

Superposition

Guiding Questions	Learning Objectives
1. What happens when waves meet an obstacle?	(e) show an understanding of the terms diffraction, interference, coherence, phase difference and path difference
2. What happens when two or more waves meet?	(a) explain and use the principle of superposition in simple applications. Two source interference (f) show an understanding of phenomena which demonstrate two-source interference using water waves, sound waves, light and microwaves (g) show an understanding of the conditions required for two-source interference fringes to be observed (h) recall and use the equation $\frac{ax}{D} = \lambda$ to solve problems for double-slit interference, where a is the slit separation and x is the fringe separation Single slit and multiple slits diffraction (i) recall and use the equation $a \sin \theta = n\lambda$ to solve problems involving the principal maxima of a diffraction grating, where a is the slit separation (j) describe the use of a diffraction grating to determine the wavelength of light (knowledge of the structure and use of a spectrometer is not required) (k) show an understanding of phenomena which demonstrate diffraction through a single slit or aperture, or across an edge, such as the diffraction of water waves in a ripple tank with both a wide gap and a narrow gap, or the diffraction of sound waves from loudspeakers or around corners (l) recall and use the equation $b \sin \theta = \lambda$ to solve problems involving the positions of the first minima for diffraction through a single slit of width b (m) recall and use the Rayleigh criterion $\theta \approx \frac{\lambda}{b}$ for the resolving power of a single aperture, where b width of the aperture.

3. What is a stationary wave and how is it produced?	(c) explain the formation of a standing (stationary) wave using a graphical method, and identify nodes and antinodes, differentiating between pressure and displacement nodes and antinodes for sound waves (b) show an understanding of experiments which demonstrate standing (stationary) waves using microwaves, stretched strings and air columns (d) determine the wavelength of sound using standing (stationary) waves
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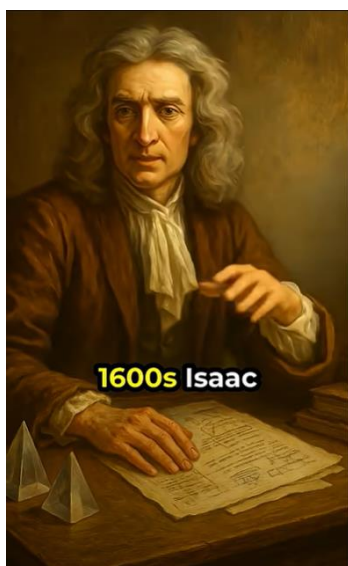
Nature of Science

Thomas Young and the Nature of Light

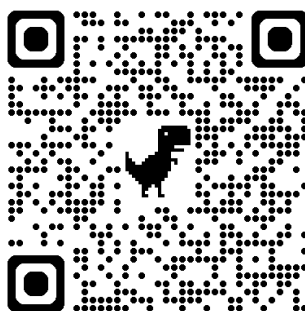
From This Month in Physics History, The American Physical Society (May 2008)
<https://www.aps.org/publications/apsnews/200805/physics/history.cfm>

The debate over whether light is a wave or a particle goes back many centuries. In the 17th century, Isaac Newton believed light was composed of a stream of corpuscles (little particles). At that time, a few scientists, most notably Dutch physicist and astronomer Christiaan Huygens, thought light was a wave vibrating in some sort of ether.

Watch the video and discover their discoveries.



<https://youtu.be/1TL8eRgeSnI?si=RBfK->



What happens when waves meet an obstacle?

1. Diffraction



Diffraction is the spreading of a wave into its geometrical shadow when it is incident on an edge of an obstacle or an aperture/opening.

The extent of spreading depends on the relative sizes of the opening and the wavelength of wave. Diffraction is most noticeable if the size of the opening is approximately equal to the wavelength of the wave.

1.1 Diffraction of Water Waves

Diffraction can be demonstrated in the lab using water waves in a ripple tank experiment.

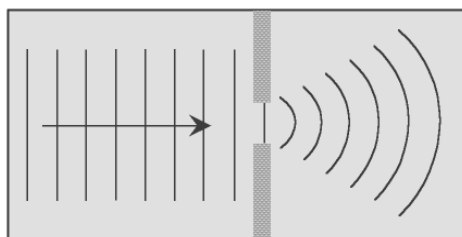


Fig. 1.1 – Schematic diagram showing water waves through a narrow gap

Fig. 1.1 shows a water wave approaching a small opening. The parallel lines (which could be shadows in the ripple tank experiment) show the wavefronts – lines joining crests of the water wave

If the wavelength is approximately the same size as the width of the opening, then the wave will diffract significantly as shown.

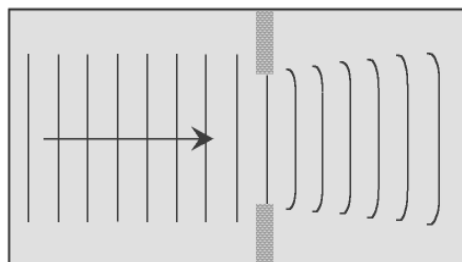


Fig. 1.2 – Schematic diagram showing water waves through a wide gap

Fig. 1.2 shows the same water wave approaching a bigger opening.

If the wavelength of the wave is much smaller than the width of the opening, only the wave that emerge around the edge experienced some low level of diffraction while the centre portion remains largely unchanged.

While Fig. 1.1 & 1.2 shows the schematic diagram that you will usually draw to illustrate diffraction, Fig. 1.3 shows the actual water waves seen in a ripple tank experiment.

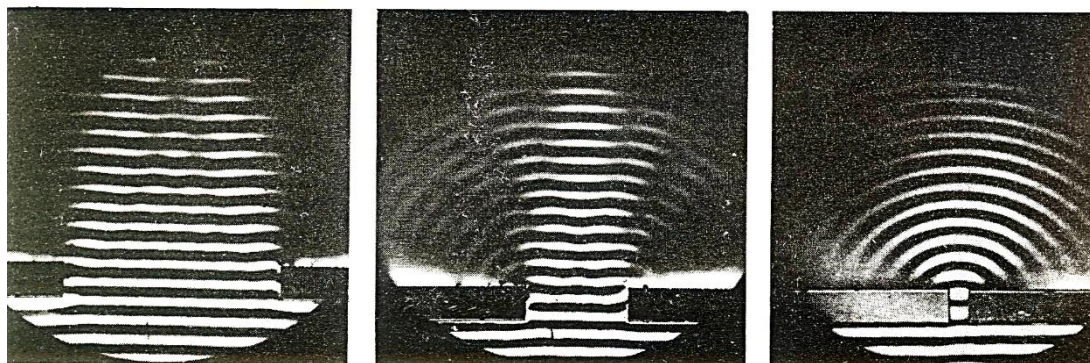


Fig. 1.3 – Actual images showing water waves through a gaps of different sizes
(source: T. Duncan – Advanced Physics)

Wavelength
572 nm

0.1 mm

10 mm

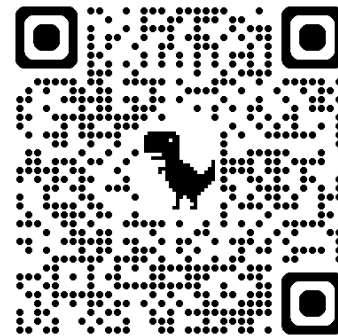
Diameter
0.04 0.22 mm 0.40

Eccentricity
0.00 0.99

Explore the simulation on diffraction. Use the slider to vary the diameter of the hole and see its effects.

1.2 Diffraction of Longitudinal Waves (Sound Waves)

Sound waves can diffract, for example, through a door opening. The width of the door (roughly a metre) is comparable to the wavelength of sound, and therefore, the sound waves diffract appreciably. This explains how we can hear someone approach the door although the person is not visually in sight.



Explore this video on diffraction of sound. You can defer the interference segment until later of this topic.

1.3 Diffraction of Transverse Waves (Electromagnetic Waves)

When visible light passes through a narrow slit (with slit width comparable to wavelength of light), it spreads beyond the narrow path defined by the slit into regions that would be in geometrical shadow. This is shown in Fig 1.4. This phenomenon will be explained further in the subsequent sections.

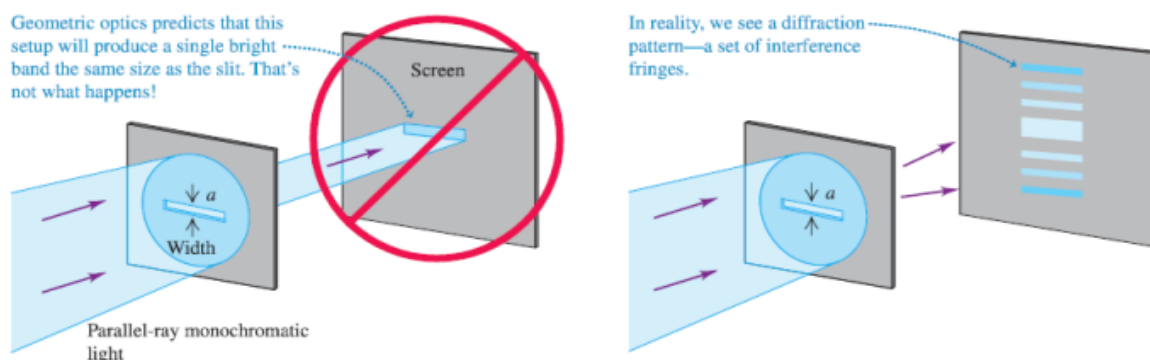


Fig. 1.4
(source: Young - College Physics)

What happens when two or more waves meet?

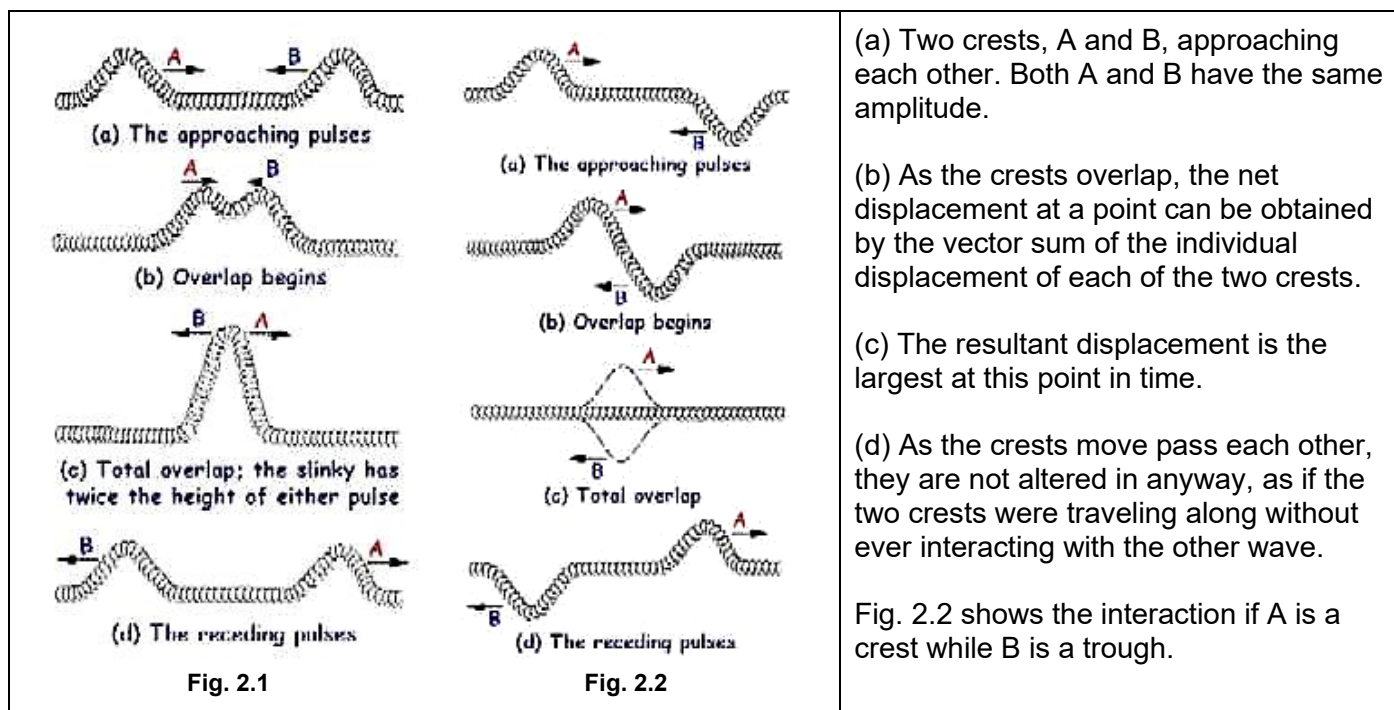
2.1 Principle of Superposition



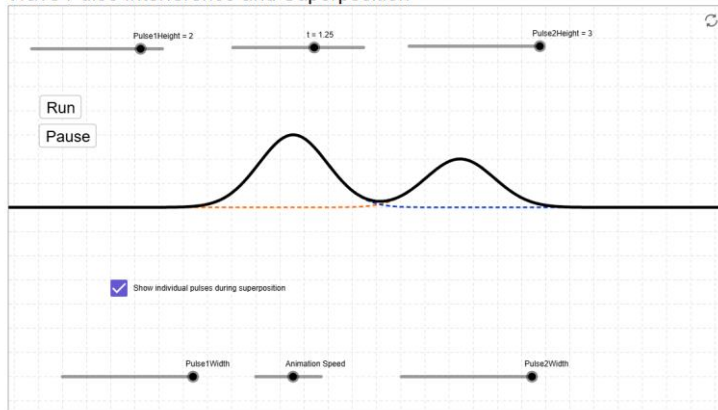
The **principle of superposition** states that the net displacement at a given place and time caused by two or more waves which traverse the same space and meet is the vector sum of the displacement which would have been produced by the individual waves separately at that position and instant of time.

2.1.1 Superposition of Two Pulses with the Same Shape and Amplitude

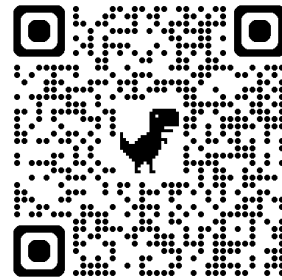
Fig. 2.1 shows the interaction between 2 crests as they approached each other while Fig. 2.2 shows the interaction between a crest and a trough.



Wave Pulse Interference and Superposition



Explore the simulation on wave interference. Use the slider to vary the amplitude of both the pulses and observe the effects.



2.1.2 Superposition of Two Pulses with Different Shapes

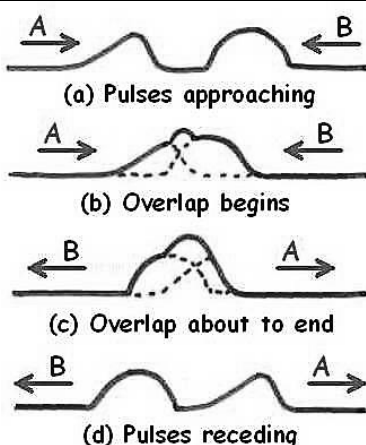


Fig. 2.3

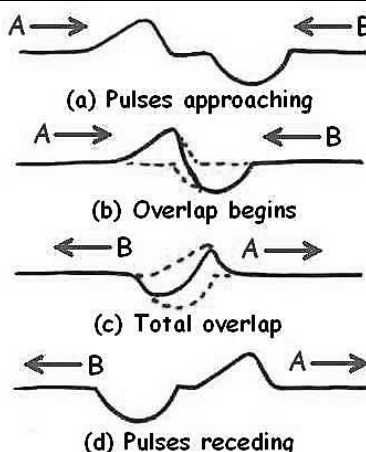
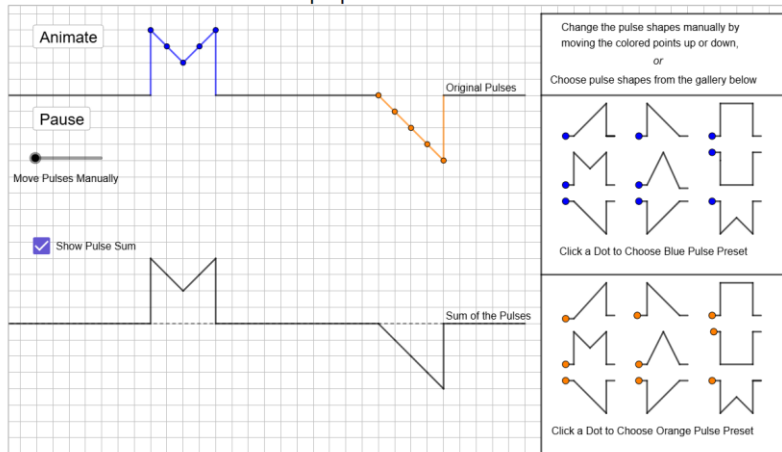


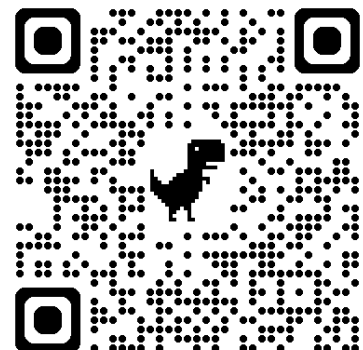
Fig. 2.4

Fig. 2.3 and 2.4 show the superposition of pulses with different shape. The principle of superposition still applies here, where at every position along the rope, the resultant displacement is the vector sum of each of the individual displacements.

Wave Pulse Interference and Superposition 2

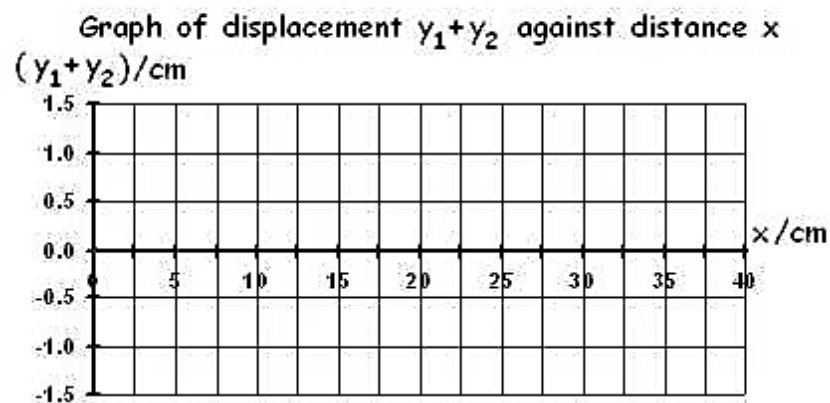
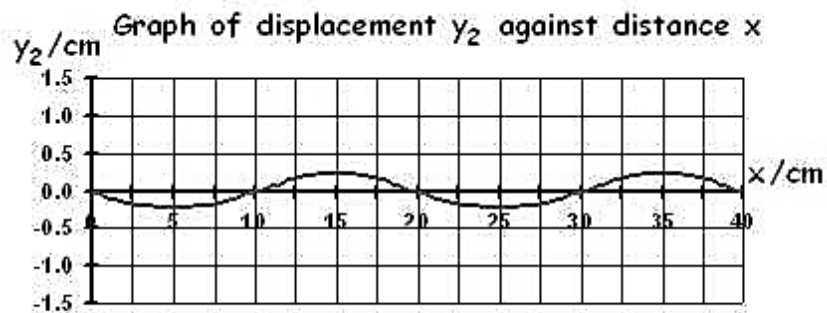
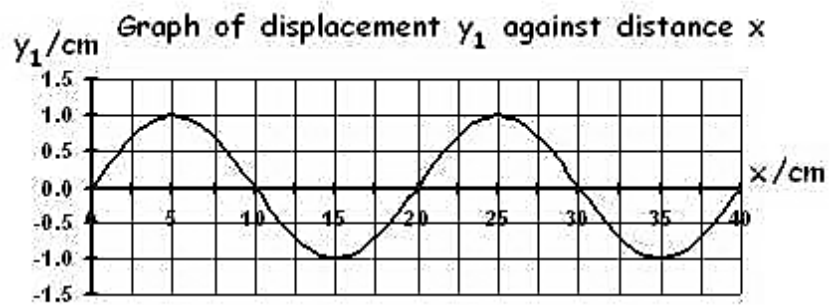


Explore the simulation on wave interference. Use the slider to vary the shape of both the pulses and observe the effects.



Example 2.1

Apply the principle of superposition to sketch the resultant waveform of $y_1 + y_2$.

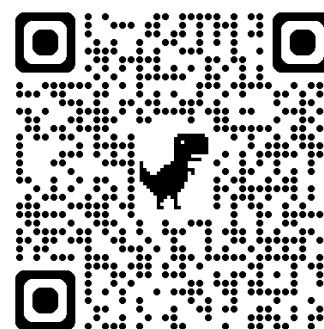
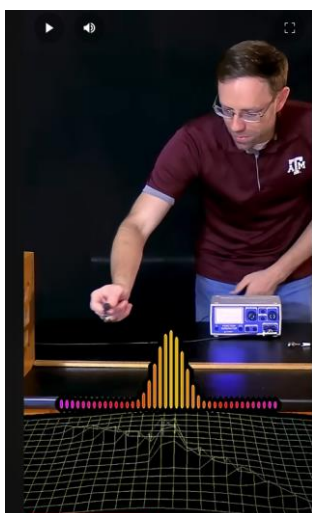


2.2 Interference

If two loudspeakers are arranged so that they are half a metre apart, and they produce the same output frequency, different intensities (loudness) of sound are heard at different positions away from and around the two loudspeakers.

When the two sets of sound waves from the two loudspeakers meet at different positions around the loudspeakers, the resultant amplitude of the combined two waves differs. This results in the different intensities of sound heard. Recall that for waves, intensity is directly proportional to the square of its amplitude.

Interference is the superposing of two or more waves which gives a resultant wave, whose resultant displacement at any point is determined by the Principle of Superposition. This results in the variation of intensity of the resultant wave.



View the video on the interference of sound waves.

2.2.1 Constructive and Destructive Interference

Interference can be either constructive or destructive.

Constructive interference occurs when two waves meet in phase (crest meets crest, trough meets trough). The magnitude of the resultant displacement at a point, which is obtained by adding the individual displacement of the 2 waves, will always be higher than that of the individual waves. This is because both the displacement of the 2 waves is always either positive or negative at any instant in time. Therefore, the resultant amplitude (and therefore intensity) when two waves meet in phase is always greater than that of the individual waves.

Destructive interference occurs when two waves meet π rad out of phase (crest meets trough and trough meets crest). The magnitude of the resultant displacement at a point will always be lower than that of the individual wave which has the higher displacement at that point. This is because the displacement of the 2 waves is always in the opposition region (one of the waves is positive while the other is negative at any instant in time). Therefore, the resultant amplitude (and therefore intensity) when two waves meet π rad out of phase is always lower than that of the individual wave which has the higher amplitude.

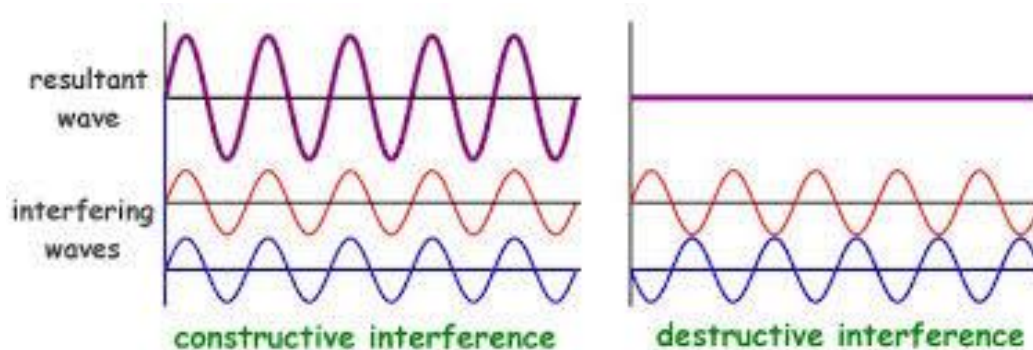


Fig. 2.5 – Visual illustration of constructive and destructive interference

(Source: <https://web2.ph.utexas.edu/>)

3. Two Source Interference

3.1 Two Source Interference of Water Waves

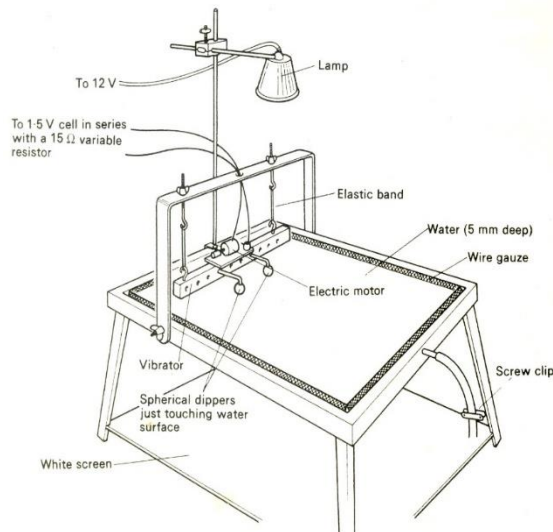


Fig. 3.1 – Ripple Tank Experiment Setup
(Source: T. Duncan – Advanced Physics)

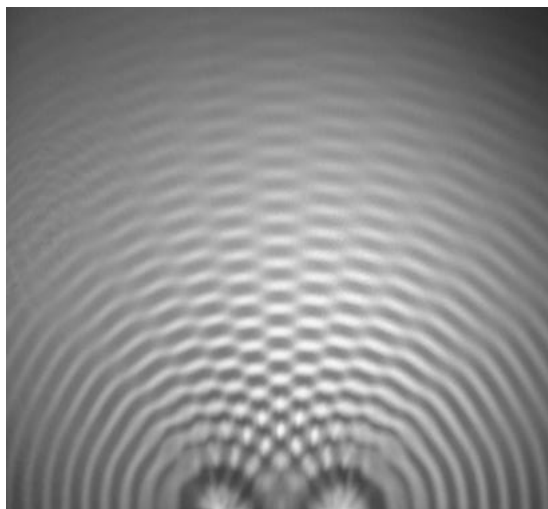
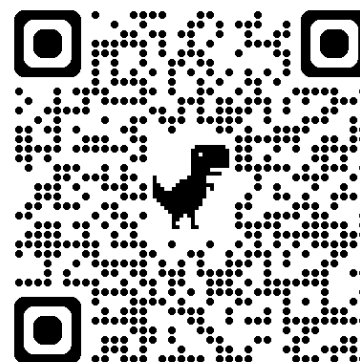


Fig. 3.2 – Shadows of ripples

Setup:

- (a) The ripple tank experiment setup is shown in Fig. 3.1.
- (b) Two prongs are attached to an oscillator.
- (c) The two prongs oscillate up and down and both are in phase.
- (d) Two set of circular ripples are generated.
- (e) These two set of ripples meet each other (interfere) as they propagate out.
- (f) Light is positioned above the water tank to cast a shadow image (Fig. 3.2) of the interference pattern onto the white screen placed below the water tank. Shadows are due to the wave's crests.
- (g) Interference pattern consists of a series of maximum (crest) and minimum (trough) in the water displacements of the combined waves due to constructive and destructive interference.



View the video on the interference of water waves.

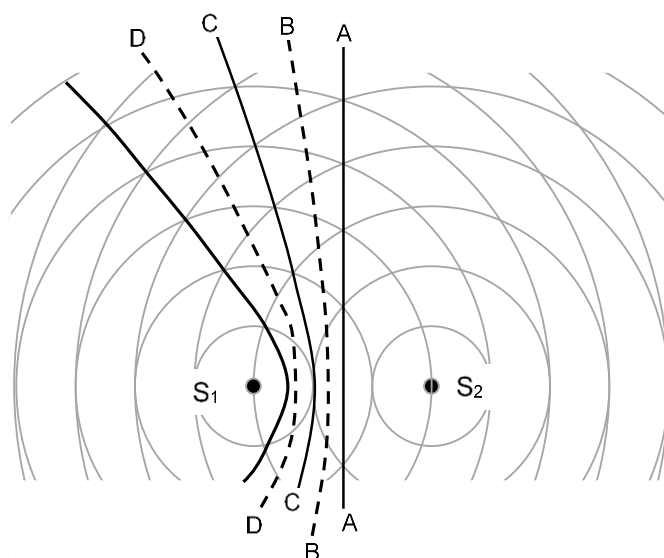


Fig. 3.3 – Schematic Diagram of Interference

Fig. 3.3 shows the wavefronts (for illustration purposes, let's assume they are the crests of the wave) produced by the two prongs S_1 and S_2 . Note that the two prongs are vibrating in phase. Analysing this schematic, there are a number of important features to know.

Line AA

- (a) At points along line AA, crests (or troughs) from S_1 arrive at the same time as crests (or troughs) from S_2 . This means that at points along AA, the two waves meet in phase with each other (phase difference is 0 rad). According to the Principle of Superposition,

Line BB

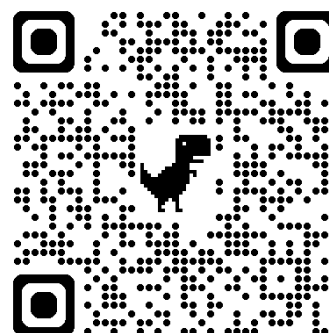
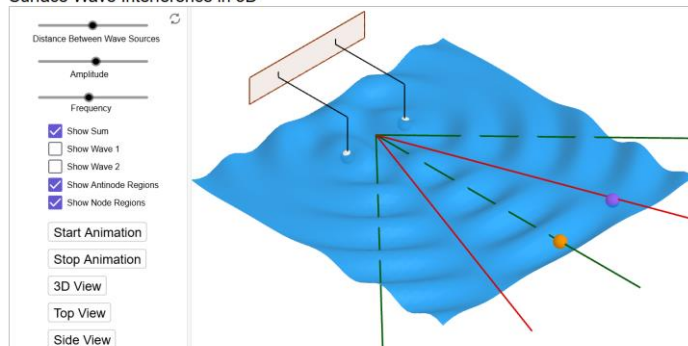
- (a) At points along line BB, crests (or troughs) from S_1 arrive at the same time as troughs (or crests) from S_2 . This means that at points along BB, the two waves meet π rad out of phase with each other (phase difference is π rad). According to the Principle of

they <u>constructively interfere</u>. Line AA is known as an <i>antinodal line</i>. (b) At every point on line AA, the path distance from S_1 and S_2 have a difference of zero wavelength. We say that there is no <u>path difference</u>.	Superposition, they <u>destructively interfere</u>. Line BB is known as a <i>nodal line</i>. (b) At every point on line BB, the distances from S_1 and S_2 has a difference of half a wavelength ($\frac{1}{2} \lambda$). We say that the <u>path difference</u> is $\frac{1}{2} \lambda$.
Line CC Waves from S_1 and S_2 arriving at points along line CC have phase difference of 2π rad. The distances from S_1 and S_2 has a difference of one wavelength (1λ). We say that the path difference is 1λ. Since the waves constructively interfere, similar to line AA, line CC is also an antinodal line.	Line DD Waves from S_1 and S_2 arriving at points along line DD have phase difference of 3π rad. The distances from S_1 and S_2 has a difference of one wavelength ($1\frac{1}{2} \lambda$). We say that the path difference is $1\frac{1}{2} \lambda$. Since the waves destructively interfere, similar to line BB, line DD is also a nodal line.

It can be shown that the separation between nodal (and antinodal) lines increases

- as the distance from S_1 and S_2 increases
- the smaller the separation of S_1 and S_2
- as the wavelength increases (the frequency of the vibrating prong decreases)

Surface Wave Interference in 3D



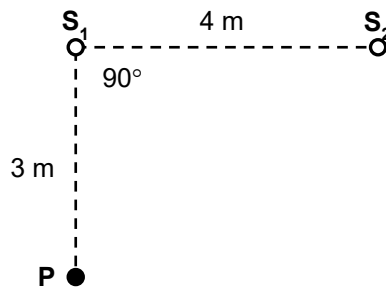
Explore the simulation to observe the interaction of 2 water waves in 3D. Activate the buttons to view nodal and antinodal lines.

3.1.1 Determining phase difference

We can see from the earlier ripple tank experiment that the phase difference when the 2 waves meet will determine if constructive or destructive interference occur. The phase difference when 2 waves meet can be determined by adding the phase difference of the sources of the 2 waves and the phase difference due to the path difference of the 2 waves.

Example 3.1

Water waves of wavelength 2 m are produced by two generators, S_1 and S_2 , placed 4 m apart. Each generator (when operated alone) produces waves of amplitude A at P , which is 3 m from S_1 .



- (a) When the generators are operating in phase, what is the amplitude of oscillation at P ?
- (b) When the generators are operating out of phase by π rad, what is the amplitude of oscillation at P ?

- (a) Distance between P and S_2 . $S_2P = \sqrt{4^2 + 3^2} = 5$ m
Path difference ($S_2P - S_1P$) = $5 - 3 = 2$ m.

Since the 2 sources are in phase, phase difference due to the 2 sources is 0 rad.

The path difference between $S_2P - S_1P$ is 2m, which is exactly one wavelength, the phase difference due to the path difference is 2π rad.

The phase difference when the 2 waves meet will be $0 + 2\pi = 2\pi$ rad.

Therefore the 2 waves meet in phase, constructive interference takes place.

Hence, the resultant amplitude = $A + A = 2A$

- (b) Since the 2 sources are π rad out of phase, phase difference due to the 2 sources is π rad.

The path difference between $S_2P - S_1P$ is 2m, the phase difference due to the path difference is 2π rad

The phase difference when the 2 waves meet will be $\pi + 2\pi = 3\pi$ rad. (Since one wavelength is equivalent to 2π rad, we can remove integer multiple of 2π rad from the phase difference and say that the phase difference when the 2 waves meet is π rad)

Therefore the 2 waves meet π rad out of phase, destructive interference takes place.

Hence, the resultant amplitude = $A - A = 0$

3.2 Two Source Interference of Visible Light (Young's Double-slit Experiment)

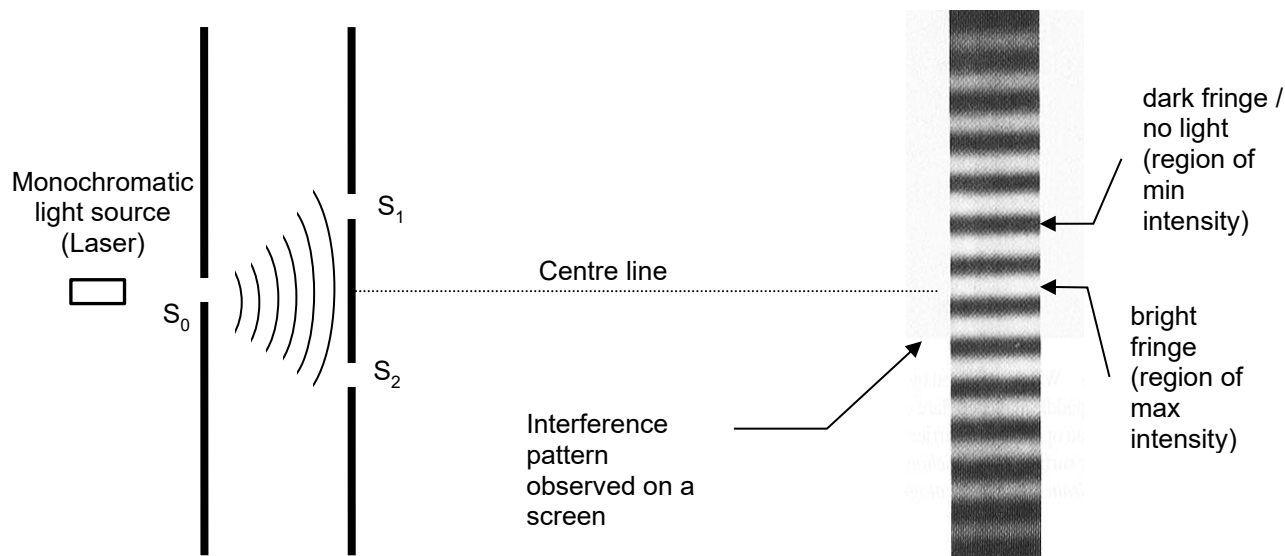


Fig. 3.4 – Young's Double-slit Experiment Setup

Setup:

- (a) S_1 and S_2 are narrow slits which act as two light sources. Each slit width is comparable to the wavelength of the monochromatic light used.
- (b) The light wave arriving at S_1 and S_2 from S_0 ensures that S_1 and S_2 act as coherent sources. If they are equidistant from S_0 , the light will emerge from them in phase.
(Coherence will be addressed in more detail at a later part of the notes)
- (c) When the light wave goes through S_1 and S_2 , diffraction takes place at the slits (sketch in Fig. 3.4).
- (d) These diffracted waves interfere in the region between the slits and the screen.
- (e) An interference pattern is observed on the screen which consists of bright and dark fringes.
- (f) The bright and dark fringes occur in a regular pattern. If the screen is placed a large distance from the slits, the spacing between two adjacent dark fringes (or two adjacent bright fringes) are the same. This distance is known as the fringe separation.

Wave Optics: Slit Experiment Simulation

Parameters

Experiment Type:

Single Slit **Double Slit** Grating

Wavelength (λ): 550 nm

Slit Width (a): 50 μm

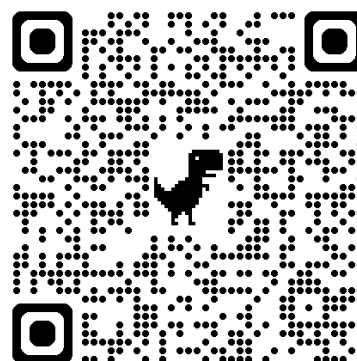
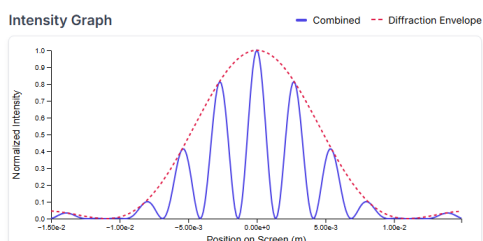
Slit Separation (d): 200 μm

Distance to Screen (L): 1.0 m

Simulation View



Intensity Graph



Explore the simulation. Change the variables to view how it impacts the image on a double slit experiment.

3.2.1 Formation of the bright and dark fringes

Consider the rays coming from S_1 and S_2 and meeting at P, the centre point of the screen.

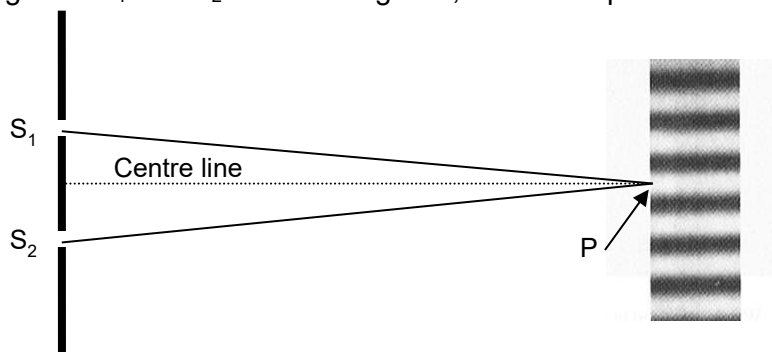


Fig. 3.5 – Formation of constructive interference

1. Drawing a ray from S_1 to P (S_1P) and from S_2 to P (S_2P), we note that the path S_1P and S_2P are the same distance.
2. Thus the path difference between S_1P and S_2P , $S_1P - S_2P = 0$.
3. Since waves come out of S_1 and S_2 in phase, and the path difference is 0, it means that waves from S_1 and S_2 arrive at P in phase with each other.
4. Their amplitude reinforced each other and there is constructive interference.
5. This results in a bright fringe at point P.

Similarly, for rays meeting at Q, a dark fringe just above the centre point on the screen, we can do the same analysis.

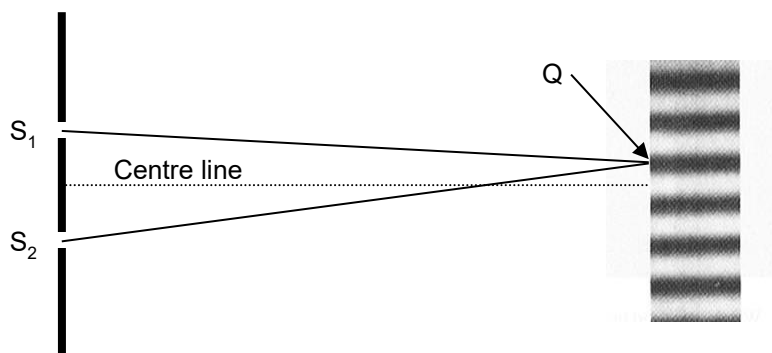
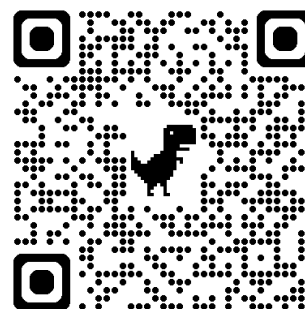
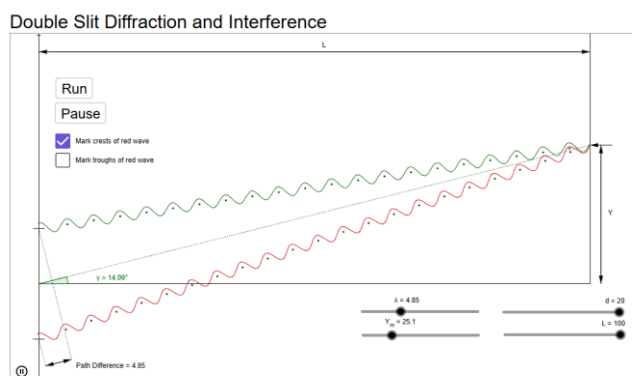


Fig. 3.6 – Formation of destructive interference

1. We can see that S_2Q is longer than S_1Q .
2. Path difference between S_2Q and S_1Q , $S_2Q - S_1Q = \frac{1}{2} \lambda$.
3. Since waves come out of S_1 and S_2 in phase, and path difference is $\frac{1}{2} \lambda$, it means that the waves arrive at Q π rad out of phase with each other.
4. Their amplitude cancelled out each other and there is destructive interference.
5. This results in a dark fringe at point Q.



Explore the simulation. Change the path difference to view how it impacts the image on a double slit experiment.

The above analysis can be extrapolated to other points on the screen in order to explain the whole pattern. The path difference and phase differences for both bright and dark fringes are illustrated in Fig. 3.7.

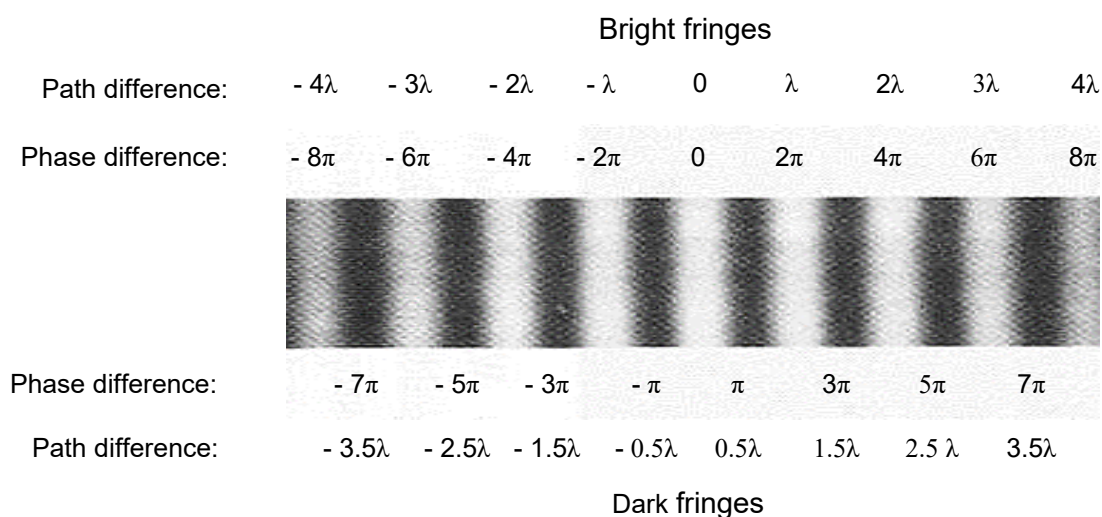


Fig. 3.7

3.2.2 Fringe Separation

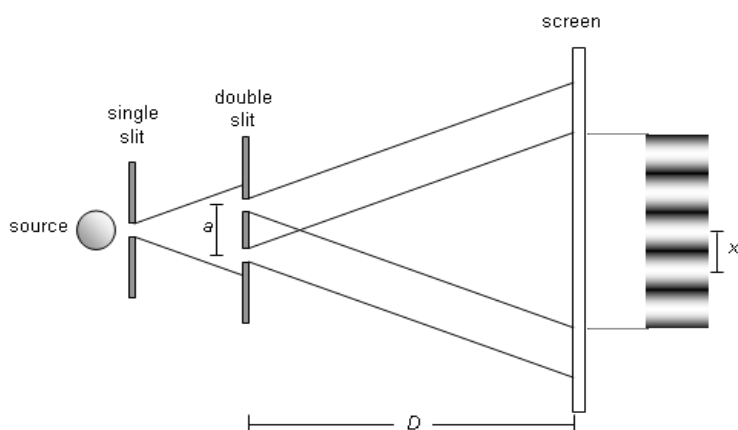


Fig. 3.8 – Variables for use in the Young's Double-slit equation

If the screen is far away from the double slit, the fringes will be quite evenly spaced. The fringe separation between adjacent bright (dark) fringes x is given by the equation

$$\lambda = \frac{ax}{D}$$

where λ is the wavelength of the monochromatic light,
 D is the distance of the fringes from the double slits,
 a is the separation of the two slits (measure from centre to centre of slits)

It is important to note that this formula is only valid for $D \gg a$.
 (Derivation of this formula is given in Annex A. Derivation of the equation is not required.)

Example 3.2 (Physics for Scientists and Engineers by Serway)

A viewing screen is separated from a double slit source by 1.2 m. The distance between the two slits is 0.030 mm. The separation between 2 consecutive bright fringes is 2.2 cm.

Determine the wavelength of the light.

[550 nm]

$$\begin{aligned} \lambda &= \frac{ax}{D} \\ &= \frac{(3 \times 10^{-5})(2.2 \times 10^{-2})}{(1.2)} \\ &= 5.5 \times 10^{-7} \text{ m} \end{aligned}$$

Example 3.3

Fringes of separation x are observed in a plane 1.00 m from a Young's double-slit arrangement illuminated by yellow light of wavelength 600 nm.

Determine the distance from the slit where fringes of the same separation x be observed when using blue light of wavelength 400 nm. [1.50 m]

$$\lambda = \frac{ax}{D}$$

$$ax = 600 \times 10^{-9} (1.00)$$

$$ax = 400 \times 10^{-9} (D)$$

$$D = 1.50 \text{ m}$$

3.3 Two source Interference of Microwaves

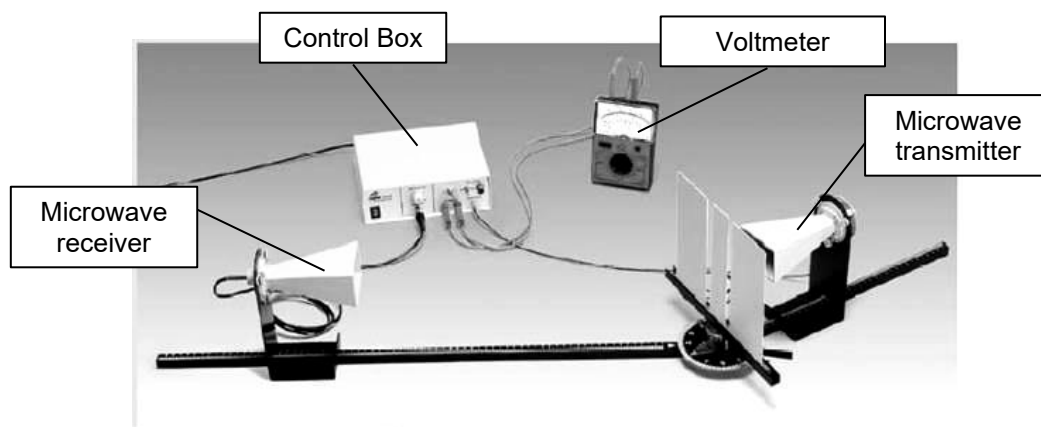


Fig. 3.9 – Setup of two source microwave interference

Setup:

- A transmitter which produces microwaves is positioned behind a double slit (metal). Each slit could be about 3 cm wide.
- Two wave-trains emerge from the two slits which act as two coherent sources.
- The receiver is connected to a control box which has a voltmeter connected to it.
- The receiver is attached to a track that can be swivelled about the slits. At positions when the receiver detects a high intensity of microwave (constructive interference due to the combined waves from the slits), the voltmeter registers a high reading.
- At positions when the receiver detects zero (or very low) intensity of microwave (destructive interference), the voltmeter registers a very low reading. At these positions, if one of the slits is covered with a metal plate, then the voltmeter reading increases since there is no more destructive interference.

3.4 Two Source Interference of Sound Waves

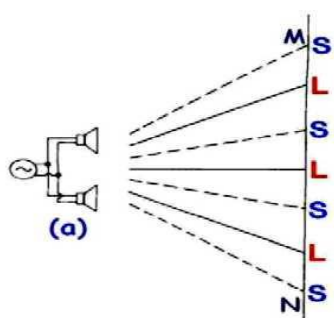


Fig. 3.10(a) – Speakers are in phase

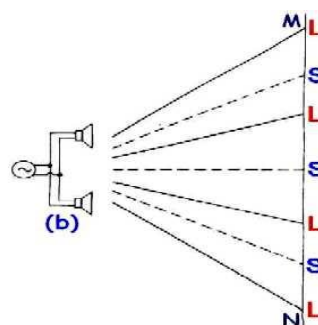


Fig. 3.10(b) – Speakers are out of phase by π

- The interference of sound waves can be demonstrated by connecting two loudspeakers in parallel to an audio-frequency oscillator as shown in Fig 3.10(a) & 3.10(b).
- As the ear or microphone is moved along the line MN, alternate loud (L) and soft (S) sounds can be heard according to whether the receiver of sound is on a line of reinforcement (constructive interference) or cancellation (destructive interference) of the sound waves.
- Fig 3.10(a) shows the case if the two speakers oscillate in phase.
- If the connections to one of the speakers are reversed, so that they oscillate π rad out of phase, then the pattern will be altered as shown in Fig 3.10(b).

3.5 Conditions for Observable Interference Fringes

Observable interference fringes implies that the bright and dark fringes are distinct from each other and they can be observed clearly (good contrast between bright and dark fringes).

- The two light sources must be coherent

Two light sources are coherent when they have constant phase relationship with each other. This implies that the sources have the same frequency.

Suppose you try to set up a demonstration with two separate light sources such as car headlights and shine it on a wall, no observable interference pattern (alternate dark and bright fringes) can be observed. The reason is that the two separate light sources do not produce two trains of light waves that have a constant phase relationship with time.

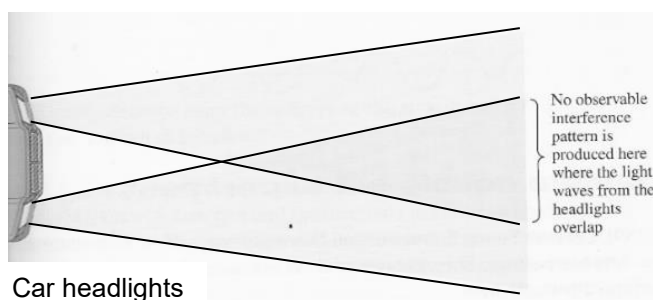


Fig. 3.11

(Source: Chris Mee - AS/A2 Physics)

The two headlights produce two different progressive waves as shown in Fig. 3.11. Light is emitted as a series of pulses and between each pulse there is an abrupt change in the phase of the waves.

Incoherent Waves

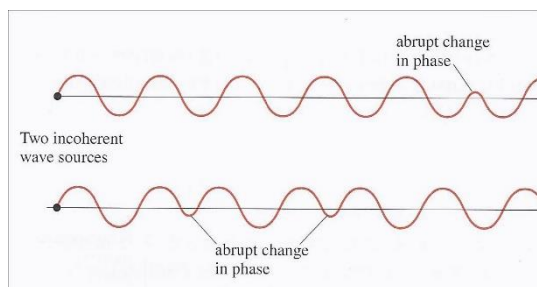


Fig 3.12 (a)

When these two waves meet at a certain point on the wall, they could be in phase one instant but out of phase in the next instant. These variations take place rapidly with continuously changing phase difference such that our eye cannot observe these variations and thus records an average effect. In other words, we would just observe uniform brightness across all points on the wall.

Coherent Waves

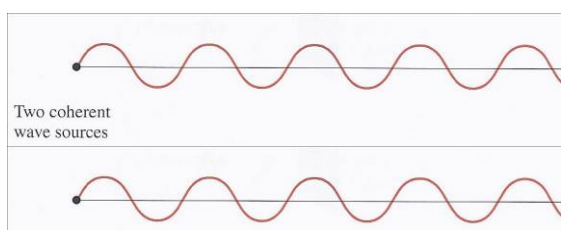


Fig 3.12 (b)

(Source: Chris Mee - AS/A2 Physics)

In section 3.2, we see an interference pattern formed using S_1 and S_2 , which act as two coherent light sources. Bright fringes will stay as bright fringes – positions where the two waves interfere constructively at all times. Dark fringes will stay as dark fringes – positions where the two waves interfere destructively at all times. Our eyes would then be able to see an observable interference pattern, that of bright and dark fringes.

(b) The amplitudes from the two sources must be approximately equal

With reference to Fig. 2.5, when the amplitudes from S_1 and S_2 are equal, the amplitudes of the waves meeting at the screen will approximately be equal in magnitude. When this happens, there is constructive interference to give a bright fringe and a destructive interference to give a dark fringe.

If the amplitudes are not equal, the dark fringe will not be completely dark as there the resultant wave will have a non-zero amplitude (there is some intensity of light). When this happens, the interference pattern will not be distinct.

(c) The two waves must be either unpolarised or polarised in the same plane

To form a distinct interference pattern, the two waves must be travelling on the same plane. This is to ensure that the two waves can cancel each other completely and form a distinct dark fringe. If the 2 waves are not unpolarised or polarised in the same plane, the dark fringe will not be completely dark as the resultant wave will have a non-zero amplitude.

(d) The slit width must be comparable to the wavelength of light

Slit width must be comparable to wavelength of light for appreciable/significant diffraction to occur. Without diffraction, the waves will not overlap and there will not be interference of the waves.

4. Single slit and multiple slits diffraction

4.1 Diffraction Grating

One of the most useful tools in the study of light is the diffraction grating. A diffraction grating is like a double slit arrangement, but consisting of many regularly spaced and narrow slits that are close to each other (on the order of more than several hundred per millimetre). When parallel, monochromatic light waves approach the slits, light is able to diffract at each of these slits as shown in Fig. 4.1.

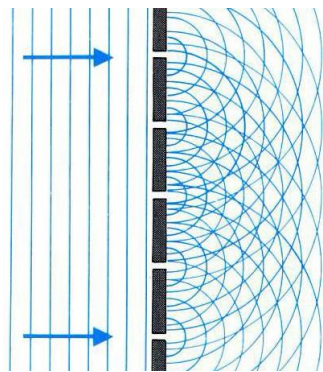


Fig. 4.1

(Source: R. Hutchings – Physics)

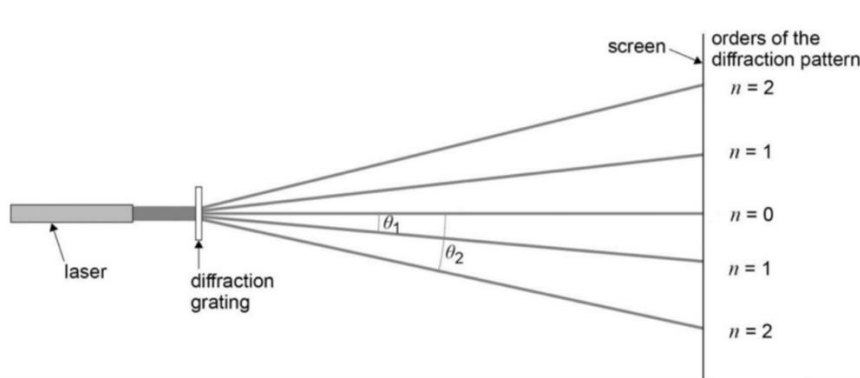


Fig. 4.2

Similar to the double-slit setup, the diffraction grating has a well-defined pattern if we place a screen far away from the diffraction grating. We can apply the same analysis of how bright fringes come about in the double-slit case to this set up. If light coming out from the first two slits have a path difference of 1λ , the light coming from the next two slits will also have a path difference of 1λ . Extrapolating this to all the other slits, it is easy to see that at a certain angle θ from the horizontal, there will be constructive interference between the light rays from all of the slits. And as we increase θ , we will again encounter the next set of light rays that will have a path difference of 2λ between rays emerging from adjacent slits.

From Fig. 4.2, it can be shown that

$$a \sin \theta = n\lambda$$

where λ is the wavelength of the monochromatic light,

θ is the angle between the horizontal axis to the point of constructive interference on the screen,

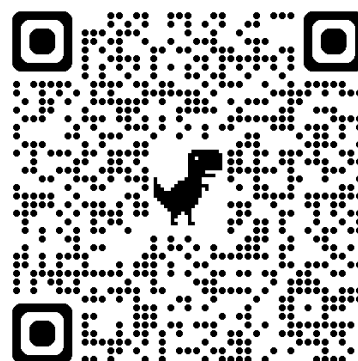
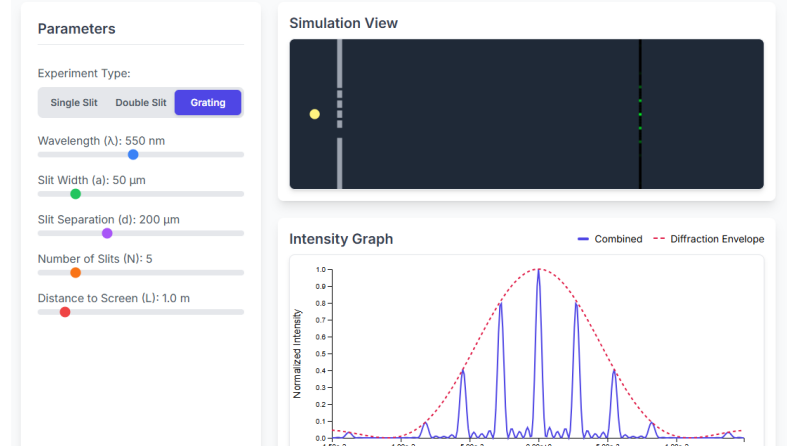
a is the separation of the two slits (measured from centre to centre of slits)

n is the order number of the bright fringes

(Derivation of this formula is given in Annex B. Derivation is not required)

It is again important to note that this formula is only valid when the distance between the grating and the screen is much larger than a .

Wave Optics: Slit Experiment Simulation



Explore the simulation. Change the variables to view how it impacts the image on a diffraction grating experiment.

Example 4.1

A diffraction grating has 400 lines per mm and is illuminated normally by monochromatic light of wavelength 600 nm. Calculate

- the grating spacing, [2.5×10^{-6} m]
- the angle to the normal at which the first order maximum is seen, [13.9°]
- the number of diffraction maxima obtained. [9]

$$\begin{aligned} \text{(a)} \quad a &= \frac{1 \times 10^{-3}}{N} \\ &= \frac{1 \times 10^{-3}}{400} = 2.5 \times 10^{-6} \text{ m} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad a \sin \theta &= n\lambda \\ 2.5 \times 10^{-6} (\sin \theta) &= (1)(600 \times 10^{-9}) \\ \theta &= 13.9^\circ \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad a \sin \theta &= n\lambda \\ \sin \theta &< 1 \\ \frac{n\lambda}{a} &< 1 \\ \frac{n(600 \times 10^{-9})}{2.5 \times 10^{-6}} &< 1 \\ n &< 4.16 \\ n &= 4 \quad \text{round down to the nearest integer} \\ \text{Number of maxima} &= 4 + 4 + 1 = 9 \end{aligned}$$

4.1.1 Interference Pattern of White Light Passing Through a Diffraction Grating

When white light is passed through a grating, the result will be as shown in Fig. 4.3.

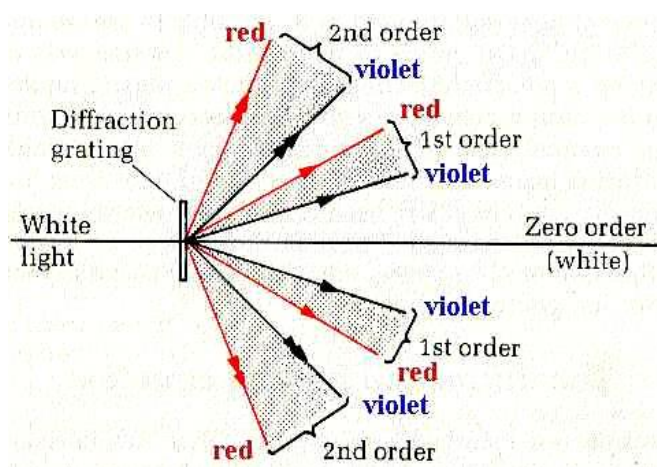


Fig. 4.3 – Effect of white light passing through a grating

For each order of diffraction, the red fringes appear on the screen at a larger angle than the violet fringe due to its larger wavelength, with the other colours falling in between.

The zeroth order fringe is white because all the colours (after passing through the grating) have the same angle of deflection for their respective diffraction pattern, so there is no separation of colours in the zeroth order fringe.

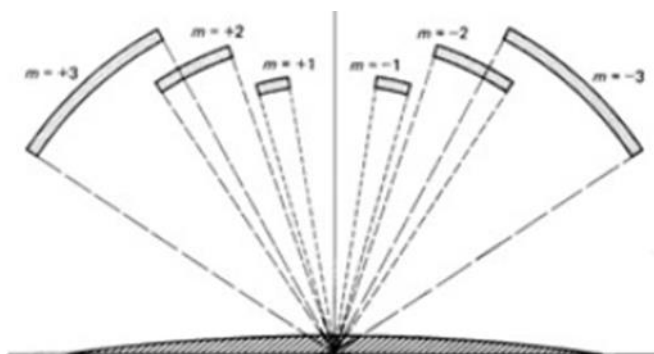
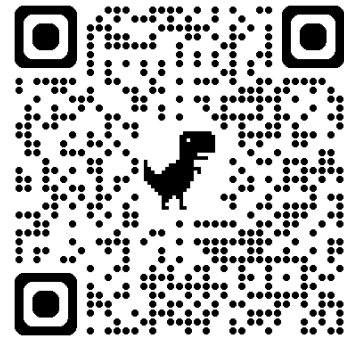
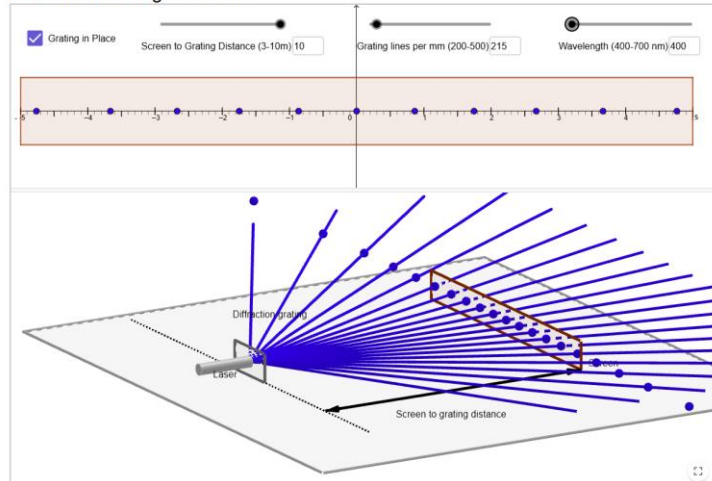


Fig. 4.4 – Overlapping of the different orders of the diffracted light

In some cases, the different orders of the diffracted lights will overlap one another as shown in Fig. 4.4.

Diffraction Grating Laser Lab



Explore the simulation. Change the wavelength and view how it impacts the outcome on a diffraction grating experiment.

4.2 Single Slit Diffraction

Let us consider a situation where light passing through a narrow opening modelled as a slit is projected onto a screen. We assume that the observing screen is far from the slit, so that the rays reaching the screen are approximately parallel. In this model, a diffraction pattern as shown in Fig. 4.5 is observed.

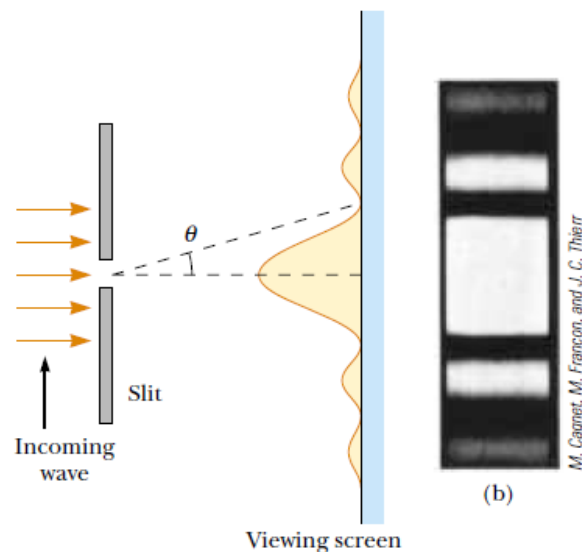


Fig. 4.5 – Single slit diffraction
(Source: *Physics for Scientists and Engineers*)

A bright fringe is observed along the axis at $\theta = 0$, with alternating dark and bright fringes on each side of the central bright fringe. Until now, we have assumed that slits are point sources of light. In this section, we will see how the finite width of slits is the basis for this diffraction.

According to Christiaan Huygens, each small portion of the slit acts as a point source of light waves. Hence, light from one portion of the slit can interfere with light from another portion, and the resultant light intensity on a viewing screen depends on the direction θ (from Fig. 4.5). Based on this analysis, we recognize that the diffraction pattern is actually an interference pattern, in which the different sources of light are different portions of the single slit.

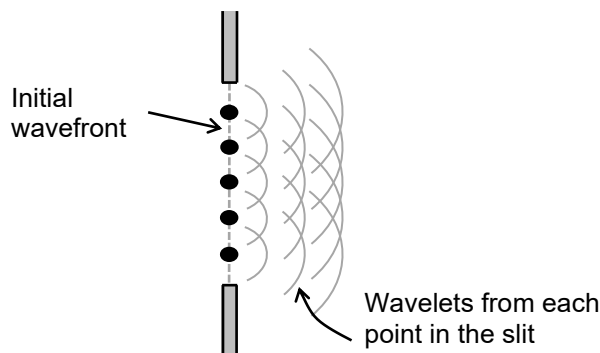


Fig. 4.6 – Greatly magnified single slit

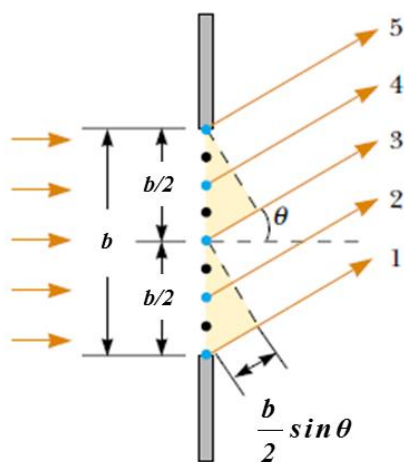


Fig. 4.7

(Source: *Physics for Scientists and Engineers*)

To understand the diffraction pattern, we divide the slit into two halves, as shown in Fig. 4.7. Note that all the waves are in phase as they leave the slit.

Consider rays 1 and 3. As these two rays travel toward a viewing screen far to the right of the figure, the two rays will interfere with each other when they hit the screen. Ray 1 travels farther than ray 3 by an amount equal to the path difference $(b/2) \sin \theta$, where b is the width of the slit. Similarly, the path difference between rays 2 and 4 is also $(b/2) \sin \theta$, as is that between rays 3 and 5. If this path difference is exactly half a wavelength, then the two waves cancel each other and destructive interference results. If this is true for two such rays, then it is true for any two rays that originate at points separated by half the slit width because the phase difference between two such points is 180° . Therefore, waves from the upper half of the slit interfere destructively with waves from the lower half when

$$\frac{b}{2} \sin \theta = \pm \frac{\lambda}{2}$$

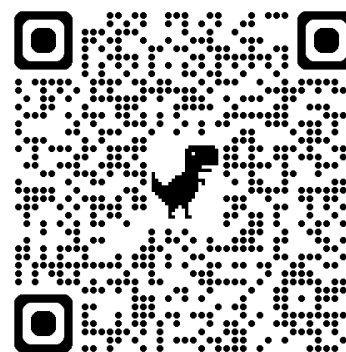
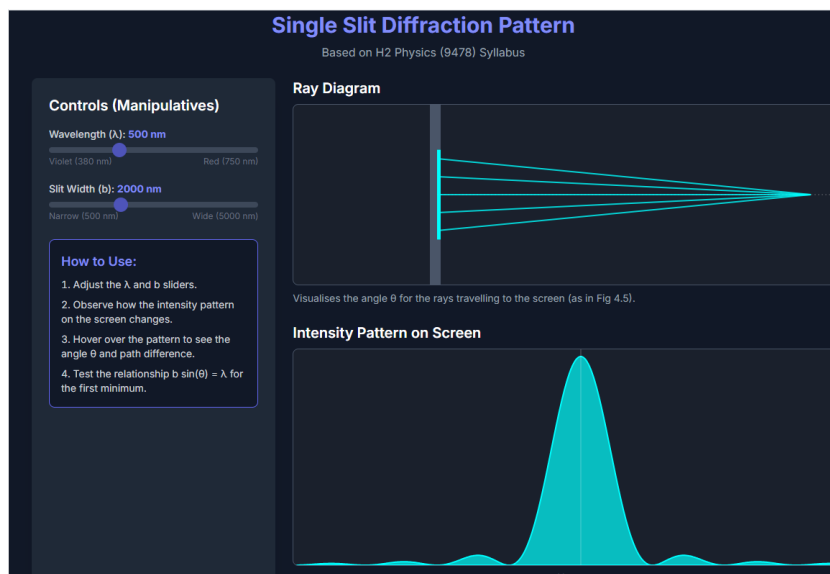
Hence the location of the first minima in the single slit diffraction pattern is given by

$$\sin \theta = \pm \frac{\lambda}{b}$$

We can further our analysis to find the other minima by dividing the slit into more divisions. The general condition for destructive interference of a single slit is

$$\sin \theta = \pm \frac{m\lambda}{b} \quad m = \pm 1, \pm 2, \dots$$

The resulting intensity-position graph on the screen will have a very bright (high intensity) central fringe. The higher order bright fringes will be dimmer (less intensity) than the central fringe.



Explore the simulation on the path difference on a single slit experiment. Use the slider to vary the wavelength and slit width. Move your cursor across the intensity graph and observe the path difference effects.

If we consider the pattern of bright and dark spots on a screen far away from the slit ($D \gg b$), we will have the intensity-distance graph as shown below.

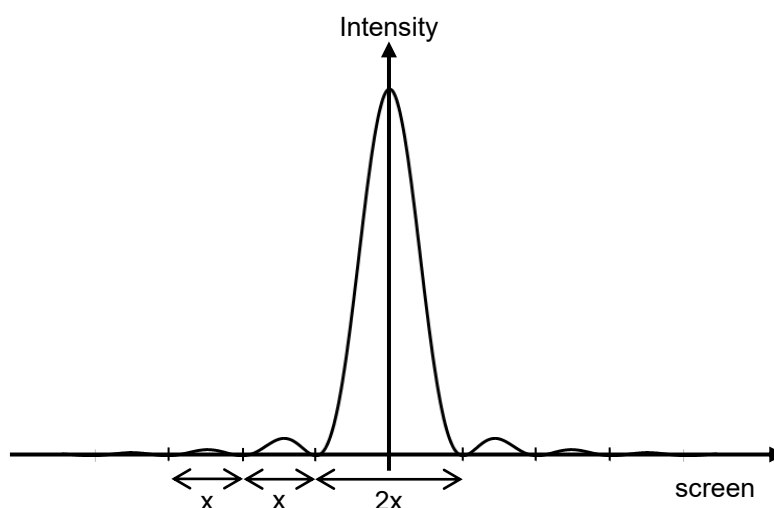


Fig. 4.8 – Intensity-distance graph for the diffraction pattern from a single slit

Three important observations can be obtained from the above graph.

1. The distance between each minimum is constant (Δx), if the distance to screen is much larger than the slit width ($D \gg b$).
2. The width of the central maximum is twice that of the secondary (smaller) peaks.
3. The central maximum is much brighter than the other peaks.

Wave Optics: Slit Experiment Simulation

Parameters

Experiment Type:

Single Slit

Double Slit

Grating

Wavelength (λ): 550 nm

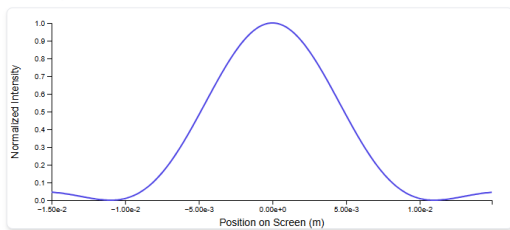
Slit Width (a): 50 μm

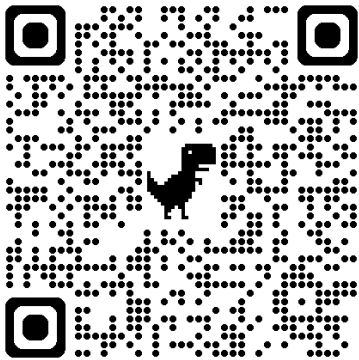
Distance to Screen (L): 1.0 m

Simulation View



Intensity Graph





▶

Explore the simulation. Change the variables to view how it impacts the image on a single slit experiment.

Example 4.2 (Physics for Scientists and Engineers, Serway)

A beam of green light is diffracted by a slit of width 0.550 mm. The diffraction pattern forms on a wall 2.06 m beyond the slit. The distance between the positions of zero intensity on both sides of the central bright fringe is 4.10 mm.

Calculate the wavelength of the laser light.

$$b = 0.550 \text{ mm}, \quad \tan \theta = \frac{\frac{1}{2}(4.10 \times 10^{-3})}{2.06}, \quad \text{find } \lambda$$

$$\sin \theta = \pm \frac{\lambda}{b}, \quad \text{for small angle } \theta, \quad \sin \theta \approx \tan \theta \approx \theta$$

$$\lambda \approx b\theta$$

$$= 0.55 \times 10^{-3} \left(\frac{2.05 \times 10^{-3}}{2.06} \right)$$

$$= 5.47 \times 10^{-7} \text{ m}$$

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4.2.1 Resolving Power of a Single Slit or Aperture (Rayleigh Criterion)

The ability of optical systems to distinguish between closely spaced objects is limited because of the wave nature of light. To understand this difficulty, consider the diagram below, which shows two light sources far from a narrow slit of width b . The sources can be two non-coherent point sources S_1 and S_2 (e.g. two distant stars). When the light from each source pass through the slit, each source is imaged as a bright central region flanked by weaker bright and dark fringe, which is a diffraction pattern. The final image observed will be the resultant of the two diffraction patterns: one from S_1 , and the other from S_2 . If the two sources are far enough apart to keep their central maxima from overlapping as in Fig. 4.9 (a), their images can be distinguished and are said to be resolved. If the sources are close together, however, as in Fig. 4.9 (b), the two central maxima overlap, and the images are not resolved.

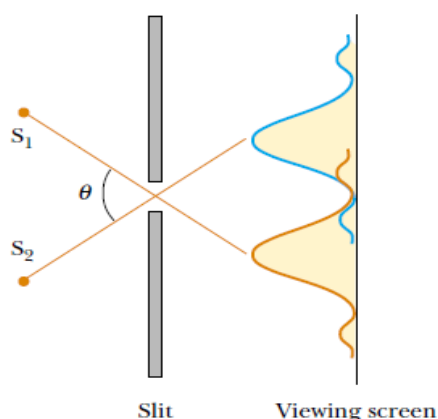


Fig. 4.9 (a) – Intensity-position of resolvable point sources

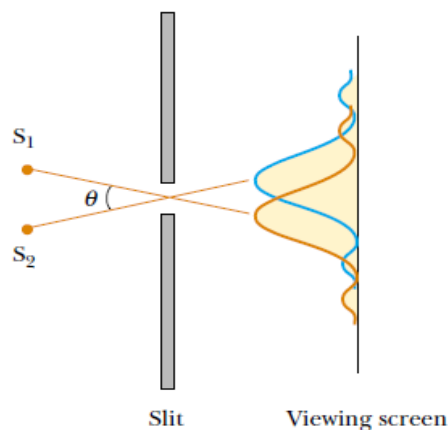


Fig. 4.9 (b) – Intensity-position of unresolvable point sources



The **Rayleigh criterion** states that the two images are just resolved when the central maximum of one diffraction pattern falls on the first minimum of the diffraction pattern.

From Rayleigh's criterion, we can determine the minimum angular separation $\theta_{\min}$ subtended by the sources at the slit for which the images are just resolved. The equation below indicates that the first minimum in a single-slit diffraction pattern occurs at the angle for which

$$\sin \theta = \frac{\lambda}{b}$$

where b is the width of the slit.

According to the Rayleigh criterion, this expression gives the smallest angular separation for which the two images are resolved. Because $\lambda \ll b$ in most situations, $\sin \theta$ is small, and we can use the approximation $\sin \theta \approx \theta$.

Therefore, the limiting angle of resolution θ_R for a slit of width b is

$$\theta_R \approx \frac{\lambda}{b}$$

Individual diffraction patterns of two point sources (solid curves) and the resultant patterns (dashed curves) for various angular separations of the sources are as shown in Fig. 4.10.

For (a), the sources are far apart, and the patterns are well resolved. For (b), the sources are closer together such that the angular separation just satisfies Rayleigh's criterion, and the patterns are just resolvable. For (c), the sources are so close together that the patterns are not resolved. The image looks as if there is only one source.

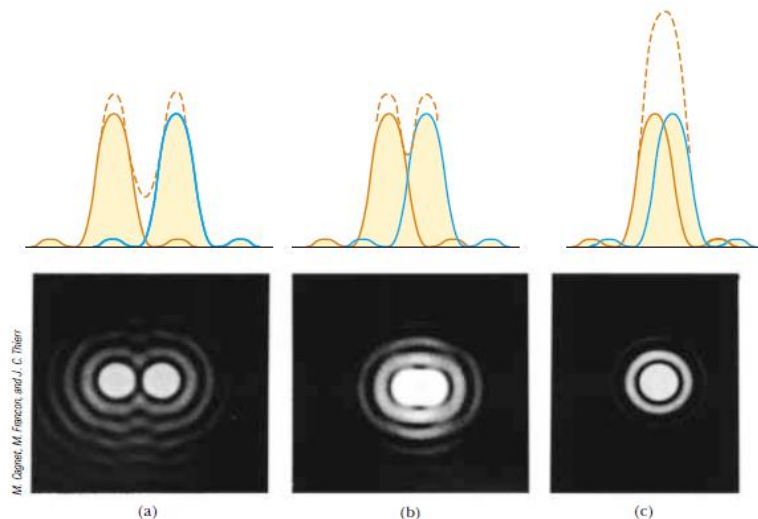


Fig. 4.10 – Diffraction pattern of circular aperture

Further (complex) mathematical analysis of diffraction from the circular aperture of diameter b shows that the limiting angle of the circular aperture obeys the equation $\sin \theta_R = 1.22 \frac{\lambda}{b}$ (for info only).

Similar to the one-dimensional slit, we use the small angle approximation $\sin \theta \approx \theta$ to obtain, $\theta_R \approx 1.22 \frac{\lambda}{b}$

Rayleigh Criterion Simulation

A simulation for H2 Physics, demonstrating the resolution of two point sources.

How it Works

Light from a point source diffracts when passing through a circular aperture (e.g., a telescope lens), creating a pattern called an Airy disc.

The Rayleigh Criterion states two sources are "just resolved" when the central maximum of one's pattern falls on the first minimum of the other.

Min. Resolvable Angle: $\theta = 1.22 \cdot (\lambda / D)$

Wavelength (λ)
400 nm ————— 700 nm

Shorter λ (blue) = smaller Airy disc = **easier** to resolve.

Aperture Diameter (D)
Small ————— Large

Larger D = smaller Airy disc = **easier** to resolve.

Source Separation
Close ————— Far

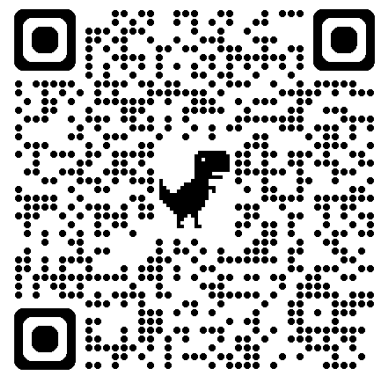
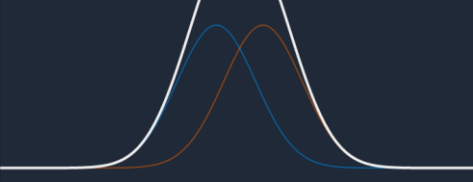
The physical distance between the two sources.

Not Resolved

Visual Simulation (Top-Down View)



Intensity vs. Position Graph



Explore the simulation. Change the wavelength, aperture width and view whether Rayleigh Criterion is resolved.

Example 4.3 (Physics for Scientists and Engineers, Serway)

A circular radar antenna on a coast guard ship has a diameter of 2.10 m and radiates at a frequency of 15.0 GHz. Two small boats are located 9.00 km away from the ship.

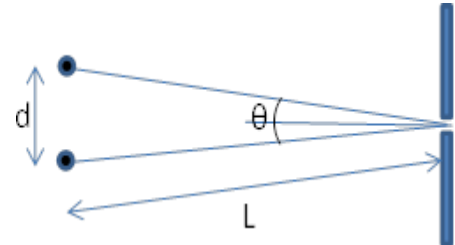
Determine the minimum distance between the boats such that they can still be detected as two separate objects.

$$\sin \theta = \pm \frac{\lambda}{b}, \quad c = f\lambda, \quad \text{for small angle } \theta, \quad \sin \theta \approx \tan \theta \approx \theta$$

$$\lambda = \frac{c}{f} = \frac{3.0 \times 10^8}{15.0 \times 10^9} = 0.02 \text{ m}$$

$$\theta_R = \frac{\lambda}{b} = \frac{0.02}{2.10} = 0.00952 \text{ rad}$$

$$\text{min distance, } d \approx r\theta = L \times \theta_R = 9000(0.00952) = 85.7 \text{ m}$$



What is a stationary wave and how is it produced?

5. Stationary Waves (or Standing Wave)

If you shake one end of a stretched string, waves will travel down it to the fixed end (incident waves) and are reflected. The incident and reflected waves interfere. In most cases, the combined waveform has a changing, jumbled appearance. But if the string is shaken at the right frequency, a steady waveform appears. The wave is stationary because the positions of the crests and troughs of the wave are not moving (along the string).

5.1 Formation of Stationary Waves



A **stationary wave** is formed when two progressive waves (waves A & B) of equal amplitude, equal frequency and same speed, meet when they travel in opposite directions.

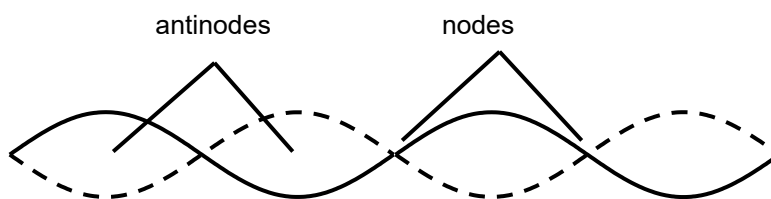


Fig. 5.1

Along the straight line where the two waves meet, there are permanent points of constructive and destructive interference, known as antinodes and nodes respectively.

The diagrams below show the superposition of two progressive transverse waves to form a stationary wave.

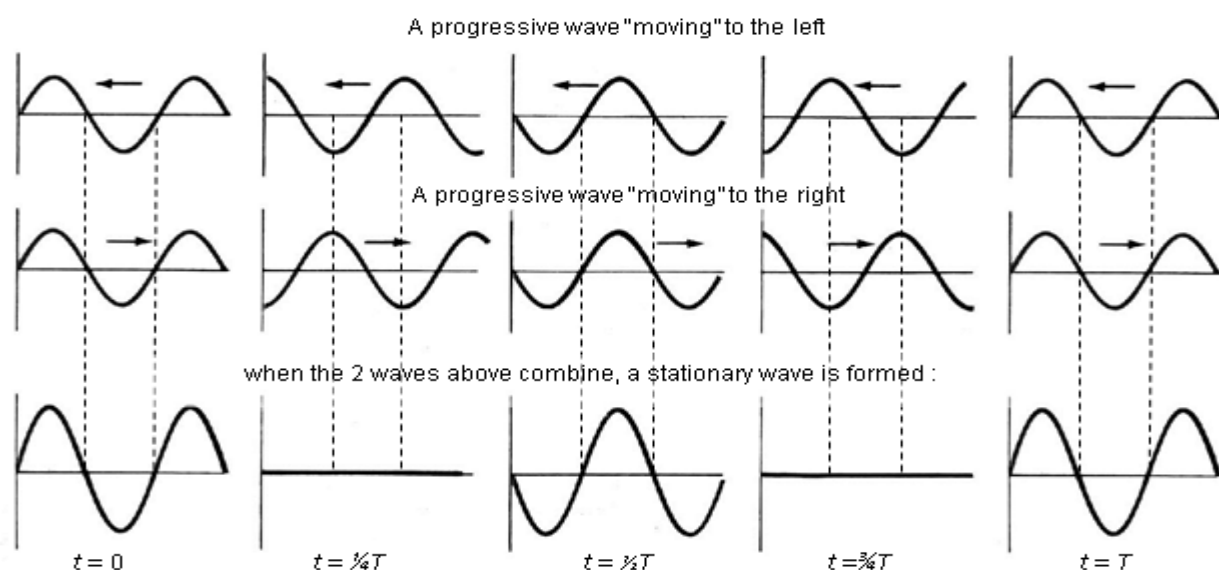


Fig. 5.2 – Diagrammatic study of the formation of stationary waves

Bear in mind that since particles in a progressive wave oscillate about their equilibrium positions, the particles making up the stationary wave also oscillate about fixed positions (except at nodes). For example, the point marked P in the stationary wave simply moves repeatedly up and down as time goes by SHM.

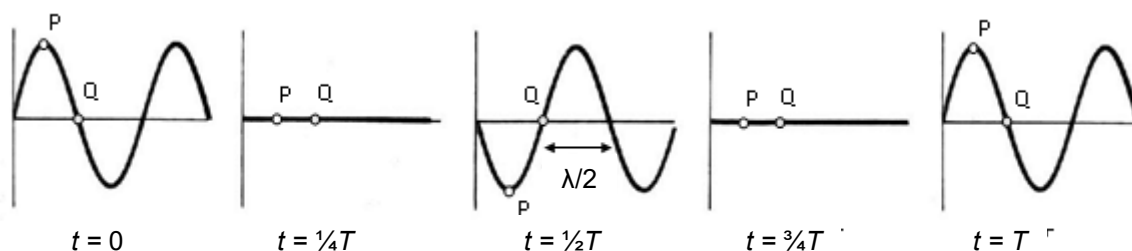


Fig. 5.3 – Motion of particles at the antinode (P) and the node (Q)

5.2 Properties of Stationary Waves

- There are positions in the stationary wave called nodes, where the displacement is always zero. Nodes are positions where the two opposite progressive waves undergo destructive interference.
- Between any two adjacent nodes, all positions on the string oscillate up and down in phase, and have different amplitudes. So all these positions have their equilibrium positions and maximum displacement simultaneously.
- Positions with maximum amplitudes are called antinodes.
- Any two points on the string on both sides of a node has a phase difference of π rad.
- The wave profile does not move along the string (the crest and trough are not moving along the string), hence the terms stationary wave or standing wave.
- There is no energy propagation unlike progressive waves.
- The distance between two successive nodes or antinodes is equal to $\frac{1}{2}\lambda$. Hence two loops make one wavelength of the stationary wave.

Comparison between Stationary Waves and Progressive Waves

	Stationary	Progressive
Amplitude	Amplitude of the points varies from zero at a node to a maximum at the adjacent antinode. All points between a node and an adjacent antinode have different amplitudes.	All points have the same amplitude.
Frequency	All points oscillate at the same frequency except points at the nodes which do not oscillate.	All points oscillate at the same frequency.
Wavelength	Wavelength of the wave is twice the distance between successive nodes or successive antinodes.	Wavelength is the distance between 2 adjacent points that are in phase.
Phase	All points within 2 adjacent nodes oscillate in phase. These points oscillate at π rad out of phase with points within the adjacent pair of nodes.	All points within 1 wavelength of each other do not oscillate at the same phase at the same instant of time.
Waveform	The waveform (wave profile) does not move.	The waveform moves in the direction of wave propagation.
Energy	Energy of the wave does not propagate. Energy is continually changing from KE to PE and then back to KE for all particles except for those at the nodes.	Energy is continuously being transferred from one point of the medium to another. Energy is continually changing from KE to PE and then back to KE for all particles.

5.3 Stationary Waves on a Stretched String

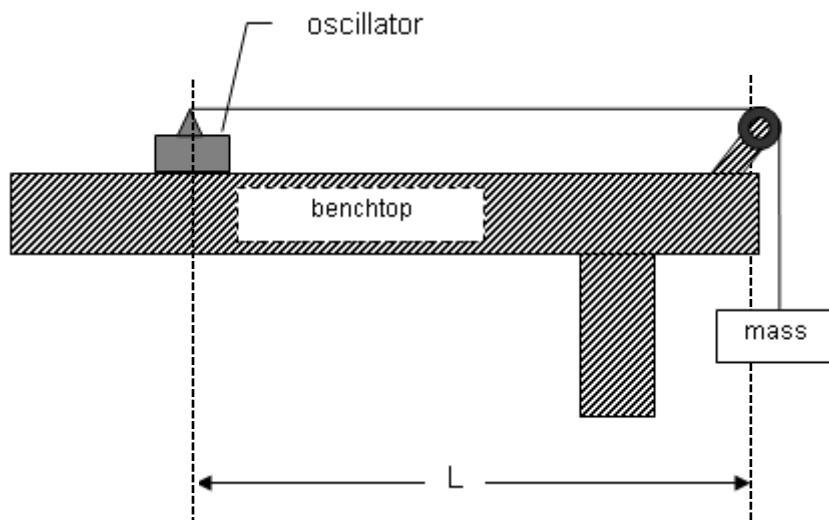


Fig. 5.4

Setup:

- A piece of light elastic string (of a certain mass per unit length) is attached to an oscillator.
- The other end of the string goes over a pulley to a mass (to provide tension to string) as shown in Fig. 5.4.
- The oscillator is switched on and it vibrates one end of the string in the vertical plane. In other words, the oscillator provides a driving frequency to the string. This causes a wave on the string to travel to the pulley end.
- This wave is reflected off the pulley end and superpose with the wave from the oscillator.
- The frequency of the oscillator is slowly increased.
- At a certain frequency (lowest frequency/fundamental frequency), a stationary wave with one loop is formed as shown:

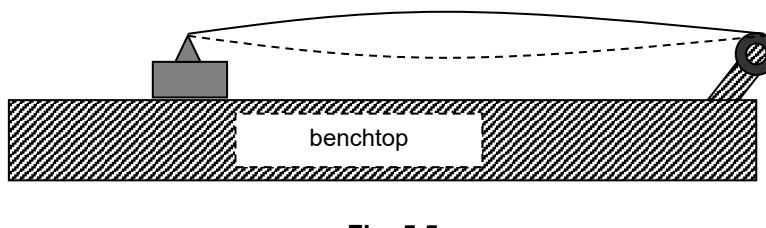
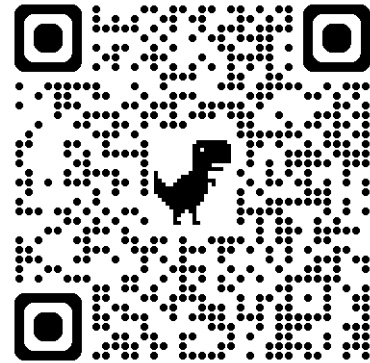
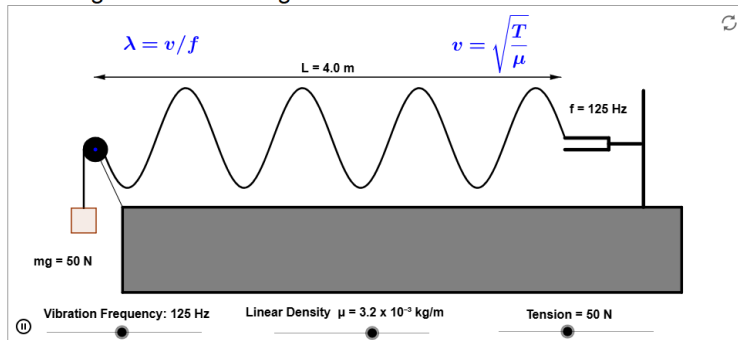


Fig. 5.5

- At certain frequencies, the superposition of the two travelling waves produces a stationary wave pattern.
- Such a stationary wave is said to be produced at resonance; the string is said to resonate (vibrate up and down at maximum amplitude) at these certain frequencies, called resonant frequencies.
- At frequencies other than the resonant frequencies, a stationary wave is not setup.

Standing Waves on Strings



Explore the simulation. Vary the frequency to see the changes.

5.3.1 Single loop diagram of a stationary wave

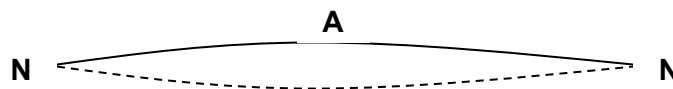
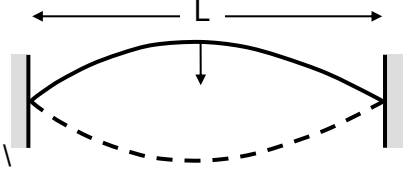
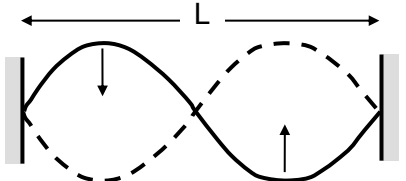
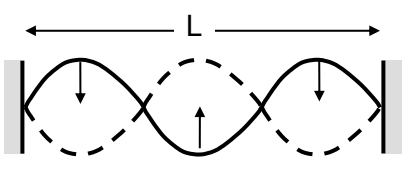


Fig. 5.6 – Diagrammatic representation of a single-loop stationary wave

- Fig. 5.6 shows the string at both its extreme displacements (one solid and one dashed, together forming a single loop). These two lines do not occur simultaneously, only one at a time.
- Point **A** represents a section on the string with the greatest amplitude of displacement. It is called the **Antinode**.
- Point **N** represents point on the string with zero displacement. It is called the **Node**.
- For this stationary wave pattern to be set up, we say that the wavelength of the progressive wave must be $2L$.
- The resonant frequency that corresponds to this wavelength would follow from $\frac{v}{\text{wavelength}}$ where v is the speed of the travelling wave.
- This is the lowest resonant frequency for the stationary wave pattern to exist (single loop).
- At higher frequency, there will be more loops formed.
- We can describe the stretched string as having a set of natural frequencies. When the frequency of the oscillator matches any of the string's natural frequencies, there is a large amplitude stationary wave vibration, resonance is said to occur.
- The speed v , of the travelling wave depends on the tension in the string and the mass per unit length of the string. Equation given by $v = \sqrt{\frac{T}{\mu}}$ where μ = mass per unit length of string and T is the tension in the string.

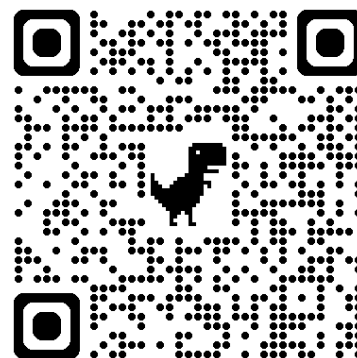
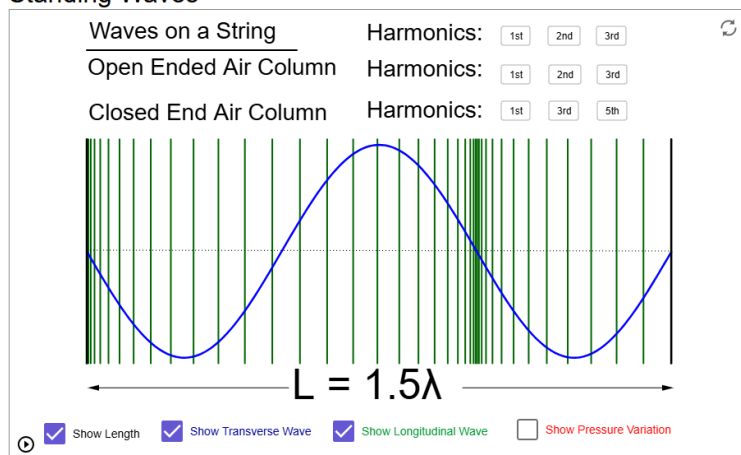
5.3.2 Stationary Wave Patterns at Different Resonant Frequencies

Recall: For a stationary wave, the wavelength is twice the distance between two successive nodes or antinodes.

Stationary Wave Pattern	Resonant Frequency	Harmonic
	wavelength = $2L$ velocity of wave, $v = f(2L)$ frequency of wave, $f_1 = \frac{v}{2L}$	1 st harmonic (fundamental)
	wavelength = L velocity of wave, $v = f(L)$ frequency of wave, $f_2 = 2\left(\frac{v}{2L}\right) = 2f_1$	2nd harmonic (first overtone)
	wavelength = $\frac{2}{3}L$ velocity of wave = $f\left(\frac{2}{3}L\right)$ frequency of wave, $f_3 = 3\left(\frac{v}{2L}\right) = 3f_1$	3rd harmonic (second overtone)

Note: The fundamental frequency of a stretched string, as well as the other harmonics, depends on the length of the string.

Standing Waves



Explore the simulation. View the simulation outcome with different scenarios and harmonics.

Example 5.1

Progressive waves of frequency 300 Hz are superimposed to produce a system of stationary waves in which adjacent nodes are 1.5 m apart.

What is the speed of the progressive waves?

Recall distance between 2 adjacent nodes = $\frac{1}{2} \lambda$

$\frac{1}{2} \lambda = 1.5 \text{ m}$, hence $\lambda = 3.0 \text{ m}$.

speed of waves $v = f \lambda = (300)(3.0) = 900 \text{ m s}^{-1}$.

Example 5.2

A taut wire is clamped at two points 1.0 m apart. It is plucked near one end. Given that the fundamental frequency is obtained with the two points at 1.0 m apart, calculate the wavelength for the fundamental frequency and also for the next two resonant frequencies.

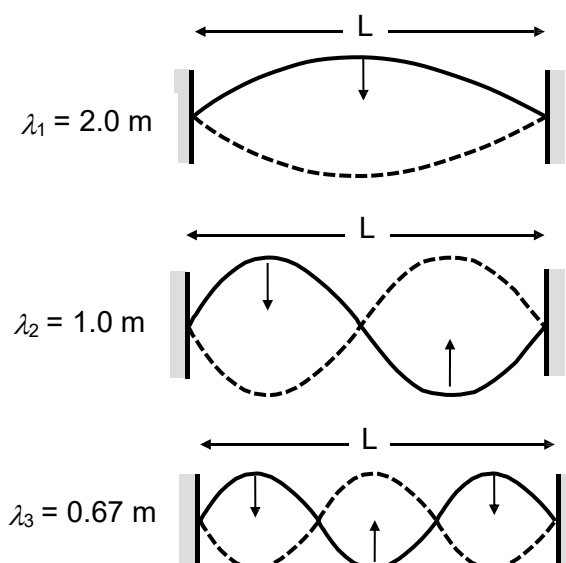
Sketch diagrams clearly, showing their waveforms clearly.

Stationary wave formed on a stretched string fixed at both ends.

Wavelength for fundamental frequency
= 2 x Distance between successive nodes
= $2L$

Wavelength for next two resonant frequencies = $L, \frac{2L}{3}$

Solving, $\lambda_1 = 2 \times 1.0 \text{ m} = 2.0 \text{ m}$, $\lambda_2 = \frac{2 \times 1.0}{3} = 0.67 \text{ m}$



5.4 Stationary Wave in an Air Column

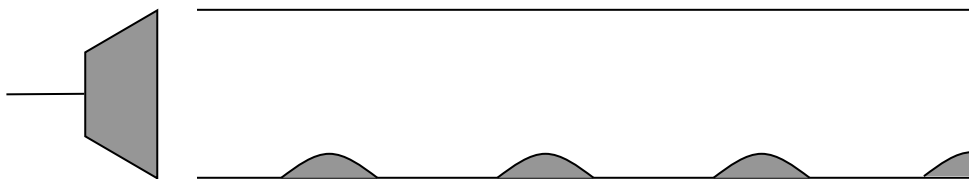


Fig. 5.7 - Kundt's Tube – experiment to illustrate stationary wave in air column

Setup:

- Powder (or Styrofoam beads) is placed evenly along the bottom length of a clear plastic tube. The tube has one open end and one closed end.
- A speaker is placed at the open end.
- The speaker generates a travelling sound wave to the right. This sound wave is reflected off the closed end and then travels to the left.
- The two travelling waves superpose to setup a stationary wave when the speaker is set at a certain frequency.
- After a while, heaps of powder gather at certain locations.
- (Think of the tube containing columns of air particles) The columns of air particles in the tube undergo oscillation in the direction of travel of the sound wave.
- The columns of air particles between the heaps of powder undergo the largest displacement (displacement antinodes) by oscillating to and fro about their equilibrium position. These air particles push the powder to its left and right as it oscillates.
- The column of air particles that is aligned with each heap does not undergo displacement (displacement nodes). In other words, these column air particles do not oscillate.

Fig. 5.8 illustrates the motion of air particles in a stationary wave at a particular point of time. The direction of the arrow indicates the movement of the air particle. The length of the arrow indicates the displacement of the air particle at that location.

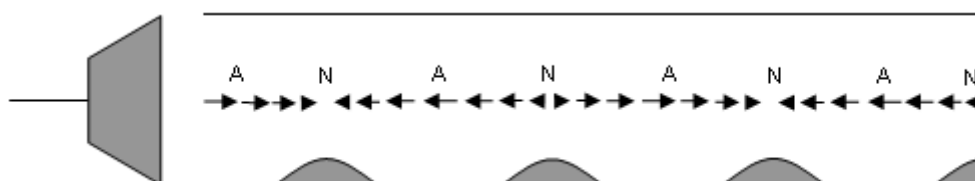
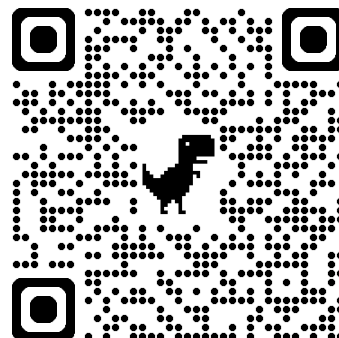
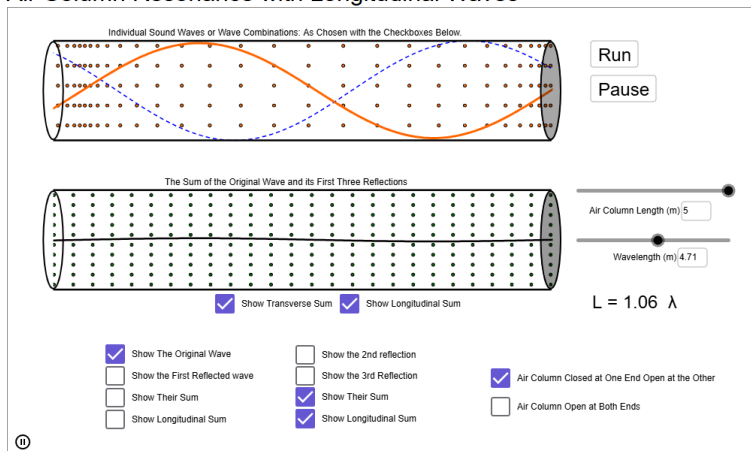


Fig. 5.8

To easily represent the stationary wave of the longitudinal wave (air column), we will represent it with the stationary wave pattern of a transverse wave.

To help you visualise the motion of the air particles in Fig. 5.8, use the following diagram as a visualisation aid. Notice that the air particles at the nodes do not move and those at the antinodes are displaced the furthest from its equilibrium position.

Air Column Resonance with Longitudinal Waves



Explore the simulation. Vary the length of the air column and wavelength. Observe the waveform, node and anti-nodes.

5.4.1.1 Relationship between the displacement-distance graph and pressure-distance graph for a longitudinal wave

Fig. 5.10 shows the displacement-distance graph for a set of air particles which are displaced. The corresponding pressure graph is also shown.

C means compression – region of higher pressure; R means rarefaction – region of lower pressure.

Particles at their equilibrium positions

Particles displaced from their equilibrium positions

Positive displacement corresponds with air particles displaced to the right of their equilibrium position.

Regions of higher pressure are where air particles are closer together. Regions of lower pressure are where air particles are farther apart.

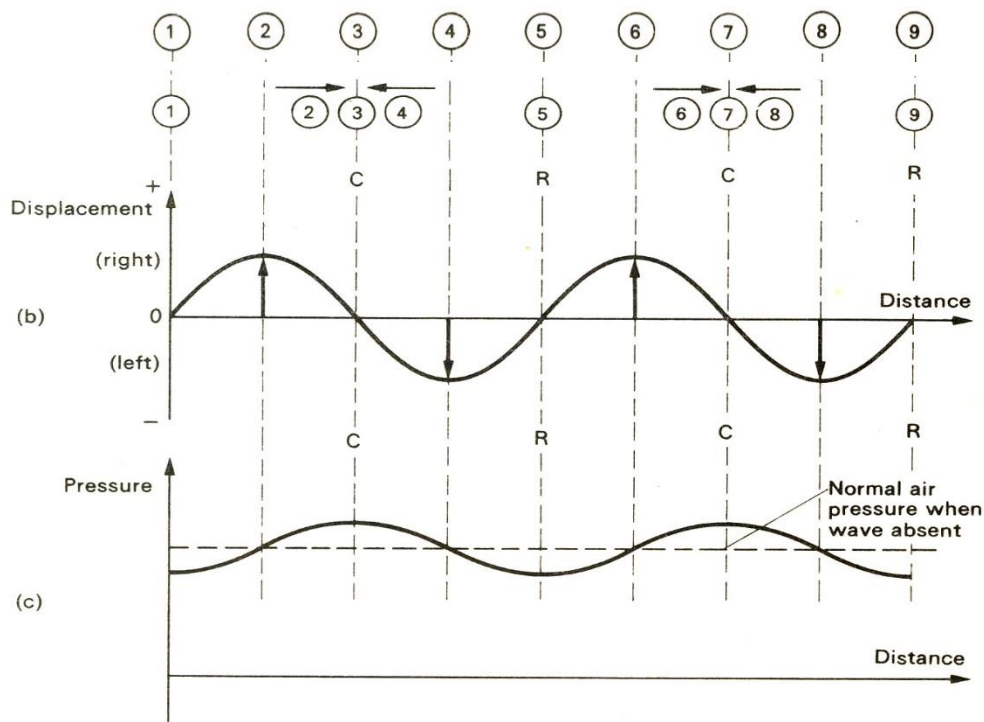
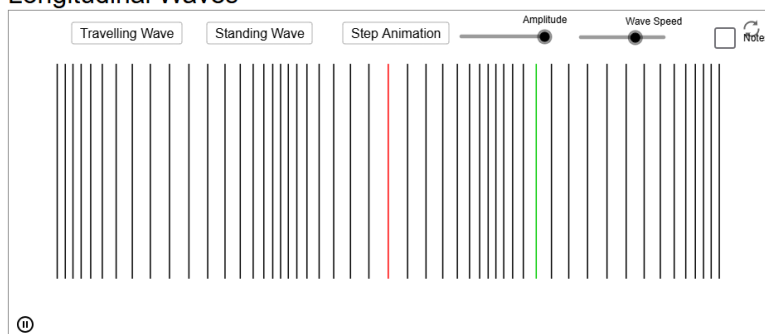


Fig. 5.10

Source: T. Duncan – Advanced Physics)

Longitudinal Waves



Explore the simulation. Switch to standing wave mode. Observe the vibrational pattern of the longitudinal wave.

5.4.1.2 Relationship to the displacement-distance graph and pressure-distance graph for stationary longitudinal waves

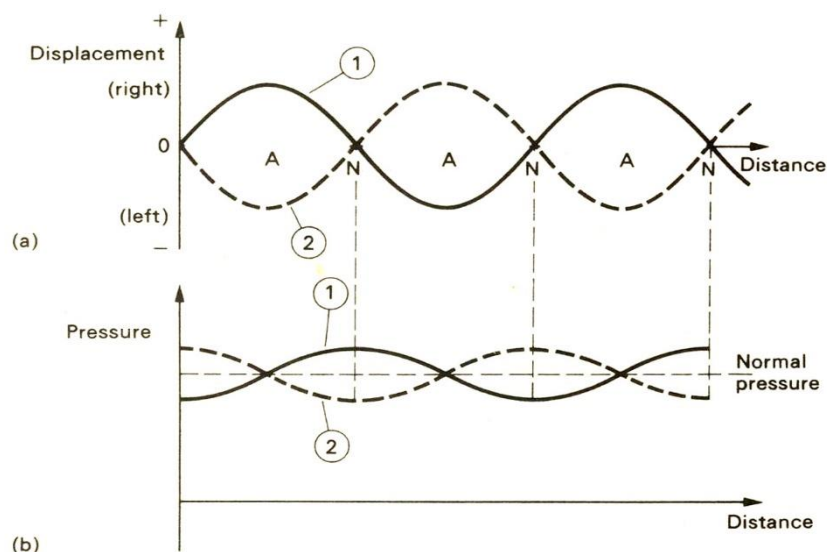


Fig. 5.11

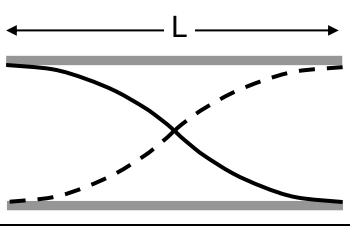
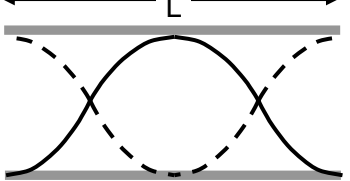
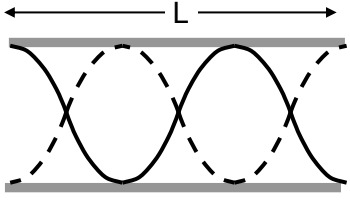
Source: T. Duncan – Advanced Physics)

As seen earlier, stationary waves can be formed in an air column if the air is disturbed – progressive longitudinal waves travel through the air to one end of the column and are reflected to interfere with the incident waves. The frequency of the sound waves is varied and resonance occurs when it matches one of the natural frequencies of vibration of the air column. There are two types of air column, categorised by their ends.

5.4.2.1 Stationary wave in a one end open and one end closed tube

Stationary Wave Pattern	Resonant Frequency	Harmonic
	wavelength = $4L$ velocity of wave = $f(4L)$ frequency of wave, $f_1 = \frac{v}{4L}$	1 st harmonic (fundamental)
	wavelength = $\frac{4L}{3}$ velocity of wave = $f\left(\frac{4L}{3}\right)$ frequency of wave, $f_2 = \frac{3v}{4L} = 3f_1$	3rd harmonic (first overtone)
	wavelength = $\frac{4L}{5}$ velocity of wave = $f\left(\frac{4L}{5}\right)$ frequency of wave, $f_3 = \frac{5v}{4L} = 5f_1$	5th harmonic (second overtone)

5.4.2.2 Stationary wave in a both ends open tube

Stationary Wave Pattern	Resonant Frequency	Harmonic
	wavelength = $2L$ velocity of wave = $f(2L)$ frequency of wave, $f_1 = \frac{v}{2L}$	1 st harmonic (fundamental)
	wavelength = L velocity of wave = $f(L)$ frequency of wave, $f_2 = \frac{v}{L} = 2f_1$	2 nd harmonic (first overtone)
	wavelength = $\frac{2L}{3}$ velocity of wave = $f\left(\frac{2L}{3}\right)$ frequency of wave, $f_3 = \frac{3v}{2L} = 3f_1$	3 rd harmonic (second overtone)

Other higher harmonics are illustrated in Fig. 5.12. AN and N refer to the displacement antinodes and nodes respectively.

	open pipe						closed pipe				
	AN	AN	AN	AN	AN		AN	AN	AN	AN	AN
		N	N	N	N		N	N	N	N	N
frequency	f_0	$2f_0$	$3f_0$	$4f_0$	$5f_0$		F_0	$3F_0$	$5F_0$	$7F_0$	$9F_0$
wavelength	$2L$	$\frac{2L}{2}$	$\frac{2L}{3}$	$\frac{2L}{4}$	$\frac{2L}{5}$		$4L$	$\frac{4L}{3}$	$\frac{4L}{5}$	$\frac{4L}{7}$	$\frac{4L}{9}$
harmonic	fundamental	2nd	3rd	4th	5th		fundamental	3rd	5th	7th	9th
possible frequency/Hz	128	256	384	512	640		64	192	320	448	576
	(a)						(b)				

Fig. 5.12
(Source: R. Hutchings - Physics)

5.4.2.3 End-correction at the open end of the tube

In practice, the air just outside the open end of a pipe is set into vibration and the displacement antinode of a stationary wave occurs a distance c (called the end correction) beyond the end. The effective length of the air column is therefore greater than the length of the pipe as shown in Fig. 5.13.

Effective length for Fig 5.13 (a) is $l + c$

Effective length for Fig 5.13 (b) is $l + 2c$

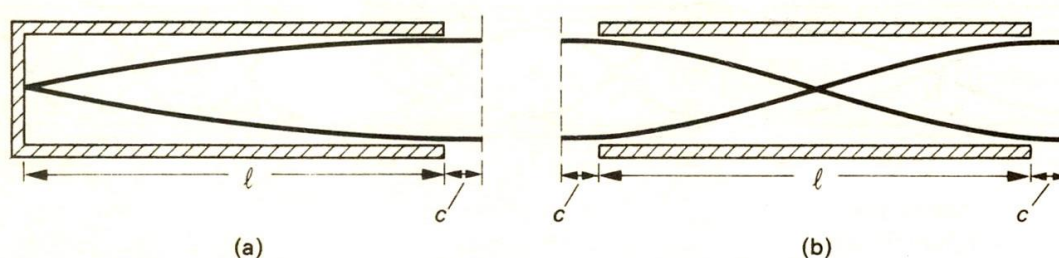


Fig. 5.13

Source: T. Duncan – Advanced Physics)

5.4.3.1 Investigation of Stationary Waves in an Air Column of Varying Length

Setup:

- Fill a tube with water.
- Place a tuning fork at the open end of the tube.
- Adjust the submerged length of the tube until a loud sound is heard as the air column resonates.
- At this resonance, a stationary wave is formed.
- Air particles move at its highest amplitude (therefore causing loud sound).
- Change the submerged length such that the air column is increased. The same loud sound can be heard again.
- Notice that since the same tuning fork is used, the frequency of the sound waves is unchanged. This means that the wavelength of the sound wave is also unchanged (speed of sound is a constant).

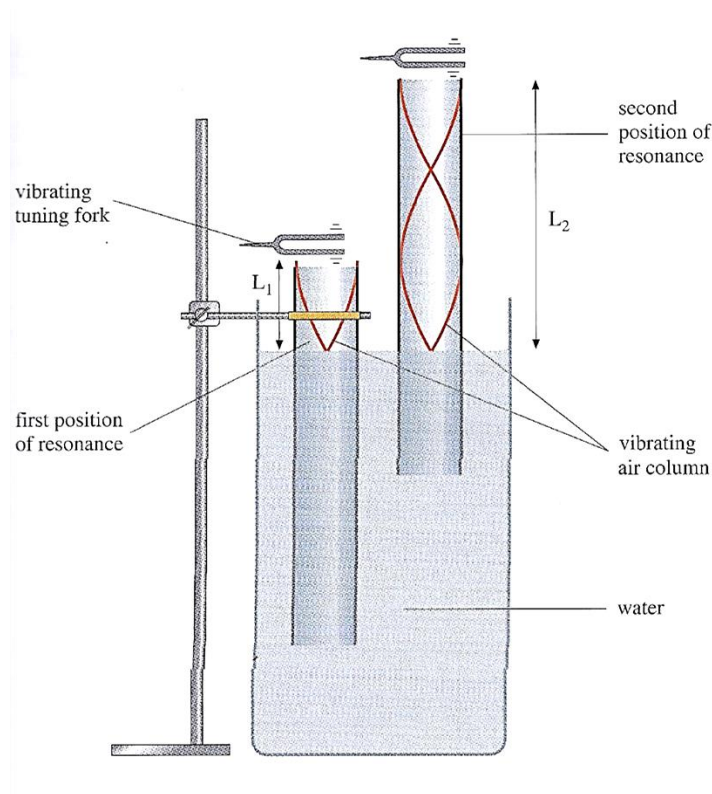
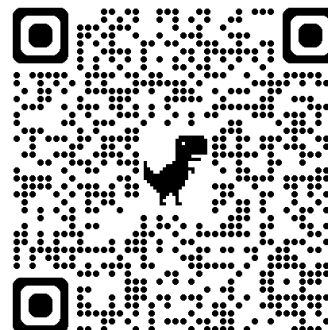
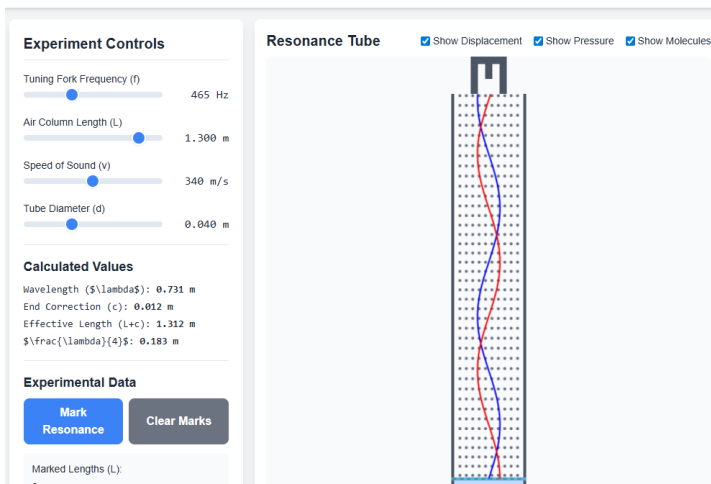


Fig. 5.14
(Source: Chris Mee - AS/A2 Physics)

H2 Physics: Resonance in Vertical Air Columns



Explore the simulation. Vary the length of the tube and observe the wave pattern, movement of air molecules and graphical output.

5.4.3.2 Determination of velocity and wavelength of sound using stationary waves

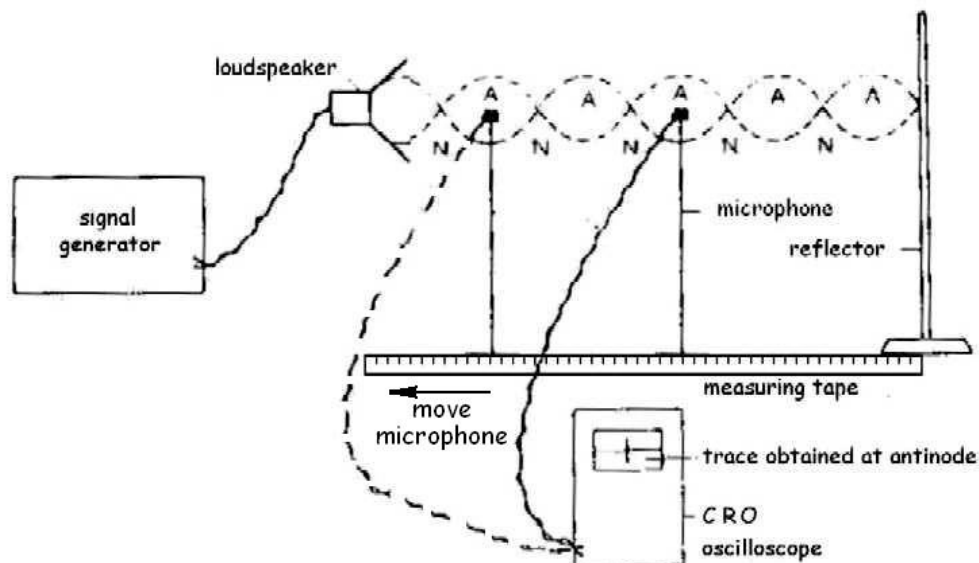


Fig. 5.15

Setup:

- Sound waves of known frequency (2 – 3 kHz is suitable) are incident on a reflector with the help of a loudspeaker. Waves emerging from the loudspeaker and the reflected wave superpose to give stationary waves.
- The detection of the pressure nodes (N) and antinodes (A) of the stationary waves can be done by means of a microphone attached to the CRO (time base is switched off).
- By moving the microphone slowly forward and backward, the vertical trace on the CRO is seen to vary.
- When the microphone is moved such that a vertical trace of maximum amplitude is obtained, the microphone is at the pressure antinode where the pressure variation is greatest. This point is also the displacement node.
- When the microphone is moved such that the amplitude of vertical trace is at a minimum, the microphone is at the pressure node where the pressure variation is the least (the non-zero amplitude is due to background noise) This point is also the displacement antinode.
- The wavelength of the sound wave can be determined since the wavelength is twice the distance between consecutive antinodes (or nodes).
- The velocity of sound wave can then be determined.

5.4.3.3 Stationary Waves of Microwaves

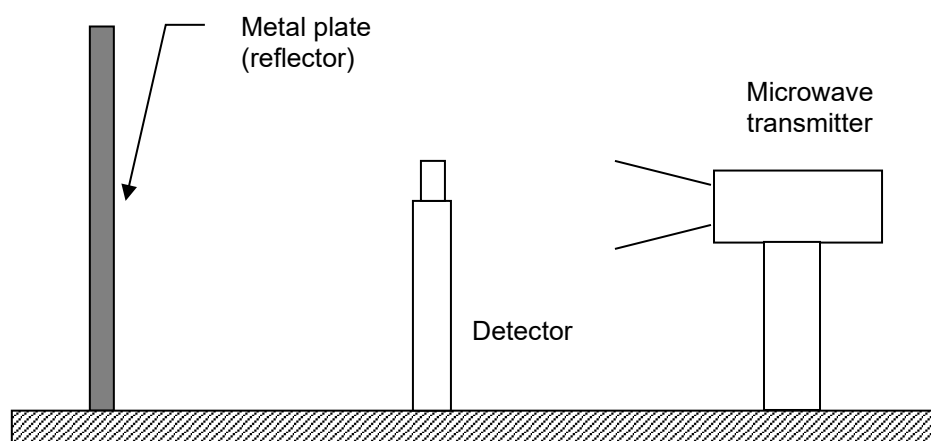


Fig. 5.16

The setup for creating stationary waves of microwaves is very similar to that of sound waves. However, instead of using a simple wall or barrier, a metal plate is required to reflect microwaves.

Setup:

- (a) Microwave from the transmitter travels to the left and reflects off the metal plate and travels to the right. The two travelling waves superpose to form a stationary wave.
- (b) A detector (connected to a control box and voltmeter) is moved along a line between the transmitter and the metal plate.
- (c) The detector registers a series of high intensity (antinodes) and low intensities (nodes) which will correspond to a high or low voltmeter reading respectively.
- (d) The wavelength of the sound wave can be determined since the wavelength is twice the distance between consecutive antinodes (or nodes).
- (e) The velocity of microwaves can also be determined.

Review Questions

NYJC/2018

- 1 (a) Explain what is meant by *polarised* light. [1]
- (b) Explain why two coherent sources of light that are polarised in planes perpendicular to each other will not produce interference fringes. [1]
- (c) Fig. 4.1 shows the setup for a Young's double slit experiment. The polarisers are initially aligned such that the light from the slits are polarised in parallel planes. The slit separation of the double slit is 0.10 mm, the distance between the double slit and the screen is 1.50 m and the separation of the fringes produced is 0.90 cm.

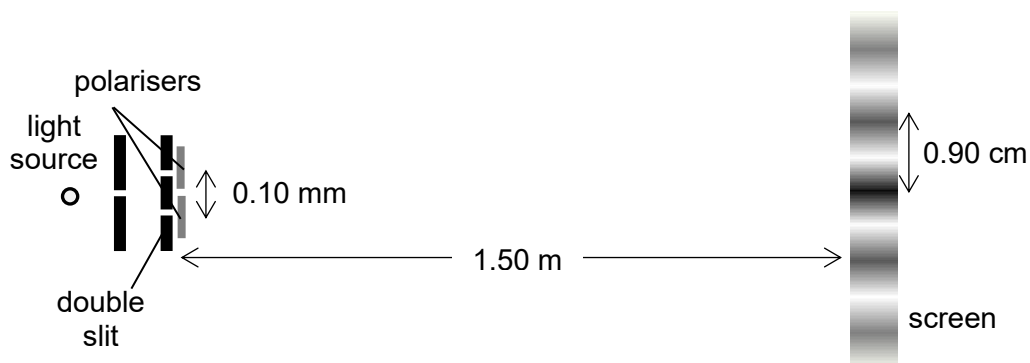


Fig. 4.1

- (i) Calculate the wavelength of the light source. [2]
- (ii) One of the polarisers is rotated by an angle of 45° about an axis parallel to the incident light. Describe the appearance of the fringes with reference to its original appearance. [2]

Answers

- 1 (a) Light in which the oscillations of the electromagnetic fields are all in a single plane
- (b) The displacements due to the two waves are in perpendicular axes, thus their vector sum will not be able to produce maxima and minima, since they can never be in the same or in opposite directions
- (c)(i) $\lambda = a \times D = 0.10 \times 10^{-3} \times 0.90 \times 10^{-2} / 1.50 = 6.00 \times 10^{-7} \text{ m}$
- (ii) Only the components of the light waves that are oscillating in parallel planes will interfere to form maxima and minima. The components that are oscillating in perpendicular planes will not be affected by each other. Thus the maxima will be less bright and the minima will be brighter.

AJC/2018

- 2 (a) Fig. 5.1 shows the arrangement for viewing a visible interference pattern on a screen.

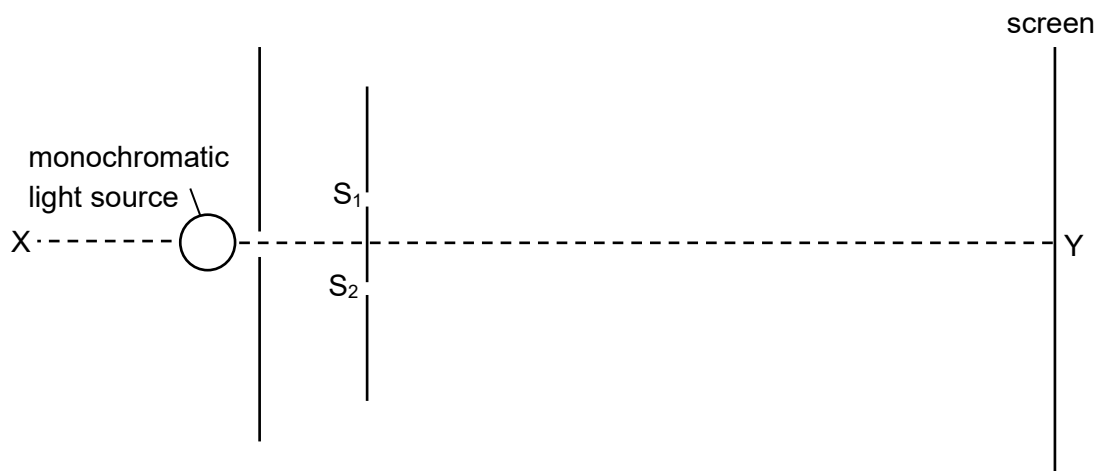


Fig. 5.1

In a darkened room, a double slit (S_1S_2) is placed in front of a narrow single slit situated in front of a monochromatic light source. Line XY is equidistant from S_1 and S_2 .

- (i) Explain why a single slit is used in Fig. 5.1. [3]
- (ii) The point Y on the screen is directly opposite to the centre of the double slit. State and explain the nature of the interference that occurs at Y. [2]
- (iii) The distance between slits S_1 and S_2 is 0.60 mm. When the screen is placed 1.8 m from the slits, the distance between the two first-order bright fringes in the interference pattern formed on the screen is 4.0 mm. Calculate the wavelength of the light.
- (iv) Suggest changes to the appearance of the fringes when each of the following changes is made separately.
 1. Moving the light source and single slit along the axis towards point X further away from S_1S_2 . [2]
 2. Shifting the single slit a small distance downwards, closer to slit S_2 . [2]

Answers

- 2 (i) For observable interference pattern, the two sources must be coherent.
At the single slit, the monochromatic light diffracts from it like a point source.
When the diffracted wave reaches S_1 and S_2 , the waves that emerge from them act as two coherent sources.
- (ii) Since the two sources are in phase and the path difference between S_1Y and S_2Y is zero, the phase difference is zero and the 2 waves arrive at Y in phase, constructive interference happens at Y.
- (iii)
$$\lambda = ax / D$$

$$= (2.0 \times 10^{-3} \times 0.6 \times 10^{-3}) / 1.8$$

$$= 6.67 \times 10^{-7} \approx 6.7 \times 10^{-7} \text{ m}$$
- (iv) 1. The fringe separation will remain the same
Intensity of bright fringe decreases, while dark fringe remains dark OR
less contrast between bright and dark fringes
2. The zeroth order will shift away from Y towards the top of the page
Fringe separation / intensity / contrast will remain the same

DH/2019

- 3 The student moves on to explore with a laser of wavelength 633 nm. The laser is placed behind a glass microscope slide that has been painted black. A single vertical slit of width 0.0800 mm has been produced by scratching through the paint with a razor blade.

Light from the laser passes through the slit and hits a wall at a distance of 5.12 m from the slit. A light sensor connected to a data logger is moved across the wall and the variation with distance moved by the sensor of the intensity of light is shown in Fig. 8.5.

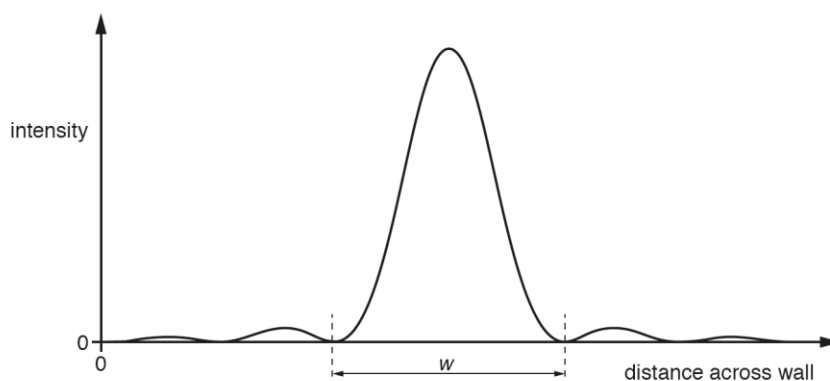


Fig. 8.5

The width w of the central patch is equal to the distance between the two minimum points on either side of the central patch where the intensity of the light is equal to zero.

- (i) Determine w .
- (ii) A second vertical slit of width 0.0800 mm is scratched across the slide. The second slit is parallel to the first and its centre is a horizontal distance of 0.240 mm away from the centre of the first slit. The slide now acts as a double slit. At the centre of the double-slit interference pattern on the wall, there are bright and dark fringes which are uniformly spaced.
 1. Some parts of the screen that were brightly lit when only the first slit was present are now dark, even though the light is still passing through the first slit in the same way. Explain what causes this to happen. [1]
 2. Determine the separation x of the bright fringes. [2]
 3. Most of the bright fringes are separated from adjacent bright fringes by a distance x . In a few places, away from the centre, however, there are separations of $2x$ and there is no light in the middle of the gap where a bright fringe might be expected. Using the results from (i) and (ii)2, explain why there is no light at such places. [2]

Answers

- 3 (i) $a \sin \theta = \lambda \Rightarrow \sin \theta = 633 \times 10^{-9} / (0.0800 \times 10^{-3})$, $\theta = 0.453^\circ$
 $w = 2 \times 5.12 \times \tan \theta = 2 \times 5.12 \times \tan(0.453^\circ) = 0.0810 \text{ m}$
- (ii) 1. light from second slit meets π rad out of phase with light from first slit or destructive interference for lights from both slits
2. $x = 633 \times 10^{-9} \times 5.12 / 0.240 \times 10^{-3} = 0.0135 \text{ m}$
- 3 At these places are single slit minimums
 No light to interfere (a factor of 3 recognised)
 (Note: $3x = 0.0405 \text{ m}$ which is equal to $w/2$)

EJC/2020

- 4 (a) Explain what is meant by the *Rayleigh criterion* for the optical resolution of two features. [2]
 (b) A 720p high-definition television (HDTV) image is composed of 720 horizontal scan lines uniformly distributed on a TV screen to form an image. Each scan line comprises pixels of varying colours and light intensities. When a person views an object, light enters the person's eyes through a small opening known as the pupil.
 (i) Explain why is it that a person standing sufficiently near the TV can distinguish between adjacent horizontal scan lines. [2]
 (ii) Fig. 3.1. shows the screen of a particular HDTV which measures 70.9 cm by 27.9 cm.

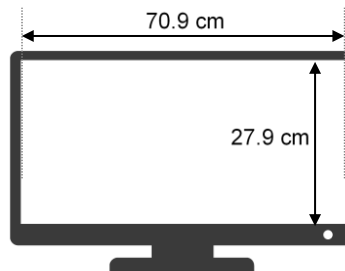


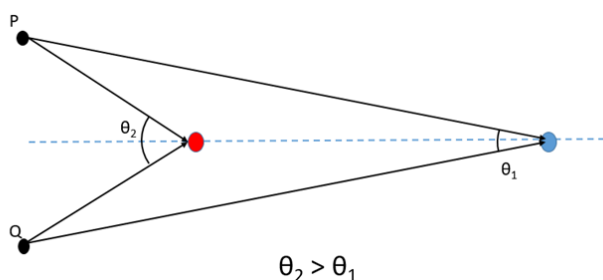
Fig. 3.1

The TV is able to output the full spectrum of visible light. The average diameter of pupils in human eyes is 3.00 mm in diameter. Determine the minimum pupil-to-screen distance necessary for the average human to no longer distinguish adjacent horizontal scan lines from one another. [3]

- (iii) The pupil of the human eye is known to dilate (i.e. increase in size) in a darker room. Explain how your answer in **(b)(ii)** will change in a darker room. [2]

Answers

- 4 (a) limit for which light reflecting off the 2 features can be just distinguished when the central maximum of one image falls on the first minimum of the other image.
 (b)(i) Diffraction pattern formed when light from horizontal scan lines enters through pupil
 Angular separation between lines increases when distance to TV screen decreases, making the two diffraction patterns distinguishable



- (ii) Distance between horizontal scan lines, $s = \frac{27.9 \times 10^{-2}}{720} = 3.88 \times 10^{-4} \text{ m}$
 Diameter of pupil, $b = 3.00 \times 10^{-3} \text{ m}$

Applying Rayleigh's criteria

$$\frac{s}{r} = \theta \approx \frac{\lambda}{b}$$

$$r = \frac{sb}{\lambda}$$

$$= \frac{(3.88 \times 10^{-4})(3.00 \times 10^{-3})}{400 \times 10^{-9}}$$

$$= 2.91 \text{ m}$$

- (iii) b increases when pupil dilate, applying Rayleigh's criteri (minimum resolvable angle decreases), r increases

- 5 A parallel beam of white light that consists of wavelengths from 350 nm to 650 nm is incident normally on a diffraction grating that has 500 lines per millimetre.
- Calculate the maximum angle in the second order spectrum. [2]
 - Calculate the minimum angle in the third order spectrum. [2]
 - Explain a problem with viewing the second or third order maxima of the white light with this diffraction grating. [1]

Answers

- 5 (a) $a \sin \theta = n\lambda$
- $$\left(\frac{1}{500,000}\right) \sin \theta = 2(650 \times 10^{-9})$$
- $$\theta = 40.5^\circ$$
- (b) $a \sin \theta = n\lambda$
- $$\left(\frac{1}{500,000}\right) \sin \theta = 3(350 \times 10^{-9})$$
- $$\theta = 31.7^\circ$$
- (c) An angular overlap of 8.8° between the second maxima and third order minima. Within this range of 8.8° , the colours are mixed up. It will be difficult to tell which colour belongs to which order of the spectrum.

- 6 (a) State two differences between stationary waves and progressive waves. [2]
 (b) A violin string has a length of 327 mm and produces a note of frequency 440 Hz. Calculate the frequency of the note produced when the same string is shortened or “stopped” to a length of 219 mm and the tension remains constant. [2]
 (c) Children often learn to play the violin on a small instrument with shorter strings. These shorter strings have to produce the same fundamental frequencies as those on the full-size instrument. State two ways in which this can be achieved. [2]

Answers

- 6 (a) In a progressive wave, all points have the same amplitude while in a stationary wave, the amplitude of the various points vary. In a progressive wave, points between nodes are in phase while in a stationary wave all points within one wavelength are out of phase with one another. For stationary waves, there is no energy transfer along the wave while in progressive waves energy is transferred in the direction of propagation. In stationary waves, there are nodes and antinodes but in progressive waves, there are no nodes and antinodes. (any 2 points)
 (b) Stationary wave formed on a stretched string fixed at both ends. Velocity of the traveling waves remains the same when the length is shortened as tension remains constant.

Wavelength for fundamental frequency = 2 x Distance between successive nodes = 2L

When L = 327 mm

Solving, $\lambda_1 = 2 \times 0.327 \text{ m} = 0.654 \text{ m}$

$$v = f\lambda_1 = 440(0.654) = 288 \text{ m s}^{-1}$$

When L = 219 mm

$$\lambda_2 = 2 \times 0.219 \text{ m} = 0.438 \text{ m}$$

$$v = f\lambda_2$$

$$288 = f(0.438)$$

$$f = 658 \text{ Hz}$$

- (c) Recall from the equation $v = \sqrt{\frac{T}{\mu}}$

To achieve the same f (fundamental frequency) using a lower λ (short length instrument), v (velocity of the traveling wave) has to be slower. This can be achieved by either lowering the tension in the string T or increasing the mass per unit length of the string μ .

Annex A – Derivation of Young's Double Slit Equation ($x = \frac{a\lambda}{D}$)

Schematic Diagram of setup (not to scale):

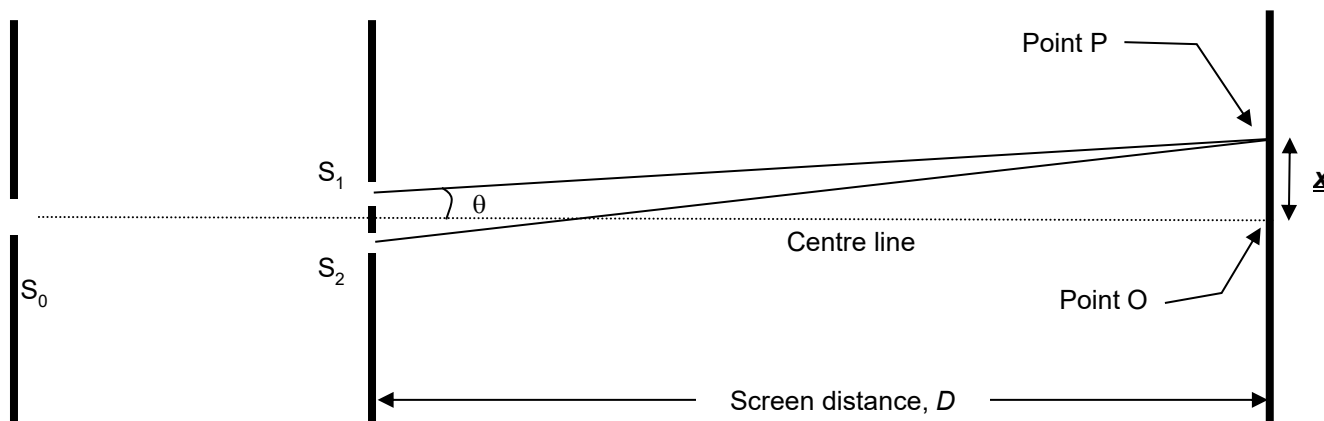


Fig A.1

The distance between the screen and the double slits D is many times more than the slit separation a , therefore, the two rays, from S_1 and S_2 , are nearly parallel to one another.

Close-up of double slits:

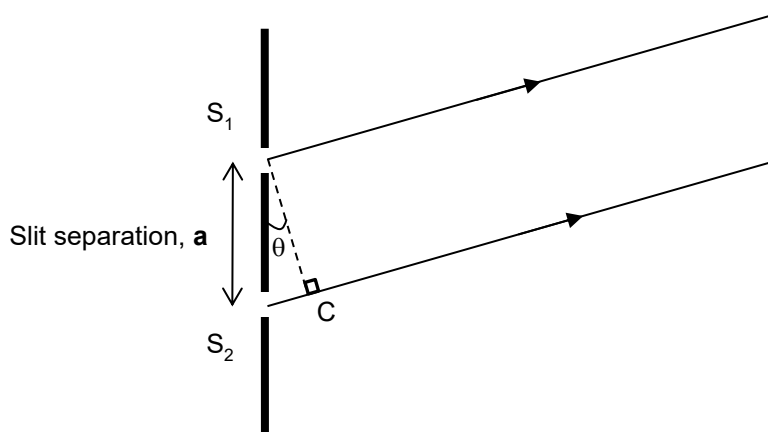


Fig. A.2

In Fig A.1, θ is approximately the same as the θ in Fig. A.2.

Hence,

$$\tan \theta = \frac{x}{D}$$

From Fig A.2,

$$\sin \theta = \frac{S_2C}{a}$$

Since θ is small, $\sin \theta \approx \tan \theta \approx \theta$,

$$\frac{x}{D} = \frac{S_2C}{a}$$

For point P to be a bright fringe, path difference, $S_2C = 1\lambda$, hence

$$\frac{x}{D} = \frac{\lambda}{a}$$

Rearranging, we get the double slit formula:

$$\lambda = \frac{ax}{D}$$

where λ is the wavelength, a is the slit separation, D is the screen distance, and, x is the fringe separation.

Annex B – Derivation of the Diffraction Grating Equation ($a \sin \theta = n\lambda$)

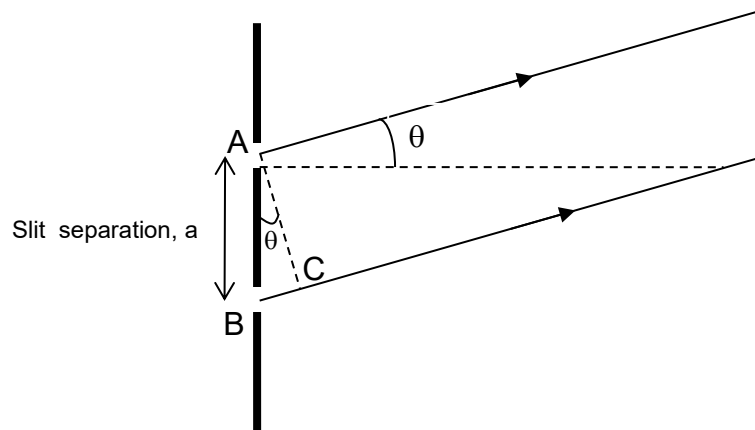


Fig. B.1

The diagram is a representation of 2 slits of a grating. For first order bright fringe, these two rays must arrive at the screen with a difference in distance of one wavelength.

Since $\sin \theta = \frac{BC}{AB}$

We have $\sin \theta = \frac{n\lambda}{a}$ where in this case, $n = 1$.

Rearranging, we get **$a \sin \theta = n\lambda$**