Secondary 3 Mathematics
WA3 Revision Time Trial — Bedok View (2023)

Duration: 66 minutes **Score: __________ / 44**

Topics tested: Graphs of Functions; Further Trigonometry; Applications of Trigonometry; Arc Length and Sector Area.

Section A: Graphs of Functions [11 marks]

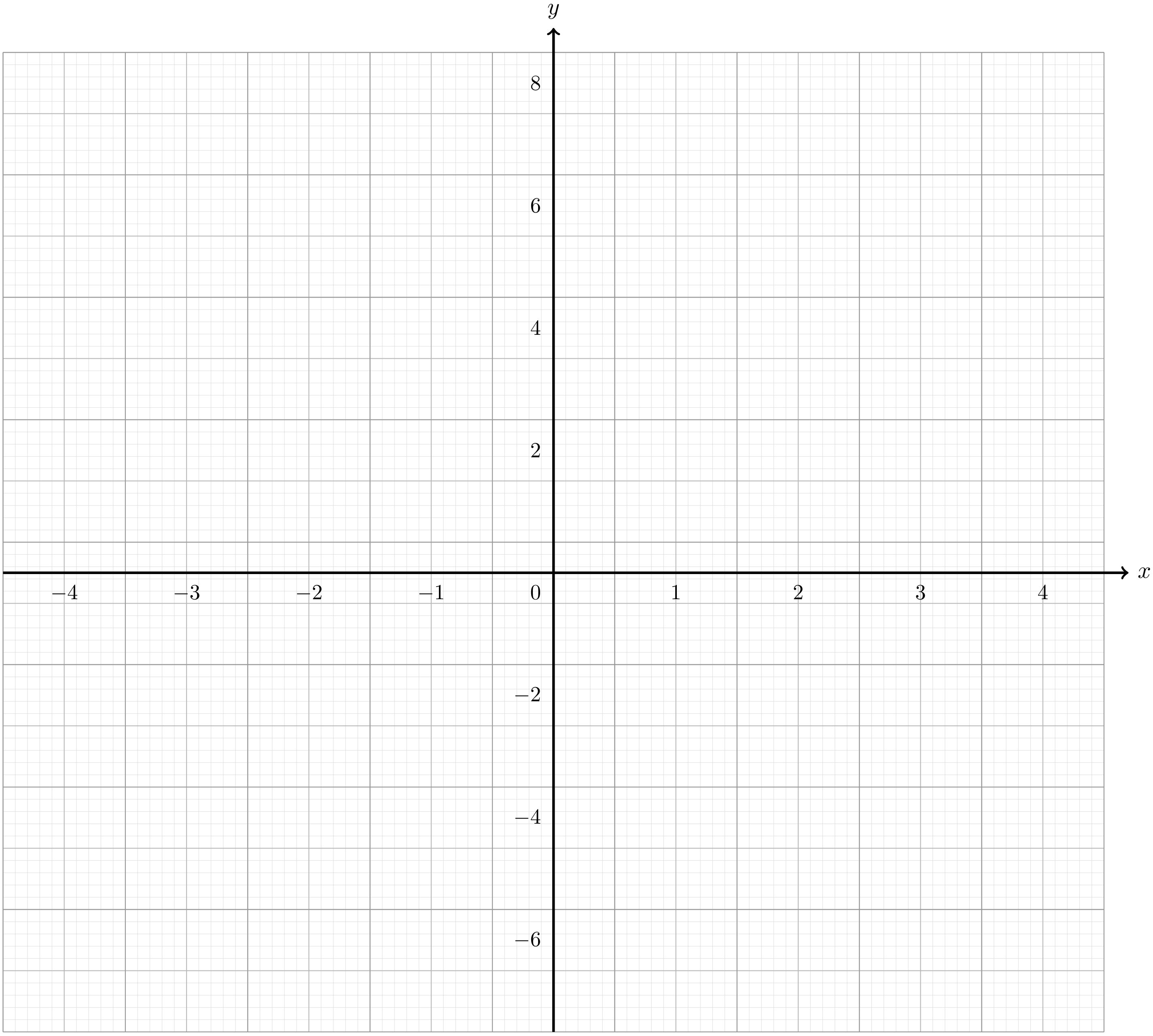
1. The variables x and y are connected by the equation $y = \dfrac{x^3}{5} - x + 2$.
Some corresponding values of x and y are given in the table below.

x	-4	-3	-2	-1	0	1	2	3
y	-6.8	p	2.4	2.8	2	1.2	1.6	4.4

(a) Find the value of p. *[1]*

Answer $p =$ ....

(b) On the grid on the next page, draw the graph of $y = \dfrac{x^3}{5} - x + 2$ for $-4 \le x \le 3$. *[3]*


y
x
8
6
4
2
0
−2
−4
−6
−4
−3
−2
−1
1
2
3
4

(c) By drawing a suitable tangent on the same grid, find the gradient of the curve when $x = 2$. *[2]*

Answer ....

(d) (i) On the same grid, draw the graph of $2y = -x + 2$. *[1]*

(ii) Using your graph, state the x-coordinate of the intersection point between the curve and the line. *[1]*

Answer $x =$ ....

(iii) The roots of the equation $2x^3 + ax^2 + bx + c = 0$ is given by the x-value of the intersection point in part **(d)(ii)**. Find the values of a, of b and of c. *[3]*

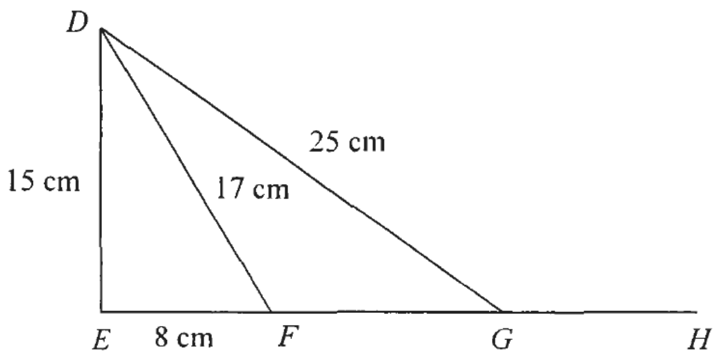
Answer $a =$ ...., $b =$ ...., $c =$ ....

Section B: Further Trigonometry [11 marks]

2. Given that $7 \sin x = 3$, find two possible values for angle x, where $0° \leq x \leq 180°$. *[2]*

Answer $x =$ .... *or* ....

3. In the diagram below, $DE = 15$ cm, $DF = 17$ cm, $DG = 25$ cm, $EF = 8$ cm and $EFGH$ is a straight line.



(a) Show that triangle DEF is a right-angled triangle. *[2]*

Answer ....

....

....

(b) Find angle DGF. *[2]*

Answer .... °

(c) Find the length of FG. *[2]*

Answer .... cm

(d) If angle $DFG = \theta$, find, without the use of a calculator,

(i) $\cos\theta$, *[1]*

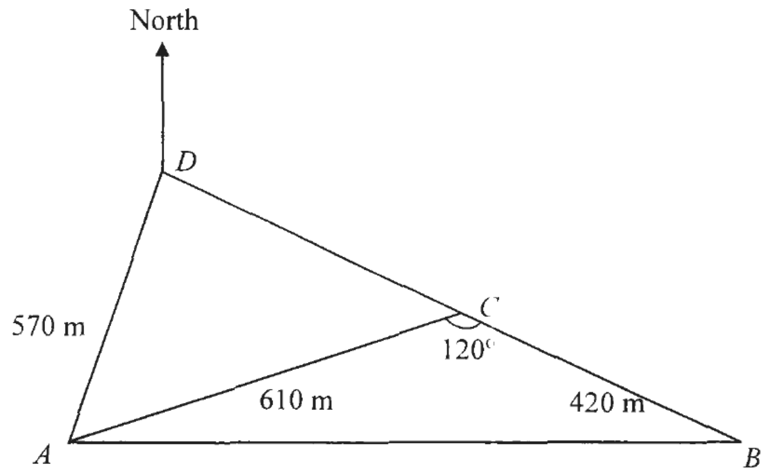
Answer ....

(ii) $\sin(90^\circ - \angle DFE)$. *[2]*

Answer ....

Section C: Applications of Trigonometry [11 marks]

4. The following figure shows a park $ABCD$. $AD = 570$ m, $AC = 610$ m and $BC = 420$ m. B is east of A and BCD is a straight line. Angle $ACB = 120°$.



(a) Calculate the length of AB. *[3]*

Answer .... m

(b) Calculate angle ADC. *[2]*

Answer .... °

(c) Calculate the bearing of D from B. *[3]*

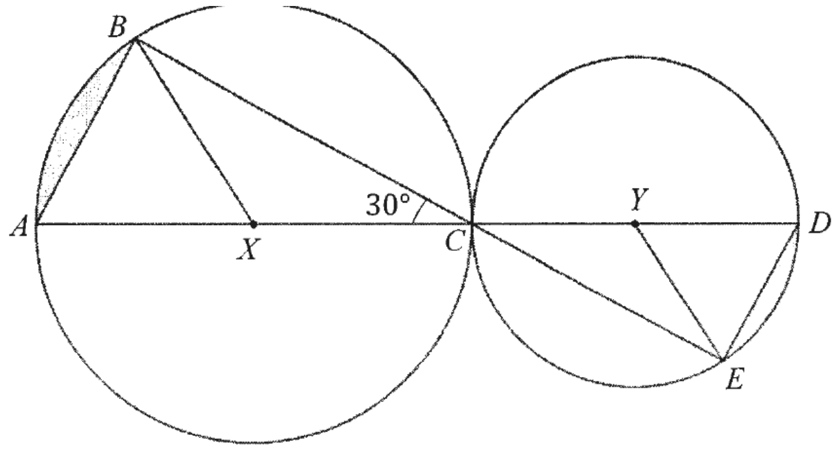
Answer .... $^\circ$

(d) A drone is spotted 70 m directly above C.
Find the greatest angle of elevation of the drone from a point along AB. *[3]*

Answer .... $^\circ$

Section D: Arc Length and Sector Area [11 marks]

5. The diagram shows two circles that touch at C. A, B and C are points on the bigger circle with centre X. C, D and E are points on the smaller circle with centre Y. BCE and $AXCYD$ are straight lines.



(a) Given that angle $BCX = 30°$, find

(i) angle CXB, *[1]*

Answer .... °

(ii) angle XAB. *[1]*

Answer .... °

(b) What type of triangle is triangle XAB? *[1]*

Answer ....

(c) The radius of the larger circle is 9 cm and the radius of the smaller circle is 6 cm.

(i) Find the area of the minor sector XAB, leaving your answer in terms of π. *[2]*

Answer .... cm^2

(ii) Calculate the percentage of the shaded areas to the total area of the two circles. *[6]*

Answer .... %

— END OF PAPER —

Answer Key
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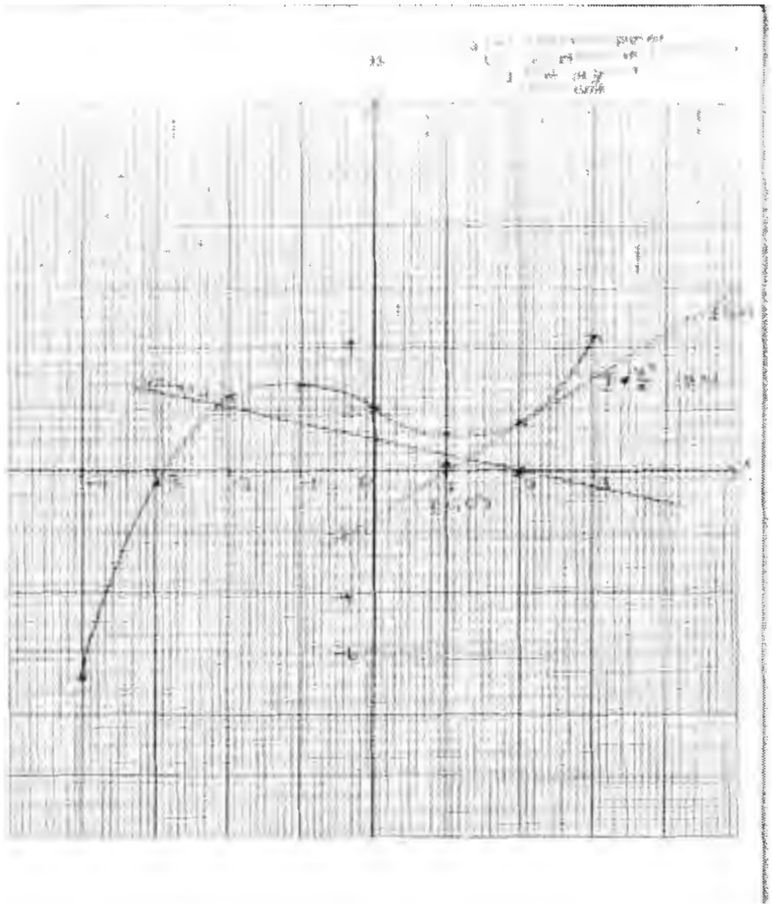
Section A: Graphs of Functions [11 marks]

1(a) At $x = -3$: $y = \dfrac{-27}{5} + 3 + 2 = -\mathbf{0.4}$. *[1]*

1(b) All eight points plotted and joined with a smooth curve: rising from $(-4,\ -6.8)$ to a local maximum near $(-1.3,\ 2.9)$, dipping to a local minimum near $(1.3,\ 1.1)$, then rising to $(3,\ 4.4)$. *[3]*



(Marking scheme graph shown, including the tangent for (c) and the line for (d)(i).)

1(c) Tangent drawn at $x = 2$.
Exact gradient: $\dfrac{dy}{dx} = \dfrac{3x^2}{5} - 1 = \dfrac{12}{5} - 1 = \mathbf{1.4}$ (accept 1.2 to 1.8). *[2]*

1(d)(i) Line $2y = -x + 2$, i.e. $y = -\dfrac{1}{2}x + 1$, drawn through $(0,\ 1)$ and $(2,\ 0)$. *[1]*

1(d)(ii) From the graph: **x** $\approx -\mathbf{2.15}$ (accept ± 0.1). *[1]*

1(d)(iii) $\dfrac{x^3}{5} - x + 2 = -\dfrac{1}{2}x + 1$
$\times 10$: $2x^3 - 10x + 20 = -5x + 10 \;\Rightarrow\; 2x^3 - 5x + 10 = 0$
$\therefore$ **a** $= \mathbf{0}$, **b** $= -\mathbf{5}$, **c** $= \mathbf{10}$. *[3]*

Section B: Further Trigonometry [11 marks]

2. $\sin x = \frac{3}{7} \Rightarrow x = 25.376\ldots°$ or $180° - 25.376\ldots°$
$\therefore x =$ **25.4**° or **154.6**° (1 d.p.). *[2]*

3(a) $DE^2 + EF^2 = 15^2 + 8^2 = 289$ and $DF^2 = 17^2 = 289$.
Since $DE^2 + EF^2 = DF^2$, by the **Converse of Pythagoras' Theorem**, triangle DEF is right-angled (at E). *[2]*

3(b) In right-angled triangle DEG: $\sin \angle DGF = \frac{DE}{DG} = \frac{15}{25}$
$\angle DGF = 36.869\ldots =$ **36.9**° (1 d.p.). *[2]*

3(c) $EG = \sqrt{25^2 - 15^2} = 20$ cm (Pythagoras).
$FG = 20 - 8 =$ **12** cm. *[2]*

3(d)(i) $\cos \theta = \cos(180° - \angle DFE) = -\cos \angle DFE = -\mathbf{\frac{8}{17}}$. *[1]*

3(d)(ii) $\sin(90° - \angle DFE) = \cos \angle DFE = \frac{EF}{DF} = \mathbf{\frac{8}{17}}$. *[2]*

Section C: Applications of Trigonometry [11 marks]

4(a) By the Cosine Rule: $AB^2 = 610^2 + 420^2 - 2(610)(420)\cos 120° = 804700$
$AB = 897.05\ldots =$ **897** m (3 s.f.). *[3]*

4(b) $\angle ACD = 180° - 120° = 60°$ (adjacent $\angle$s on straight line BCD).
By the Sine Rule in triangle ACD: $\dfrac{610}{\sin \angle ADC} = \dfrac{570}{\sin 60°}$
$\sin \angle ADC = \dfrac{610 \sin 60°}{570} = 0.9268\ldots \Rightarrow \angle ADC =$ **67.9**° (1 d.p.). *[2]*

4(c) By the Sine Rule in triangle ABC: $\dfrac{\sin \angle CBA}{610} = \dfrac{\sin 120°}{897.05\ldots}$
$\angle CBA = 36.079\ldots°$
D lies on CB produced, so bearing of D from $B = 270° + 36.079\ldots° =$ **306.1**° (1 d.p.). *[3]*

4(d) The angle of elevation is greatest at the point on AB nearest to C (foot of the perpendicular from C).
Shortest distance $= 420 \sin 36.079\ldots° = 247.33\ldots$ m.
$\tan\theta = \dfrac{70}{247.33\ldots} \Rightarrow \theta =$ **15.8**° (1 d.p.). *[3]*

Section D: Arc Length and Sector Area [11 marks]

5(a)(i) $XB = XC$ (radii), so $\angle XBC = \angle BCX = 30°$.
$\angle CXB = 180° - 30° - 30° =$ **120**° ($\angle$ sum of isosceles $\triangle$). *[1]*

5(a)(ii) AXC is a straight line, so $\angle AXB = 180° - 120° = 60°$.
$XA = XB$ (radii), so $\angle XAB = \dfrac{180° - 60°}{2} =$ **60**°. *[1]*

5(b) All three angles are $60°$: **equilateral triangle**. *[1]*

5(c)(i) Area of minor sector $XAB = \dfrac{60°}{360°} \times \pi(9)^2 = \mathbf{13\frac{1}{2}}\pi$ cm^2. *[2]*

5(c)(ii) Area of triangle $XAB = \dfrac{1}{2}(9)(9)\sin 60° = 35.074$ cm^2.
Shaded segment (big circle) $= 13.5\pi - 35.074 = 7.338$ cm^2.
Area of minor sector $YDE = \dfrac{60°}{360°} \times \pi(6)^2 = 6\pi$ cm^2; area of triangle $YDE = \dfrac{1}{2}(6)(6)\sin 60° = 15.588$ cm^2.
Shaded segment (small circle) $= 6\pi - 15.588 = 3.261$ cm^2.
Percentage $= \dfrac{7.338 + 3.261}{\pi(9)^2 + \pi(6)^2} \times 100\% = \dfrac{10.599}{367.56} \times 100\% =$ **2.88**% (3 s.f.). *[6]*