Tutorial 7A: Sequences and Series

Section A (Basic Questions)

1 Write down the series given by the summation notation, showing the first three terms and the last two terms whenever possible.

(a) $\sum_{r=1}^{8}\left(\frac{4}{r}+\frac{r}{4}\right)$ **(b)** $\sum_{r=4}^{30}(5-3r)$ **(c)** $\sum_{j=5}^{15}(m+3j)$, where m is a constant

Solutions

(a)	$\sum_{r=1}^{8}\left(\frac{4}{r}+\frac{r}{4}\right)=4\sum_{r=1}^{8}\frac{1}{r} + \frac{1}{4}\sum_{r=1}^{8}r$ $=4\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\ldots\frac{1}{7}+\frac{1}{8}\right)+\frac{1}{4}(1+2+3\ldots+7+8)$
(b)	$\sum_{r=4}^{30}(5-3r)=(-7)+(-10)+(-13)+\ldots+(-82)+(-85)$
(c)	$\sum_{j=5}^{15}(m+3j)$, where m is a constant $=m+3(5)+m+3(6)+m+3(7)+\ldots+m+3(14)+m+3(15)$

2 For each of the following series, write down the rth term and express the series using the summation notation.

(a) $3+6+9+...+369$ **(b)** $(-4)+(-1)+(2)+...+(53)$

(c) $4+2+1+\frac{1}{2}+\frac{1}{4}+...$ **(d)** $\frac{1}{2^2}-\frac{2}{3^2}+\frac{3}{4^2}-\frac{4}{5^2}+...+\frac{7}{8^2}-\frac{8}{9^2}$

Solutions

(a)	$3+6+9+...+369=3+6+9+...+3(r)+...+369$ $=\sum_{r=1}^{123}(3r)$
(b)	$(-4)+(-1)+(2)+...+(53)$ $=(-4)+(-1)+(2)+...+(3r-7)+...+(53)$ $=\sum_{r=1}^{20}(3r-7)$ or $\sum_{r=0}^{19}(-4+3r)$ $u_1=-4+3\,(0)$ $u_2=-4+3\,(1)$ $u_3=-4+3\,(2)$ $\vdots$ $u_r=-4+3\,(r-1)=3r-7$ $\vdots$ $u_{20}=53=-4+3\,(19)$
(c)	$4+2+1+\frac{1}{2}+\frac{1}{4}+...$ $=4+2+1+\frac{1}{2}+\frac{1}{4}+...+2^{3-r}+...$ $=\sum_{r=1}^{\infty}2^{3-r}$ or $\sum_{r=-2}^{\infty}\frac{1}{2^r}$ $u_1=4$ $u_2=4\left(\frac{1}{2}\right)$ $u_3=4\left(\frac{1}{2}\right)^2$ $\vdots$ $u_r=4\left(\frac{1}{2}\right)^{r-1}$ $\vdots$
(d)	$\frac{1}{2^2}-\frac{2}{3^2}+\frac{3}{4^2}-\frac{4}{5^2}+...+\frac{7}{8^2}-\frac{8}{9^2}$ $=\frac{1}{2^2}-\frac{2}{3^2}+\frac{3}{4^2}-\frac{4}{5^2}+...(-1)^{r+1}\frac{r}{(r+1)^2}+\frac{7}{8^2}-\frac{8}{9^2}$ $=\sum_{r=1}^{8}\left[(-1)^{r+1}\frac{r}{(r+1)^2}\right]$

3 Justify whether each of the following statements is true or false.

(a) $\sum_{r=1}^{2n} r^4 = \sum_{r=0}^{2n-1} (r+1)^4$ **(b)** $\sum_{r=1}^{n} 1 = \frac{1}{2}n(n+1)$

(c) $\sum_{r=1}^{n} r^2 = \left(\sum_{r=0}^{n} r\right)^2$ **(d)** $\sum_{r=1}^{3n} r^5 = \sum_{r=0}^{3n} r^5$

Solutions

(a)	$\sum_{r=1}^{2n} r^4 = \sum_{r=0}^{2n-1} (r+1)^4$ is true because $\text{LHS} = \sum_{r=1}^{2n} r^4 = 1^4 + 2^4 + \ldots + (2n)^4$ $\text{RHS} = \sum_{r=0}^{2n-1} (r+1)^4 = 1^4 + 2^4 + \ldots + (2n)^4$
(b)	$\sum_{r=1}^{n} 1 = \frac{1}{2}n(n+1)$ is true for $n = 1$. For $n \neq 1$, $\sum_{r=1}^{n} 1 = \frac{1}{2}n(n+1)$ is false because $\text{LHS} = \sum_{r=1}^{n} 1 = n(1) = n$ $\text{RHS} = \frac{1}{2}n(n+1) \neq \text{LHS}$
(c)	$\sum_{r=1}^{n} r^2 = \left(\sum_{r=0}^{n} r\right)^2$ is true for $n = 1$. For $n \neq 1$, $\sum_{r=1}^{n} r^2 = \left(\sum_{r=0}^{n} r\right)^2$ is false because $\text{LHS} = \sum_{r=1}^{n} r^2 = 1^2 + 2^2 + \ldots + n^2$ $\text{RHS} = \left(\sum_{r=0}^{n} r\right)^2 = (0+1+2+\ldots+n)^2 \neq \text{LHS}$
(d)	$\sum_{r=1}^{3n} r^5 = \sum_{r=0}^{3n} r^5$ is true because $\text{LHS} = \sum_{r=1}^{3n} r^5 = 1^5 + 2^5 + 3^5 \ldots + (3n)^5$ $\text{RHS} = \sum_{r=0}^{3n} r^5 = 0^5 + 1^5 + 2^5 + 3^5 \ldots + (3n)^5 = \text{LHS}$

4 It is given that $\sum_{r=1}^{n} u_r = 5n + 2^n$, where u_r is an expression in terms of *r*. Find the value of

(i) $\sum_{r=1}^{10} u_r$ **(ii)** $\sum_{r=5}^{10} u_r$ **(iii)** u_{10}

[(i) 1074 **(ii)** 1038 **(iii)** 517**]**

Solutions

(i)	$\sum_{r=1}^{10} u_r = 5(10) + 2^{10} = 1074$
(ii)	$\sum_{r=5}^{10} u_r = \sum_{r=1}^{10} u_r - \sum_{r=1}^{4} u_r$ $= 5(10) + 2^{10} - \left[5(4) + 2^4\right] = 1038$
(iii)	$u_{10} = \sum_{r=1}^{10} u_r - \sum_{r=1}^{9} u_r$ $= 5(10) + 2^{10} - \left[5(9) + 2^9\right] = 517$