{r} -\frac{Qq}{4\pi\varepsilon_o r^2}\,\mathrm{d}r$$

$$= \left[\frac{Qq}{4\pi\varepsilon_o r}\right]_{\infty}^{r}$$

$$= \frac{Qq}{4\pi\varepsilon_o r}$$

[Note: this proof is not required in the syllabus]

This work done W is converted as electric potential energy *U* at the point.

The ***electric potential energy*** *U* at P is

$$\boxed{U = \frac{1}{4\pi\varepsilon_o}\frac{Qq}{r}}$$

Since $V = \frac{W}{q}$, the ***electric potential*** *V* at P is

$$\boxed{V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r}}$$

Note that
1. Electric Potential *V* is a scalar quantity.
2. Electric potential *V* is defined to be **zero at infinity**.
3. Depending on the work done by external force from infinity to the point, electric potential can be positive or negative. It is positive if charge Q is positive, negative if Q is negative.

The total electric potential at some point *P* due to several point charges is the algebraic sum (or scalar sum) of the potentials due to the individual charges.

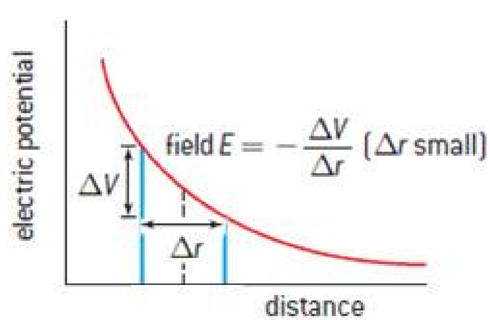
5.2 <u>Relationship between *E* and *V*</u>

The relationship between *E* and *V* is $\boxed{E = -\frac{dV}{dr}}$

An electric field exists between 2 points which are at a different electric potential. In fact the strength of the electric field depends on how the potential changes with distance.

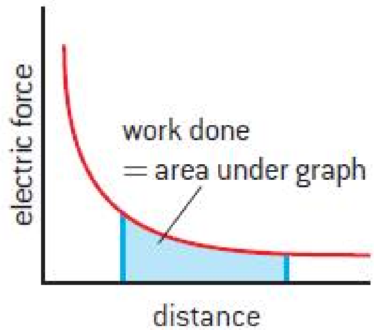
The electric field strength *E* at a point is numerically equal to the potential gradient at that point and the direction of the *E* field points in the direction of decreasing potential *V*.

In graphical representation, the gradient of V – x gives the magnitude of the electric field strength E at that point.



5.3 Graphical interpretations of electric field strength and potential

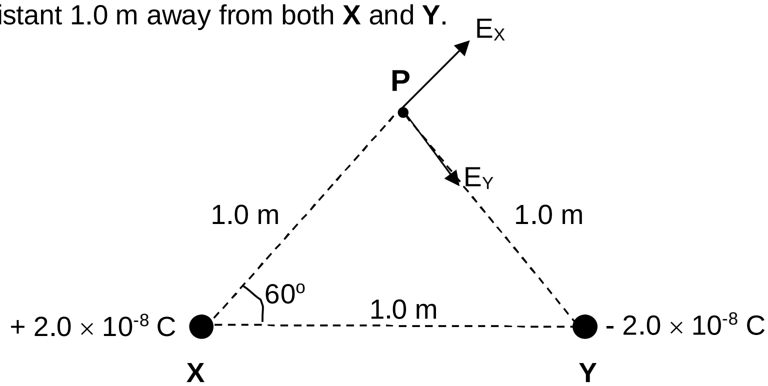
If an electric force varies with distance (shown in figure below) then the work done is the area under the graph.



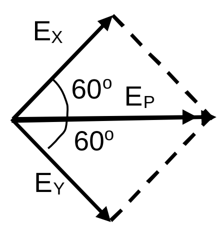
Electric field strength is the force *per unit positive charge,* and so the area under a graph of electric field strength against distance is equal to the work done *per unit charge* – in other words the change in potential ΔV.

Example 6

Two point charges **X** and **Y** are 1.0 m apart in air and carry charges of + 2.0×10^{-8} C and - 2.0×10^{-8} C respectively. Find (a) the electric field strength and (b) the electric potential at point **P**, distant 1.0 m away from both **X** and **Y**.



Solution



(a) Since electric field strength is a vector, the resultant field strength is obtained by **vector addition** of E_X and E_Y.

As the triangle is equilateral $\Rightarrow$ all sides are equal

$$E_P = |E_X| = |E_Y|$$

$$= \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2} = \frac{1}{4\pi\varepsilon_o}\frac{2.0x10^{-8}}{1.0^2}$$

$= 180\ \text{NC}^{-1}$ towards the right

(b) Since electric potential is a scalar quantity, the resultant is obtained by **scalar addition** of the two potentials.

$V_P = V_X + V_Y = 0$ (since the 2 charges are equal in magnitude but of opposite sign)

5.4 Electric field and potential due to an isolated charged conducting sphere

Any charge q placed on an isolated conducting sphere of radius R resides entirely on its outer surface because of repulsive forces between them. **Since there is no charge within the conductor, the electric field is zero at every point inside the charged conductor.**

The equation $E = -\dfrac{dV}{dr} = 0$ implies that all points inside the conductor are at constant electric potential,

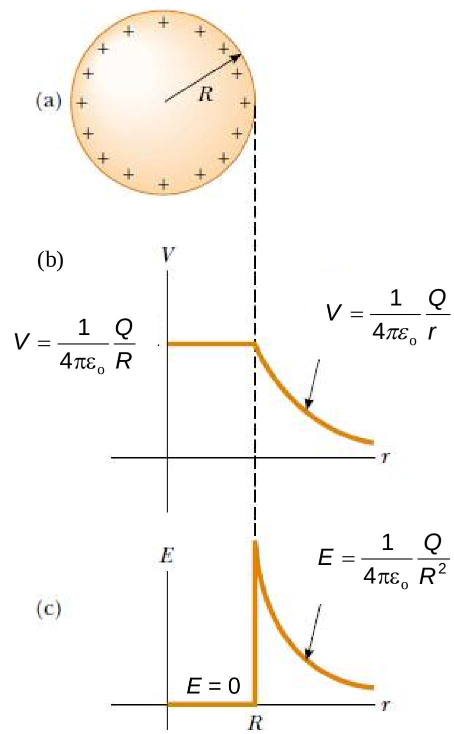
Thus for $r < R$, Q = 0 $\Rightarrow$ $E = 0$

$$\Rightarrow V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{R} \text{ is a constant.}$$

At points outside a conducting sphere, the charge behaves as though it were all concentrated as a point charge at the centre of the sphere.

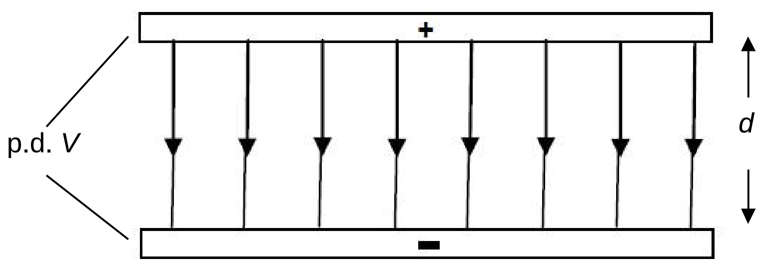
Thus for $r \geq R$, $E = \dfrac{1}{4\pi\varepsilon_o}\dfrac{Q}{r^2}$

$$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r}$$



5.5 Uniform Electric Field in charged parallel plates

The electric field between a pair of oppositely charged parallel plates is uniform. It consists of parallel lines which are equally spaced.

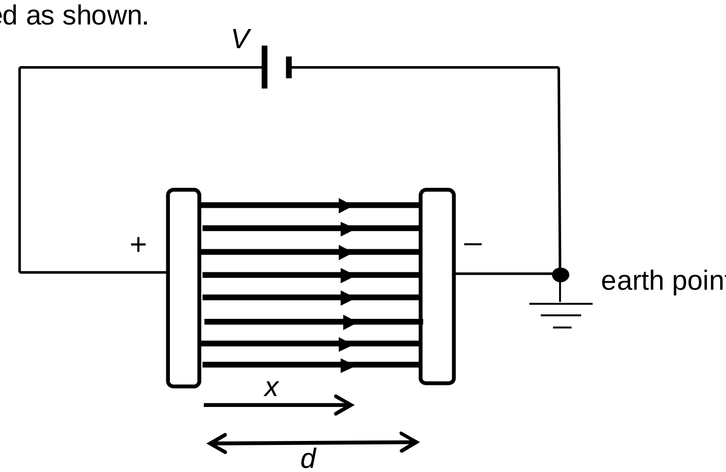


In this case, we can write $E = -\dfrac{dV}{dr}$ as

$$\boxed{E = \frac{\text{potential difference across the plates}}{\text{separation of the plates}} = \frac{V}{d}}$$

Note: The equation $E = \dfrac{1}{4\pi\varepsilon_o}\dfrac{Q}{r^2}$ cannot be applied to a uniform field as it is only applicable for electric fields due to **point charges** or **spherical conductors**.

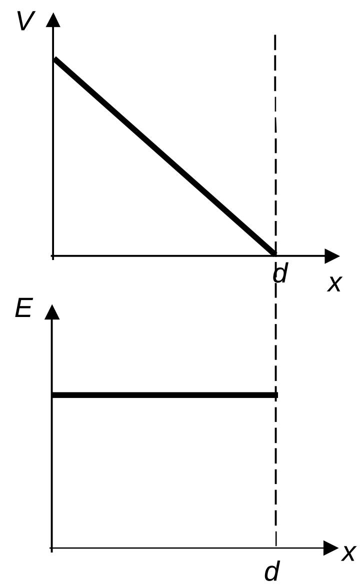
Consider a pair of parallel plates connected directly across a cell of p.d. V. The negative plate is earthed as shown.



Since the field is uniform, the graph of E vs x is a horizontal constant line. The graph of V vs x graph should be a straight line with constant gradient dV/dx.

Electric potential

Electric field strength



5.6 Relationships between *F, E, V, U*

	F	**V**
E	$F = qE$	$E = -\dfrac{dV}{dr}$
U	$F = -\dfrac{dU}{dr}$	$U = qV$

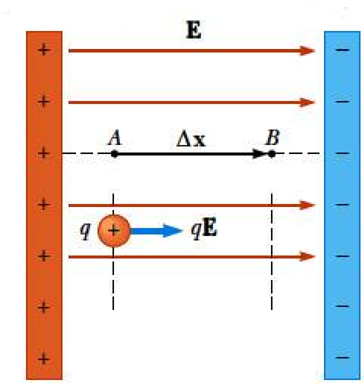
5.7 Motion of charge particles in a uniform electric field

Consider a charge in an electric field between two parallel plates. We are required to describe the motion of the charge when its initial motion is:
1. parallel to the uniform field
2. perpendicular to the uniform field between the two parallel plates.

The following two examples serve to illustrate the two kinds of motion.

Example 7 (Motion of a charge particle whose initial motion is parallel to the field)

A proton of mass 1.66 x 10^{-27} kg, initially at rest, is moved in a uniform field between two parallel plates from A to B, through a distance Δx = 0.0078 m and potential difference of V_{AB} = 2.1 kV. The charge of the proton is +1.60 x 10^{-19} C.



(a) Calculate the electric field strength *E* between the plates.
(b) Calculate the electric force *F* that acts on the proton.
(c) Describe the motion of the proton from A to B.
(c) What is the final speed of the proton when it reaches B?
(Note: for this question, there is no other external force, only electric force is present.)

Solution

(a) $E = \left|\frac{\Delta V}{\Delta x}\right| = \frac{2100}{0.0078} = 2.7 \times 10^{5}\ \text{Vm}^{-1}$

(b) $F = qE = 1.60 \times 10^{-19} \times 2.7 \times 10^{5} = 4.3 \times 10^{-14}$ N

(c) Since the electric force F is a constant, the proton moves with constant acceleration from A to B.

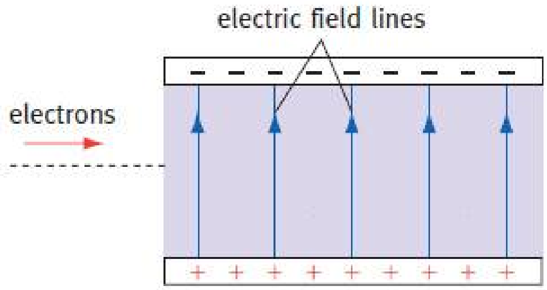
(d) $a = \frac{F}{m} = \frac{4.3 \times 10^{-14}}{1.66 \times 10^{-27}} = 2.6 \times 10^{13}\ \text{ms}^{-2}$

$v^2 = 0 + 2a\,\Delta x = 2 \times 2.6 \times 10^{13} \times 0.0078 \Rightarrow v = 6.4 \times 10^{5}\ \text{ms}^{-1}$

Alternatively, we can use work done by electric force = gain in KE of proton
i.e. $W = F.\,\Delta x = ½\, mv^2$ to obtain final speed *v*.

Example 8 (Motion of charged particles projected at right angles to an uniform *E*-field)

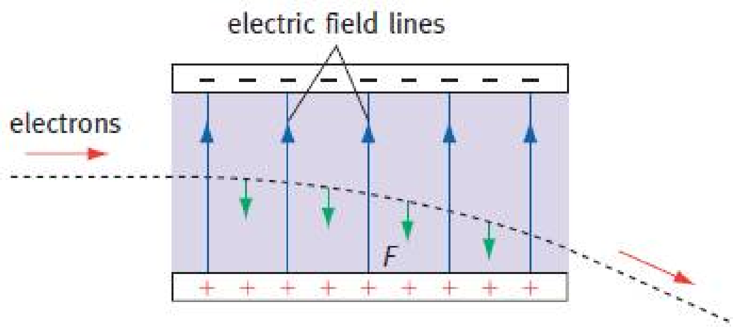
A beam of electrons enters the electric field between two charged parallel plates, as shown.



What is the subsequent motion of the beam?

Solution

Inside the field, a constant electric force $F = qE$ acts vertically downwards on the electron. The electron's acceleration is downward, so the vertical velocity is increasing at a constant rate. There is no horizontal acceleration and the horizontal velocity does not change.



The subsequent motion of the electron will follow a parabolic path towards the positive plate.

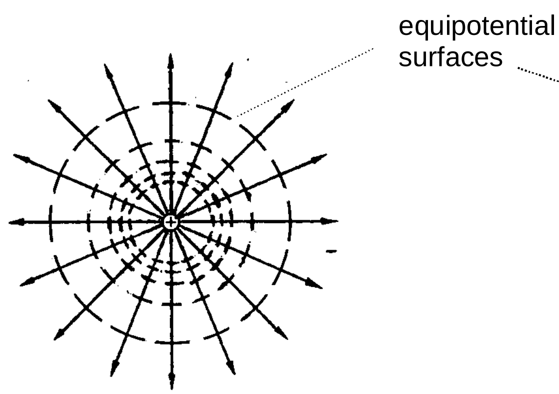
6. Equipotential Surfaces

Equipotential surfaces are surfaces on which the potential is the same at all points.

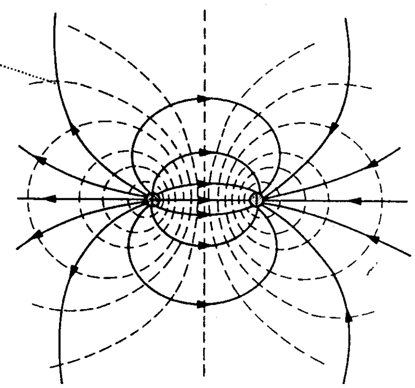
The potential difference between any two points is zero. Hence, **no work is required to move a charge at constant speed on an equipotential surface**. **The electric field at every point of an equipotential surface is perpendicular to the surface**. If the electric field had a component parallel to the surface, that component would produce an electric force on a charge placed on the surface. This force would do work on the charge as it moved from one point to another, in contradiction to the definition of an equipotential surface.

Equipotential surfaces can be represented on a diagram by drawing equipotential contours (dotted lines). Note that the equipotentials are perpendicular to the electric field lines at all points.

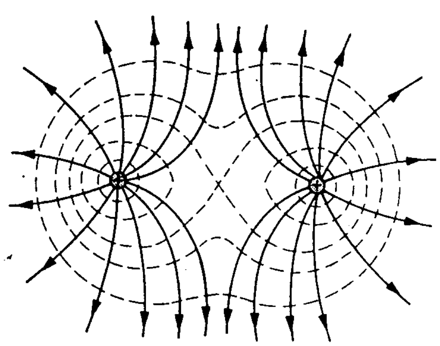
6.1 Examples of equipotential surfaces (dotted lines) and E field lines (solid lines) for four arrangements of charges



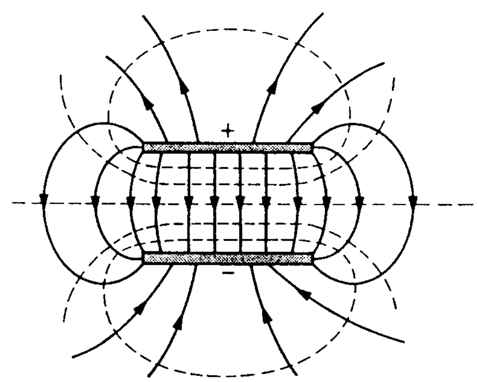
isolated point charge (positive)



two equal charges;
one positive and one negative



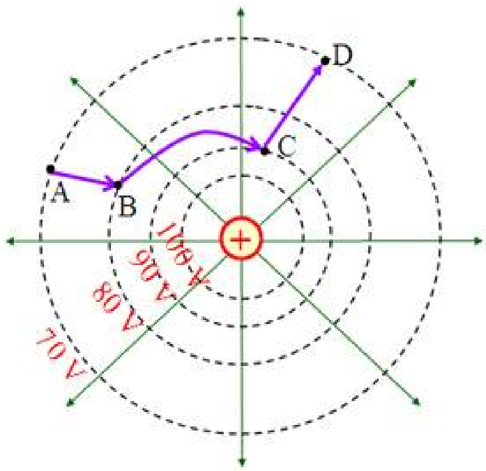
two equal positive charges



two plane parallel plates
with equal and opposite charges

<u>Example 9</u> (Work done on Equipotential Surfaces)

A +2.0 μC charge is moved from one equipotential surface to another as shown in the diagram. The equipotentials are labelled 100 V, 90 V, 80 V, 70 V respectively. Calculate the work done to move it from (a) A to B, (b) B to C, (c) C to D and (d) D to A.



(a) Work is done by an external agent

$$W = \Delta U$$

$$W_{AB} = \Delta U = q\,\Delta V = q\,(V_B - V_A)$$
$$= 2.0 \times 10^{-6}\,(80 - 70)$$
$$= 2.0 \times 10^{-5}\ \text{J}$$

(b) $W_{BC} = q\,(V_C - V_B)$
$= 2.0 \times 10^{-6}\,(90 - 80)$
$= 2.0 \times 10^{-5}$ J

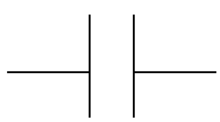
(c) $W_{CD} = q\,(V_D - V_C)$
$= 2.0 \times 10^{-6}\,(70 - 90)$
$= -4.0 \times 10^{-5}$ J

(d) $W_{DA} = q\,(V_D - V_A) = 0$ since it moves along an equipotential surface.

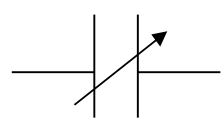
7.1 Capacitance

A capacitor is a device for storing electric charges. Basically, it consists of two parallel metal plates with an insulator known as dielectric present between the plates.

Below shows the schematic symbols used for capacitors.



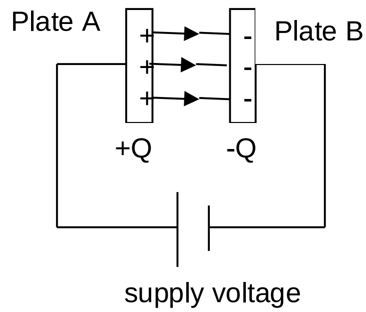
fixed capacitor



variable capacitor

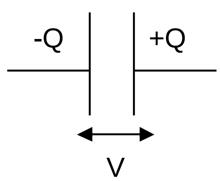
Capacitance is the ability to store charge. If more charge are stored for a given applied voltage, the capacitance will be larger.

A capacitor can be charged by connecting a battery across the conducting plates, as shown below.



A fully charged capacitor has the following characteristics:

(a) It stores equal amount of positive charges (+Q) and negative charges (-Q) on the 2 plates.
(b) The charges remain on the conducting plates even after the battery is disconnected.
(c) There is a potential difference between the 2 oppositely charged conducting plates. When the capacitor is fully charged, the voltage across the plates id equal to the supply voltage from the battery. This causes the flow of direct current onto the capacitor to cease.
(d) The electric field set up within the capacitor stores electrical potential energy.



The capacitance *C* of a capacitor is defined as the ratio of the charge stored to the potential difference. Mathematically it is given by

$$\text{Capacitance } C = \frac{\textit{charge on any one plate of capacitor}}{\textit{p.d. across the plate}} = \frac{Q}{V}$$

Since a conductor such as an insulated metal sphere is able to store charge on its surface, the term capacitor is also applicable to a conductor.

The capacitance of a conductor is C

The unit for capacitance is the farad (F).

Example 10 : An isolated metal sphere of radius r carries a charge $+Q$. The charge may be assumed to be concentrated at the centre of the sphere.

(a) State, in terms of Q and r, the electric potential V at the surface of the sphere. Identify any other symbols you use.

(b) Write down the relationship between the capacitance C, charge Q and potential V.

(c) Hence, show that the capacitance C of the sphere is given by

$$C = 4\pi\varepsilon_o r$$

(a) Electric potential at the surface of sphere, $V = \dfrac{1}{4\pi\varepsilon_o}\dfrac{Q}{r}$

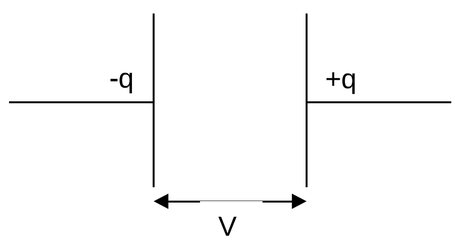
(b) $C = \dfrac{Q}{V}$

(c) Rearranging (a) $\dfrac{Q}{V} = 4\pi\varepsilon_o r$

$$\Rightarrow \quad C = 4\pi\varepsilon_o r$$

7.2 Energy stored in a capacitor

A charged capacitor stored energy in the form of electric potential energy.



Consider a parallel plate capacitor as shown above, when the p.d across the plate is V, the charge on each plate is given by

$$q = CV$$

When a small charge +dq is moved from the negative plate to the positive plate, the charge on the positive plate becomes (q + dq) and the charge on the negative plate increases to –(q+dq).

The work done in moving the charge +dq through a p.d. of V is

$$dW = Vdq$$

Since $V = \dfrac{q}{C}$, $\Rightarrow \quad dW = \dfrac{q}{C}dq$

Hence the work done in charging a capacitor from q=0 to q=Q is

$$W = \int_{q=0}^{q=Q} dW = \int_{q=0}^{q=Q} \frac{q}{C}dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{1}{2}\frac{Q^2}{C}$$

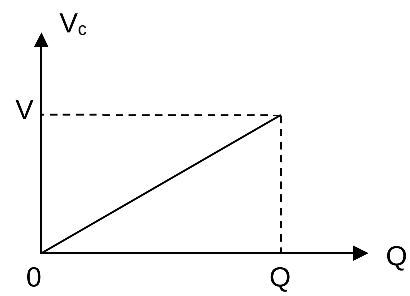
This work done in charging the capacitor is converted into electric potential energy U stored in the capacitor.

Therefore, the electric potential energy stored in a charged capacitor is

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

7.3 The Potential difference V_c – Charge Q graph

From the equation $Q = CV$, the graph of V_c versus Q is a straight line passing through the origin as shown below.



The total work done and hence the total electric potential energy U stored is also given by the area under the V_c – Q graph,

i.e. $U = W = \textit{area under the } V_c - Q \textit{ graph}$

$$= \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$$

Example 11 :

A 3.0 μF capacitor is charged by a battery of e.m.f. 800 V.

Determine

(a) the magnitude of the charge stored in each plate of the capacitor;
(b) the energy supplied by the battery;
(c) The energy stored in the capacitor.

Account for the difference in energy in (b) and (c).

(a) Using $Q = CV = 3.0 \times 10^{-6} \times 800 = 0.00240$ C
(b) $\varepsilon = QV = 0.00240 \times 800 = 1.92$ J
(c) $U = \frac{1}{2}QV = \frac{1}{2} \times 1.92 = 0.960$ J

The loss in half of the energy supplied by the battery escapes in the form of heat energy in the connecting wires and the internal resistance of the battery.

8. Comparison between electric and gravitational fields

(a) General cases

Electrical Quantity	Gravitational Quantity
$E = \frac{F}{q}$	$g = \frac{F}{m}$
$V = \frac{W}{q}$	$\phi = \frac{W}{m}$
$E = -\frac{dV}{dr}$	$g = -\frac{d\phi}{dr}$

(b) For point charges and point masses

Electrical Quantity	Gravitational Quantity
$F = \frac{Q_1 Q_2}{4\pi\varepsilon_o r^2}$	$F = \frac{GM_1 M_2}{r^2}$
$E = \frac{Q}{4\pi\varepsilon_o r^2}$	$g = \frac{GM}{r^2}$
$V = \frac{Q}{4\pi\varepsilon_o r}$	$\phi = -\frac{GM}{r}$

Differences:

- Electric force acts on charges whereas gravitational force acts on masses.
- Electric force can be attractive or repulsive depending on the signs of the charges whereas gravitational force is always attractive.
- Gravitational force between two masses does not depend on the medium in which they are situated whereas electric force between two charges does.

Similarities:

- Both the gravitational force and the electrical force obey the inverse square law.
- Both forces are examples of action-at-a-distance.
- Both forces are conservative forces. This means that the work done by these forces depends only on the difference in potentials between the initial and final positions and not on the path taken. No energy is dissipated.

Terminology

- point charge - A charge that is assumed to be concentrated at a point which does not affect the E-field it is place in.
- test charge - a point charge with a very small amount of electric charge
- field of force - a region of space where an object placed anywhere in it experiences a force
- isolated charge - a charge placed so far away from all other charges that it does not experience any force from them

Electric Fields Tutorial

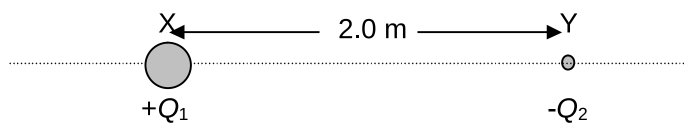
Solutions to Q3, 5, 10 are provided
(ε_o= 8.85 x 10^{-12} F m^{-1}, m_e = 9.11 x 10^{-31} kg, e = -1.60 x 10^{-19} C)

Electric Force and Coulomb's Law

1 (a) Two charged objects are separated by a distance. The first charge is larger in magnitude than the second charge. We therefore conclude that the first charge exerts a larger force on the second charge. True or false?

(b) Two charged objects attract each other with an electric force. If the charges on both objects are doubled and the separation is also doubled, what happens to the force between them?

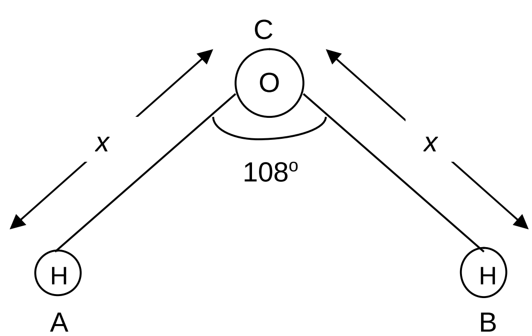
2 Two small point charges $+Q_1$ and $-Q_2$ are placed at X and Y respectively and are separated by a distance r = 2.0 m as shown below. The charges of Q_1 and Q_2 are 4.0 x 10^{-6} C and -2.0 x 10^{-6} C respectively.



(a) Draw on the figure the direction of the electric field which Q_1 causes at Y.
(b) What is the electric field strength which Q_1 produces at Y?
(c) What is the force which Q_1 causes on Q_2?
(d) Sketch on the figure the pattern of the electric field around the charges.

[9.0 kNC^{-1}, 18 mN]

3 The diagram shows a simple model of a water molecule where A and B are hydrogen atoms and C is an oxygen atom. The oxygen atom acts as a point charge of magnitude –1.1 × 10^{-19} C and each of the hydrogen atoms acts as a point charge of magnitude +0.55 × 10^{-19} C. The distance x between the oxygen atom and each hydrogen atom is 1.0 × 10^{-10} m.
Calculate
(a) the force exerted by each hydrogen atom on the oxygen atom, and
(b) the magnitude and direction of the resultant force acting on the oxygen atom due to the two hydrogen atoms.



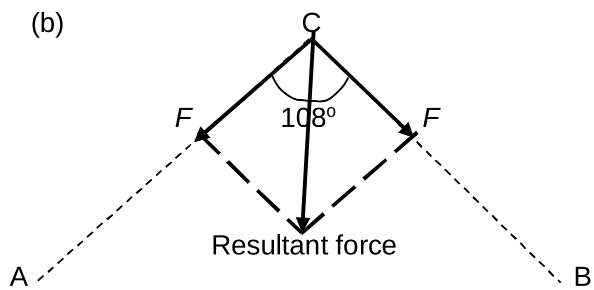
Solution:

(a) $$F = \frac{Q_A Q_C}{4\pi\varepsilon_o x^2}$$

$$= \frac{0.55\times10^{-19}\times(-1.1\times10^{-19})}{4\pi\times8.85\times10^{-12}\times(1.0\times10^{-10})^2} = -5.4\times10^{-9}\text{ N}$$

The minus sign indicates that the force is attractive.

(b)



Resultant force = $2F\cos 54^\circ$

$= 2\times5.4\times10^{-9}\times\cos 54^\circ$

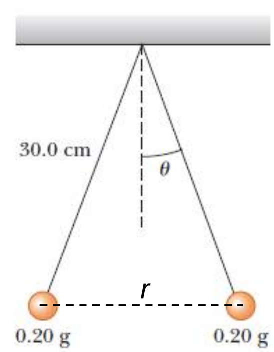
$= 6.4\times10^{-9}$ N at an angle of 54° to force F

4 Two small metallic spheres, each of mass m = 0.20 g, are suspended as pendulums by light strings from a common point as shown in the figure. The spheres are given the same electric charge q, and it is found that they come to equilibrium when each string is at an angle θ = 5.0° with the vertical. Each string is 30.0 cm long.

(a) Sketch the forces acting on each sphere.

(b) What is the magnitude of the charge on each sphere?

[7.2×10^{-9} C]



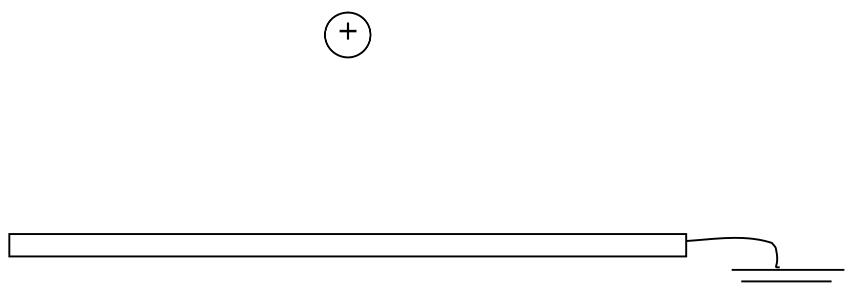
(c) Discuss what happens to angle θ if
1. Both of equal mass m, charges are 2q and q.
2. Both of equal charge q, masses are 2m and m.

Electric Field Strength

5 (a) What do you understand by an *electric field*?
(b) When defining the electric field, why is it necessary to specify that the magnitude of the test charge be very small?

> (a) Electric field is a region in space whereby a small test charge placed in it experiences a force.
> (b) The test charge must be small so that it will not affect the magnitude of the original field.

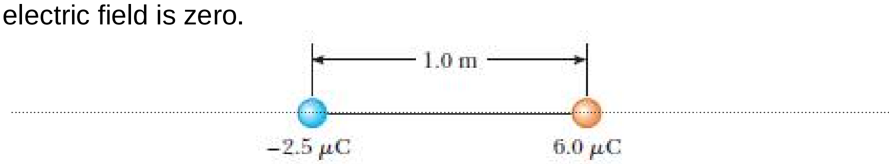
6 The figure below shows a positively charged metal sphere placed above an earthed metal plate. As the sphere is moved closer to the earthed plate, the plate becomes negatively charged.



(a) Suggest how the negative charge on the earthed plate arises.

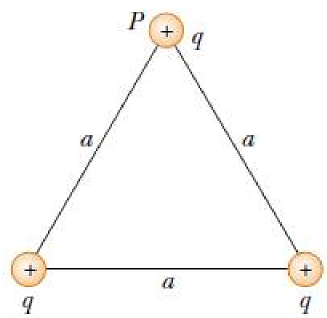
(b) Draw the electric field pattern between the sphere and the plate.

7 In the figure below, determine the point **P** (other than infinity) at which the resultant electric field is zero.



[1.8 m to the left of the -2.5μC charge]

8 Three equal positive charges q = 5.00 nC are at the corners of an equilateral triangle of side a = 20.0 cm, as shown in the figure.

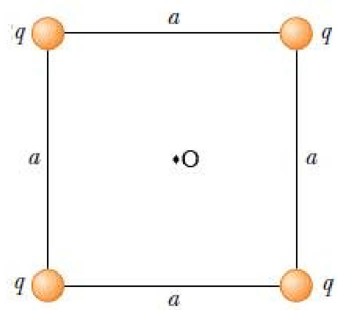


(a) What are the magnitude and direction of the electric field at *P* due to the two charges at the base?
(b) What is the electric force acting on the charge q placed at P?

[1950 NC^{-1} upwards, 9.75 x 10^{-6} N]

Electric Potential and Potential Energy

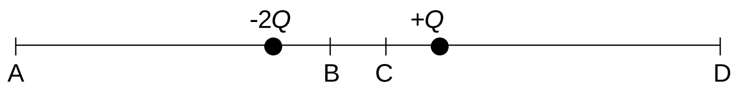
9 Four equal charges *q* are placed at the corners of a square of side *a*.



(a) Show that the electric field is zero at the centre O.
(b) Show that the electric potential at the centre is non-zero and determine its magnitude in terms of q and a.

[$\frac{\sqrt{2}q}{\pi\varepsilon_o a}$]

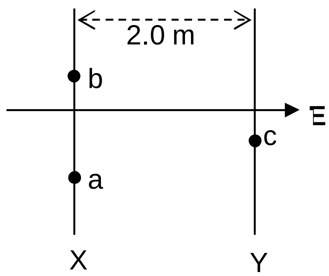
10 (a) Two point charges -2*Q* and +*Q* are situated as shown. At which point A, B, C or D, could the resultant electric field strength due to these charges be zero?



(b) Give a physical explanation of the fact that the electric potential near an isolated positive charge is positive whereas the potential near an isolated negative charge is negative.

(a) D
(b) V = *W/q. The work done W by an external agent to bring unit positive test charge q from infinity to a point near an isolated <u>positive</u> charge is <u>positive</u> because of <u>repulsion</u> between them. Hence the potential at that point is <u>positive</u>.*
However*, the work done W by an external agent to bring unit positive test charge q from infinity to a point near an isolated <u>negative</u> charge is <u>negative</u> because of <u>attraction</u> between them. Hence the potential at that point is <u>negative</u>.*

11 The figure shows two equipotential lines X and Y. The uniform electric field strength between X and Y is 5.0 V m^{-1} and points in the direction shown. The potential of X is –10.0 V. The distance between X and Y is 2.0 m.

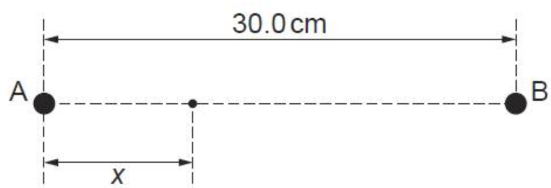


(a) Which is at a higher electric potential, X or Y? Hence find the potential of Y.

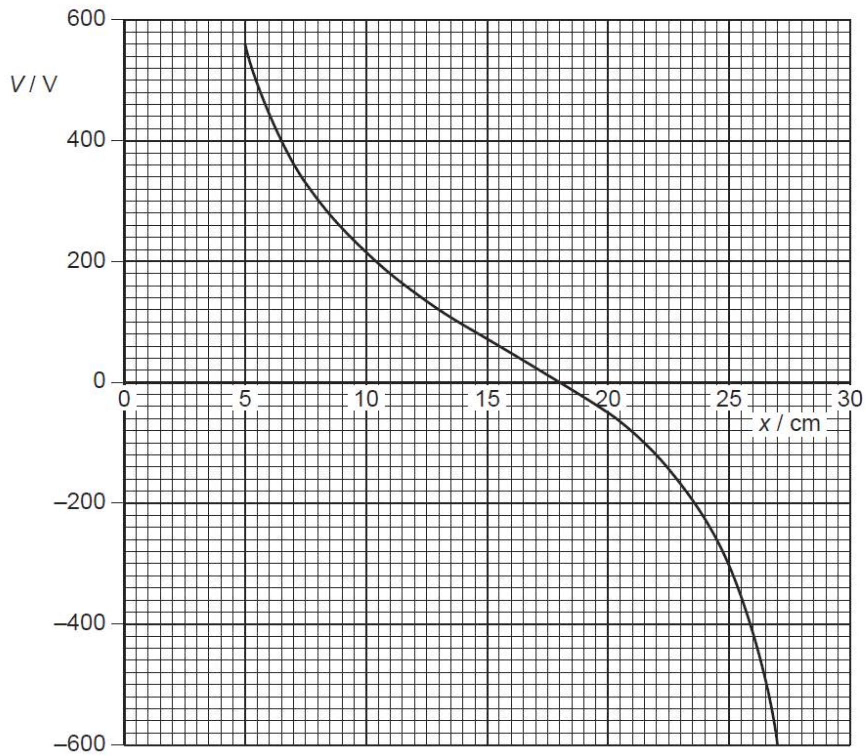
(b) Calculate the work done by external agent to bring a -5.0 μC charge from

(i) a to b, and (ii) a to c. [- 20.0 V, 0 J, 50 μJ]

12 Two isolated point charges A and B are separated by a distance of 30.0 cm, as shown in the figure.



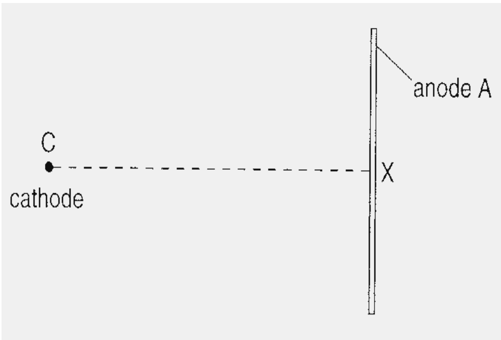
The charge at A is + 3.6 nC.
The variation with distance x from A along AB of the potential V is shown in the figure below.

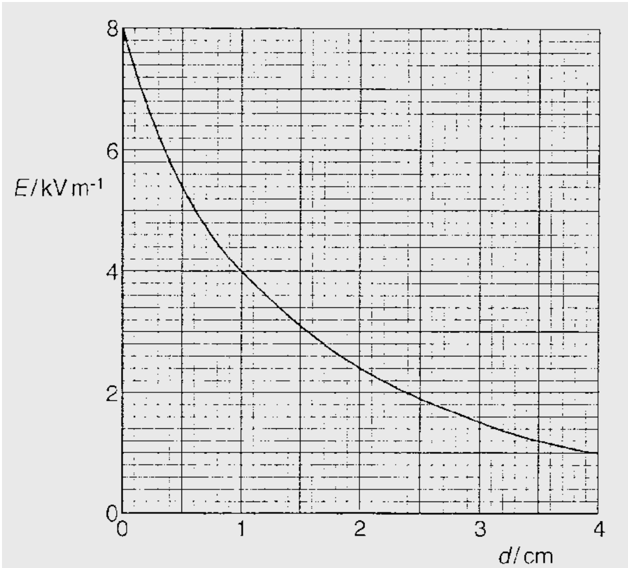


(a) State the value of *x* at which the potential is zero.
(b) Use your answer in (a) to determine the charge at B.
(c) A small test charge is now moved along the line AB from x = 5.0 cm to x = 27 cm. State and explain the value of *x* at which the force on the test charge will be maximum.
(d) If the charges of A and B are both positive, give a rough sketch of the resultant V-x graph

[18.0 cm, -2.4 nC,]

13





Electrons are emitted from a cathode C and are accelerated towards an anode A as shown in the figure above. The anode is earthed. CX is a line drawn from C normal to the anode A. The distance CX is 4.0 cm. The variation with distance *d* from C along CX of the magnitude of the electric field strength *E* is shown in the graph above.

(a) On the figure, mark with an arrow the direction of the electric field along CX.

(b) Use the graph to determine the force *F* on an electron at a point mid-way between C and X.

(c) (i) A student assumes that the force *F* on the electron remains constant as the electron moves from C to X. Use the value of *F* calculated in (b) to estimate, on the basis of this assumption, the potential difference between C and X.
(ii) Suggest, with a reason, whether the magnitude of the potential difference calculated in (i) will be an over-estimate or an under-estimate of the actual potential difference.

(Ans: 3.84×10^{-16} N, 96 V)

<u>Capacitance</u>

14 The capacitance of a certain variable capacitor may be varied between limits of 1×10^{-10} F and 5×10^{-10} F by turning a knob attached to the movable plates. The capacitor is set to 5×10^{-10} F, and is charged by connecting it to a battery of e.m.f. 200 V.

(a) What is the charge on the plates?

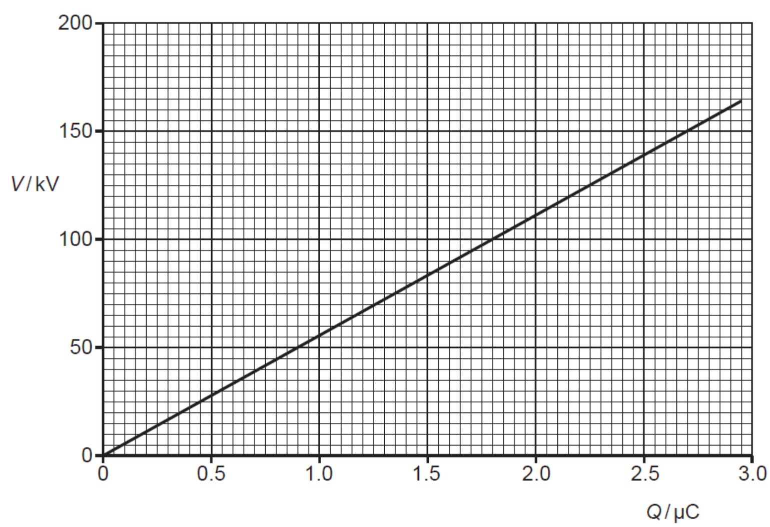
The battery is then disconnected and the capacitance is changed to 1×10^{-10} F.

(b) Assuming that no charge is lost from the plates, what is now the potential difference between them ?

(c) How much mechanical work is done against electrical forces in changing the capacitance?

(Ans: 1×10^{-7} C, 1000 V, 40 μJ)

15 The variation of the potential V of an isolated metal sphere with charge Q on its surface is shown below.



An isolated metal sphere has capacitance *C*.

Use the graph above to determine

(a) (i) the capacitance of the sphere;
(ii) the electric potential energy stored on the sphere when charged to a potential of 150 kV.

(b) A spark reduces the potential of the sphere from 150 kV to 75 kV. Calculate the energy loss from the sphere.

(Ans: 1.8×10^{-11} F, 0.20 J, 0.15 J)