

Name: _____ ()

Class: _____



華僑中學

HWA CHONG INSTITUTION

Paper E

HIGHER MATHEMATICS

Level : Secondary Three

Duration : 1 hour

Do not open this booklet until you are told to do so.

INSTRUCTIONS TO CANDIDATES

Write your name, class and index number on all the work you hand in.

Answer **4 out of 7** questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 40.

You are expected to use an electronic calculator to evaluate explicit numerical expressions. **The use of a graphing calculator is not permitted.**

You are reminded of the need for clear presentation in your answers. Omission of essential working will result in loss of marks.

This question paper consists 5 printed pages, including this page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. TRIGONOMETRY

Mensuration

Arc Length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

Answer 4 out of 7 questions

1 **(a)** Given that $x + y = 1$, find the value of $x^3 + 3xy + y^3$. [3]

(b) Solve the inequality $\frac{(x-1)^2(x+2)}{x^2-7x+12} \leq 0$.

Hence, solve $\frac{(\sin x - 1)^2(\sin x + 2)}{\sin^2 x - 7\sin x + 12} \leq 0$ for $0 \leq x \leq 2\pi$. [7]

2 **(a)** Given that the equation $x^2 + 2k + 1 = (k + 2)x$ has two real roots and the sum of squares of the two roots is 11, find the value of k . [5]

(b) The graph $y = x^2 - 11x + m + 30$ intersects the x -axis at two points, and both points are on the right of the point $(5, 0)$. Find the range of values that m can take. [5]

3 **(a)** Express the following simultaneous equations in matrix form, and hence solve for the value of x and of y . [4]

$$4x + 5y = 2$$

$$2x - 7y = 20$$

(b) A polynomial $px^7 + qx^3 + rx + 4$ has a remainder of 3 when it is divided by $x + 7$. Find the remainder when it is divided by $x - 7$. [3]

(c) A standard catenary has the equation $y = a \left(\frac{e^{\frac{x}{a}} + e^{-\frac{x}{a}}}{2} \right)$, where a is a real

constant, and is symmetrical about the y -axis.

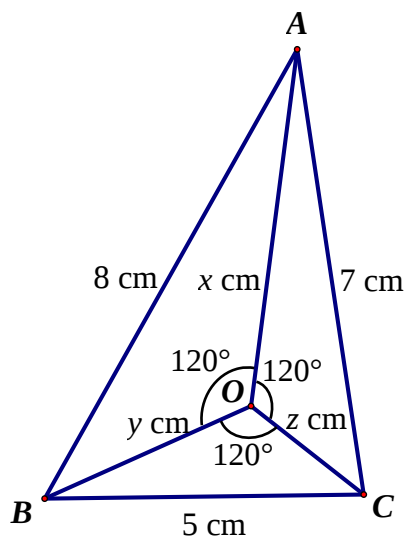
Given that the equation of a transformed catenary curve is $y = \frac{e^{x+5} + e^{-x-5}}{4} - 2$,

find the coordinates of the catenary's turning point. [3]

[Turn Over]

- 4 (a) Simplify $\sqrt{8-2\sqrt{15}}$, leaving your answer in the form $\sqrt{a}-\sqrt{b}$, where a and b are positive integers. [2]
- (b) Given that a , b and c are the sides of a triangle ABC , simplify $\sqrt{(a+b-c)^2} + \sqrt{(a-b+c)^2} + \sqrt{(a-b-c)^2}$. [3]
- (c) Solve the equation $\sqrt{\log_x \sqrt{3x}} \times \log_3 x = 1$. [5]
- 5 Three points have coordinates $A(4, 3)$, $B(5, 2)$ and $C(1, 0)$. Find
- (i) the equation of the circle that passes through the points A , B and C , [6]
- (ii) the equation of the line that is a tangent to the circle and passes through the origin. [4]
- 6 (a) Functions f and g are defined by
- $$f : x \mapsto 2x + a,$$
- $$g : x \mapsto -2x + b.$$
- Given that $f^2(4) = 1$ and $g(2) = g^{-1}(2)$, find
- (i) the value of a and of b , [3]
- (ii) the value of $fg(3)$. [2]
- (b) The function f is defined by $f : x \mapsto \frac{x}{x+1}$, $x \neq -1$.
- On two separate diagrams**, sketch the following graphs, indicating clearly the x -intercepts, the y -intercepts and the asymptotes, if any.
- (i) $y = |f(x)|$ [3]
- (ii) $y = f(|x|)$ [2]

- 7 In $\triangle ABC$, $AB = 8$ cm, $BC = 5$ cm and $AC = 7$ cm. O is a point in the triangle such that $\angle AOB = \angle AOC = \angle BOC = 120^\circ$. It is also given that $OA = x$ cm, $OB = y$ cm and $OC = z$ cm.



- (i) Find the area of $\triangle ABC$, leaving your answer in exact form.
(Hint: use Heron's formula, $\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$.) [1]
- (ii) Show that $x^2 + xy + y^2 = 64$.
Hence, write down the value of $y^2 + yz + z^2$ and of $x^2 + xz + z^2$. [3]
- (iii) Find the value of $xy + yz + xz$. [3]
- (iv) Hence, find the value of $(x + y + z)^2$. [3]

End of Paper