9749 H2 Physics Topic 13

DUNMAN HIGH SCHOOL

Electric Fields

Year 6 (2025)

Guiding Questions

- What is electric charge? How do charges interact?
- What do field lines represent? Do field lines represent similar things for gravitational fields and electric fields?

Content

- Concept of an electric field
- Force between point charges
- Electric field of a point charge
- Uniform electric fields
- Electric potential

Learning Outcomes
Students should be able to:

Concept of an electric field	(a)	show an understanding of the concept of an electric field as an example of a field of force and define electric field strength at a point as the electric force exerted per unit positive charge placed at that point
	(b)	represent an electric field by means of field lines
	(c)	recognise the analogy between certain qualitative and quantitative aspects of electric field and gravitational field
Electric force between point charges	(d)	recall and use Coulomb's law in the form $F = Q_1Q_2/4\pi\varepsilon_o r^2$ for the electric force between two point charges in free space or air
Electric field of a point charge	(e)	recall and use $E = Q/4\pi\varepsilon_o r^2$ for the electric field strength of a point charge in free space or air
Uniform electric fields	(f)	calculate the electric field strength of the uniform field between charged parallel plates in terms of potential difference and separation
	(g)	calculate the forces on charges in uniform electric fields
	(h)	describe the effect of a uniform electric field on the motion of charged particles
Electric potential	(i)	define the electric potential at a point as the work done per unit positive charge in bringing a small test charge from infinity to that point
	(j)	state that the field strength of the electric field at a point is numerically equal to the potential gradient at that point.
	(k)	use the equation $V = Q/4\pi\varepsilon_o r$ for the electric potential in the field of a point charge, in free space or air.

(c) recognise the analogy between certain qualitative and quantitative aspects of electric field and gravitational field.

1. Analogy between electric field and gravitational field

field:	**electric**	**gravitational**
force [*discussed in detail in learning outcome (d)*]	$\propto \frac{1}{r^2} \rightarrow F_e = k\left(\frac{Q_1Q_2}{r^2}\right)$ [Coulomb's law] • between two point charges Q_1 and Q_2 • SI units: N • a vector	$\propto \frac{1}{r^2} \rightarrow F_g = k'\left(\frac{Mm}{r^2}\right)$ [Newton's law of gravitation] • between two point masses M and m • SI units: N • a vector
force acts on:	object with charge only	object with mass only
field strength [*discussed in detail in learning outcome (e)*]	$E \equiv \frac{F}{q}$ (definition) $\propto \frac{1}{r^2} \rightarrow E = k\left(\frac{Q_1}{r^2}\right)$ • due to a point charge Q_1 [in a vacuum (free space) or air] • SI units: N C^{-1} (point charge) or V m^{-1} (parallel plate) • a vector	$g \equiv \frac{F}{m}$ (definition) $\propto \frac{1}{r^2} \rightarrow g = k'\left(\frac{M}{r^2}\right)$ • due to a point mass M • SI units: N kg^{-1} • a vector
field strength depends on medium?	Yes, $k = \frac{1}{4\pi\varepsilon_0\kappa}$. κ : dielectric constant of material; for vacuum, $\kappa = 1$; for air (1 atm), $\kappa = 1.00059$ ε_0 : permittivity of free space $= 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2} (or F m^{-1})	No, $k' = G$.
field generated by:	charge	mass
example of uniform field:	between charged parallel plates having a p.d. across them	on surface of the Earth
potential [*discussed in detail in learning outcome (k)*]	$V \equiv \frac{W}{q}$ $\propto \frac{1}{r} \rightarrow V = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{Q_1}{r}\right)$ • sign depends on type of point charge • V is zero at infinity • SI units: J C^{-1} • a scalar	$\phi \equiv \frac{W}{m}$ $\propto \frac{1}{r} \rightarrow \phi = -G\left(\frac{M}{r}\right)$ • sign is always negative • ϕ is zero at infinity • SI units: J kg^{-1} • a scalar
potential energy of two point charges or masses	$\propto \frac{1}{r} \rightarrow EPE = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{Q_1Q_2}{r}\right)$ • r : distance between Q_1 and Q_2 • positive (negative) if Q_1 and Q_2 have the same sign (opposite signs) • SI units: J • a scalar	$\propto \frac{1}{r} \rightarrow GPE = -G\left(\frac{Mm}{r}\right)$ • r : distance between M and m • it is always negative • SI units: J • a scalar
relationship between field strength and potential	$E = -\frac{dV}{dr}$	$g = -\frac{d\phi}{dr}$

(d) **recall and use Coulomb's law in the form $F = Q_1Q_2/4\pi\varepsilon_0 r^2$ for the force between two point charges in free space or air.**

2. Coulomb's law

Coulomb's law states that the electric force *F* acting between any two point charges Q_1 and Q_2 is directly proportional to the product of the charges and inversely proportional to the square of their separation *r*. The direction of the force is along the line joining the two point charges.

i.e. $$F \propto \frac{Q_1Q_2}{r^2} \quad \rightarrow \quad F = k\frac{Q_1Q_2}{r^2} \quad \rightarrow \quad \boxed{F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}}$$

where the constant of proportionality $k = 1/4\pi\varepsilon_0$ = 9.0 × 10^9 N m^2 C^{-2}

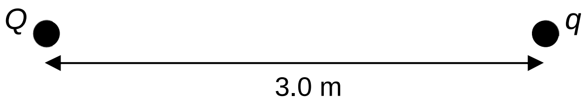
The permittivity of free space, ε_0 = 8.85 × 10^{-12} C^2 N^{-1} m^{-2} (or F m^{-1}) (given in data list)

Note:

This relation is applicable only to point charges. For practical purposes, two charged bodies can be treated as point charges if they are separated far apart such that the size of each of the charged bodies is negligible compared to their separation.

Example 1

Two point charges *Q* (+2.0 μC) and *q* (+1.0 μC) are separated by a distance of 3.0 m.



(a) Determine the magnitude and direction of the force of *Q* on *q*, F_q.

(b) Determine the magnitude and direction of the force of *q* on *Q*, F_Q.

(c) If *q* is negatively charged with twice its original magnitude, determine the new magnitude and direction of the force of *Q* on *q*, F_q'.

Solution

(a) Since both charges are positive, F_q is repulsive and is directed **away from** *Q*, along the line joining *Q* and *q*.

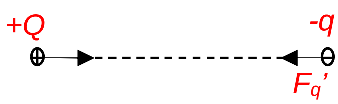
Using Coulomb's law,

(b)

F_Q is directed **away from** *q*, along the line joining *q* and *Q*.

(c) Since both charges are oppositely charged, the force is attractive.

F_q' is **directed towards** *Q*, along the line joining *q* and *Q*.



Using Coulomb's law,

The magnitude of the force is

(a) show an understanding of the concept of an electric field as an example of a field of force and define electric field strength at a point as the electric force exerted per unit positive charge placed at that point

3.1 Electric Field: concept of a field revisited

A ***field of force*** [1] is a region of space within which there could be a non-contact force acting on an object placed in that field.
Specifically, an **electric field** is a region of space within which there is an **electric force** acting on a **charged object** placed in that field.

Electric field strength at a point in an electric field is defined as the

exerted on a **small test charge**[2] placed at that point.

Mathematically, electric field strength E at a point can be expressed as

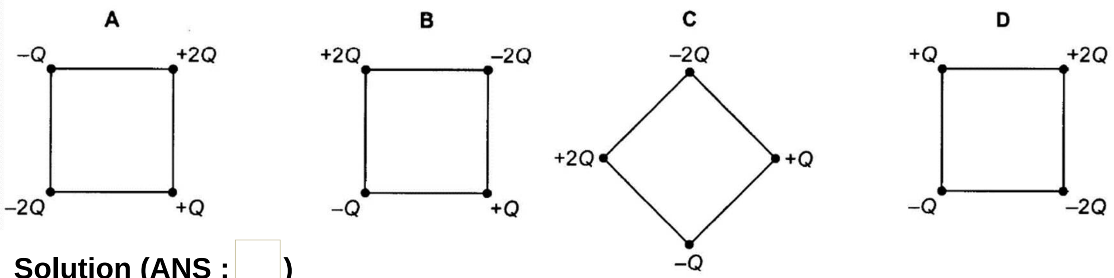
$$\boxed{E = \frac{F}{q}}$$

where F is the electric force acting on the charge (unit: newton, symbol: N)
q is the electric charge (unit: coulomb, symbol: C).

Notes:
- $F = qE$
- Electric field strength, E is a vector and has units: N C^{-1} (or V m^{-1}).
- The direction of electric field strength at a point in an electric field is the same as that of the force acting on a small positive test charge, q placed at that point.
- Since there are two types of electric charge (positive and negative), the word "positive" must be stated when defining electric field.

<u>Example 2 (2010/H2/P1/Q24)</u>
<u>(Electric Fields with Static Equilibrium)</u>

Four point charges of magnitude $+2Q$, $+Q$. $-2Q$ and $-Q$ are fixed at the corners of a rigid square resting on a frictionless surface.
A uniform electric field is applied in the direction from left to right in the plane of the surface.
In which situation will the square be in equilibrium?



<u>Solution</u> (ANS :)

The conditions for static "equilibrium" are that

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[1] An object placed in an ordinary space without fields of force (like in deep outer space) would not have any force acting on it. Other examples of fields of force include:
- the *gravitational field* (within which an object with *mass* would have a *gravitational force* acting on it)
- the *magnetic field* (within which an object with "*magnetic property*" *may* have a *magnetic force* acting on it)

[2] Analogous to gravitational fields, the test charge is small, so that it does not distort the electric field.

(a) the resultant force acting on the system is equal zero in any direction i.e. $\sum F = 0$ (translational equilibrium) and

(b) the resultant torque acting on the system about any axis of rotation is equal to zero i.e $\sum \tau = 0$ (rotational equilibrium).

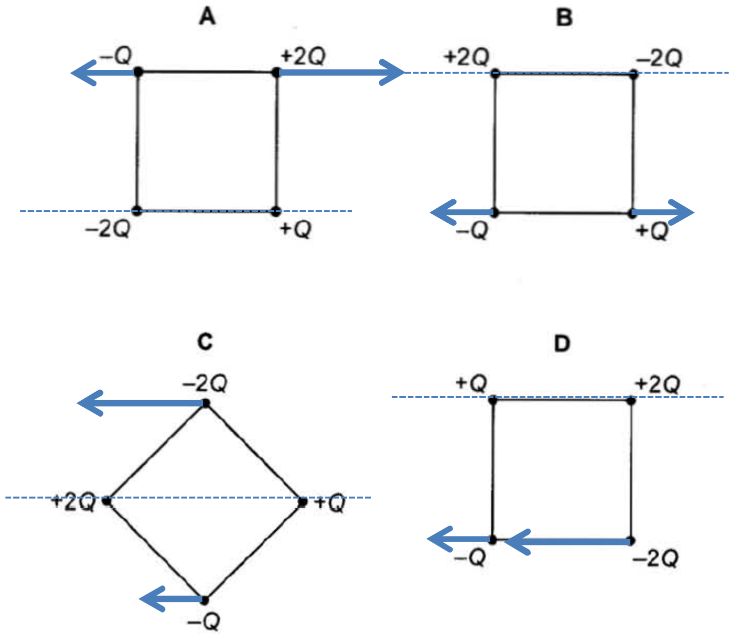
In this case, the net charge is	
Thus, from $F = qE$,	

To check for rotational equilibrium, a suitable pivot needs to be chosen for each option. The dotted lines represent the loci of possible pivotal positions for each option. If *r* is the length of the square and *E* is the magnitude of the uniform electric field, then

for option **A**,	
for option **C**,	
for option **D**,	

Inquiry 01:

Draw partial free-body diagrams for each system of charges shown in the options, ignoring any forces lying on the dotted lines because they have zero moment arms:

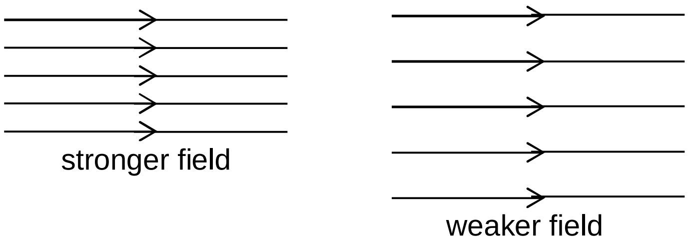


(b) represent an electric field by means of field lines

Electric field lines are used to help us visualise electric fields[3]. An electric field line is an imaginary lines or curves drawn through a region of space so that its tangent at any point is in the direction of the electric-field vector at that point.

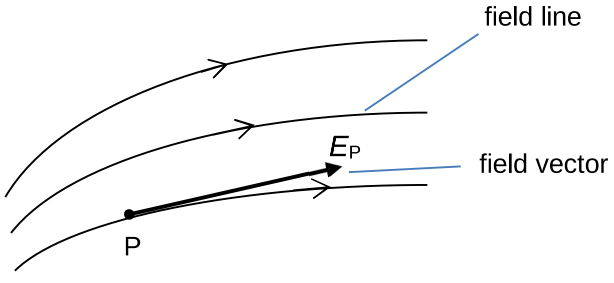
3.1.1 Basic features of electric field lines

1. The direction of the field is the tangent to the field line at each point. It is the direction of the force a small stationary positive test charge would experience if it is placed at that point.
2. The spacing between the lines indicates the *strength* of the field. A stronger field would have more closely spaced lines.
3. Electric field lines either originate on positive charges or come in from infinity, and either terminate on negative charges or extend out to infinity.
4. The number of lines originating from a positive charge or terminating on a negative charge must be proportional to the magnitude of the charge.
5. They do not intersect (since this would imply that at the cross-over point, the field would have two different directions). Every single location in space has its own electric field strength and direction associated with it.
6. In a uniform field, the field strength is the same at all points and could be represented as parallel lines that are equally spaced as shown below:

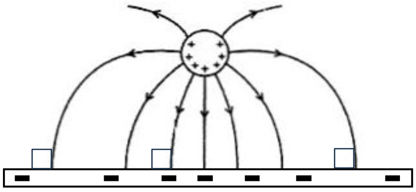


The field is stronger if the lines are closer to each other.

7. In a non-uniform field, the field lines are drawn such that the tangent to a field line at a point gives the direction of electric field strength E at that point as shown:



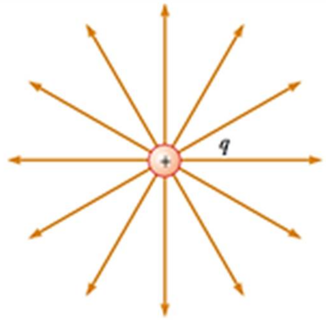
8. Field lines touch surface of charged conductors at 90°.



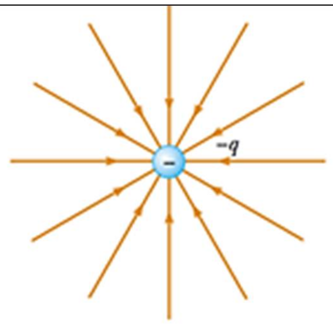
[3] Michael Faraday (1791–1867) introduced field lines as a way to represent fields. This creative conceptual model provides us with a powerful tool for thinking and communication about interaction at a distance.

3.1.2 Electric field line patterns:

I. A positive point charge



II. A negative point charge



Figures I and II (top): The closeness of the field lines is interrelated to the strength of the electric field.

A positive test charge placed anywhere will experience a force in the direction of the field line; the magnitude of this force will be directly proportional to the density of the field lines (e.g. being greater nearer the charge).

III. An isolated **positively charged** spherical conductor

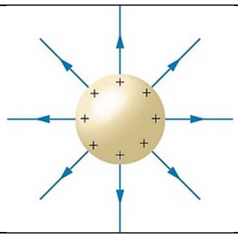


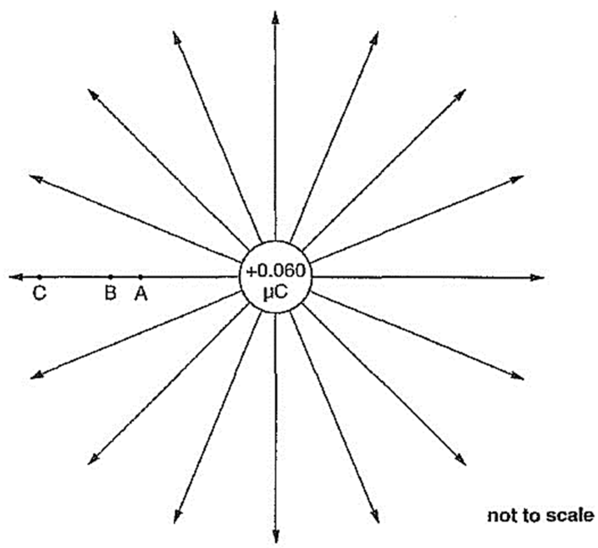
Figure III (left): The mutual repulsion of excess positive charges on a spherical conductor distributes them uniformly on its surface. In electrostatics, the electric field at every point within the material of the conductor is zero.

The resulting electric field acts outside the conductor and is perpendicular to the surface of the conductor.

Outside the conductor, the electric field is equivalent to that of a point charge with its excess charge located at its centre.

<u>Example 3 (N2008/P3/Q5(d)(i))</u>

A conducting sphere of radius 0.10 m carries a charge of + 0.060 μC. The electric field around the sphere is shown in the figure to the right. Suggest why it appears as if the charge is concentrated at the centre of the sphere.



<u>Solution</u>

As the conductor is a sphere,

The electric field lines directed away from these charges are also

Hence the radial field lines would appear to originate from a charge that is positioned at the centre of the sphere.

IV. A pair of charged parallel metal plates having a potential difference between them

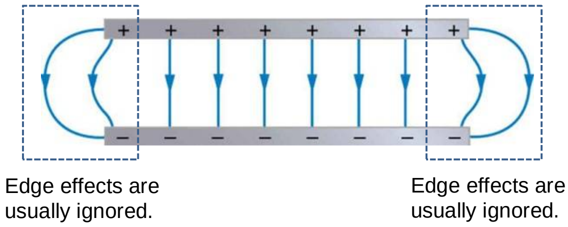
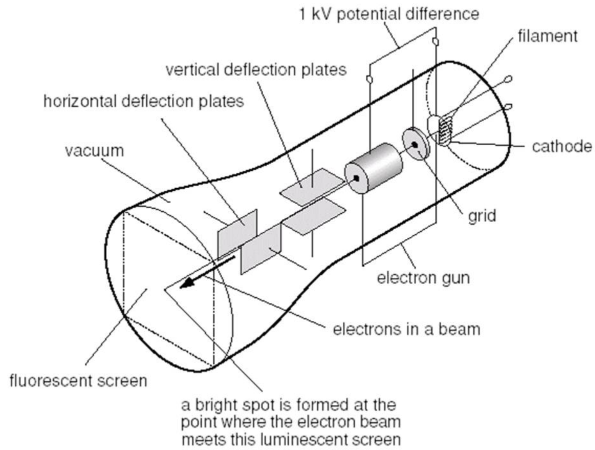


Figure IV (top): Two metal plates with equal, but opposite, excess charges.

The electric field between them is *uniform* in strength and direction except near the edges. This is known as edge effects, and the non-uniform fields near the edge are called the *fringing fields*.



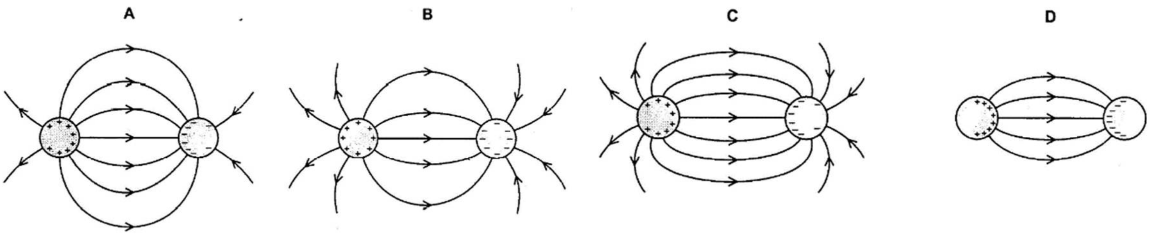
In **Figure IV**, the electric field lines are drawn incorporating edge effects. However, in the subsequent sections involving charged parallel plates, such effects will be ignored and in each case the assumption is an idealized situation, in which electric field lines between the charged parallel plates are straight lines, and zero outside.

Edge effect is usually ignored in A-level as well, i.e. the electric field strength is taken to be constant anywhere between the plates.

One use of such a field is to produce uniform acceleration of charges between the plates, such as in the electron gun of a TV tube or, more commonly, Cathode Ray Tube (CRT) - **Figure V (top right)**. A black-and-white CRT TV has a single electron gun, while a color CRT TV needs three guns because each pixel on the screen is made of a red, a green and a blue dot.

Example 4 (2010/H2/P1/Q25)

Two small metal spheres have equal but opposite charges.
Which diagram shows a possible distribution of charge on the spheres and the electric field near the spheres?



Solution (**Ans :**)

Option **D**

In option **B**,

Option **A** is a better answer than option **C** since the density of the field lines decreases

I.

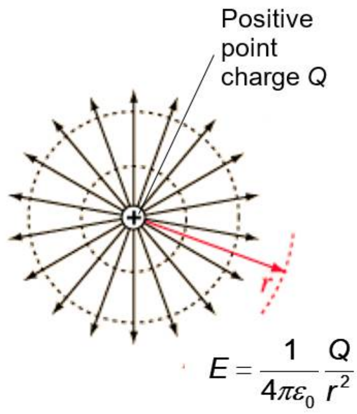
II.

(e) recall and use $E = Q/4\pi\varepsilon_o r^2$ for the field strength of a point charge in free space or air

3.2.1 Electric field strength due to a point charge

Consider a small positive test charge q at a distance r from a point charge Q. By Coulomb's law, the charge q will experience a force, $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$. From the definition of electric field strength, $E = \frac{F}{q}$, the electric field strength E at the point occupied by q is

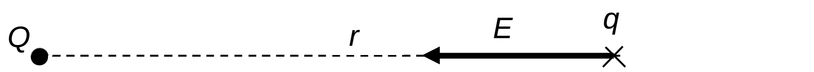
$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2}}$$



Note:

- This point charge Q is the one which creates the electric field in its surrounding region of space. If Q is positive, then the field strength is radially outward Q in all directions. If Q is negative, then the field strength is radially inward towards Q. (The dotted circles represent spherical equipotential surfaces.)

(Direction of electric field strength due to a positive point charge Q)

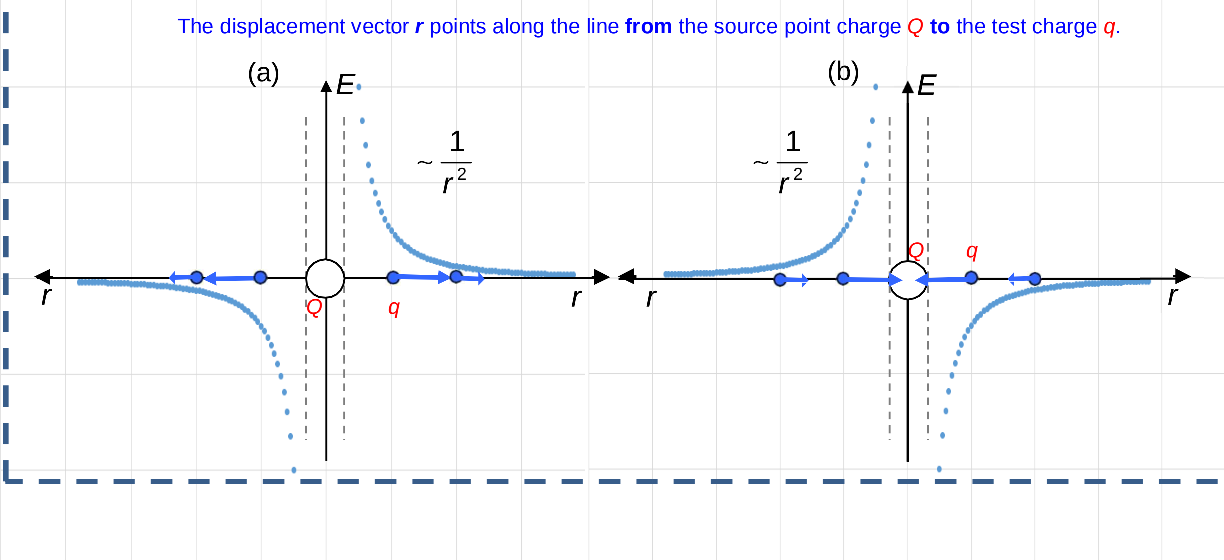


(Direction of electric field strength due to a negative point charge Q)

- To find the electric field strength at a point due to more than one point charge, apply vector addition of individual fields.

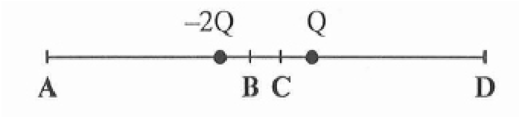
Inquiry 02:

Sketch the variation of electric field strength E with distance r from a (a) positive point charge Q, and (b) negative point charge Q:



Example 5 (N96/1/16)

Two point charges −2Q and +Q are situated as shown. At which point could the resultant electric field due to these charges be zero?

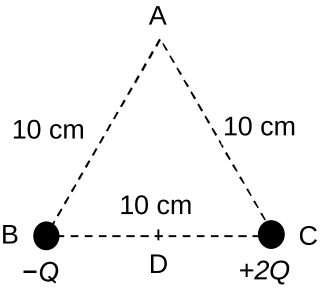


Solution (Ans :)

Example 6

(Electric field strength and force - multiple charges)

The figure shows two point *charges –Q and +2Q* fixed at the points B and C and at a distance of 10 cm apart.



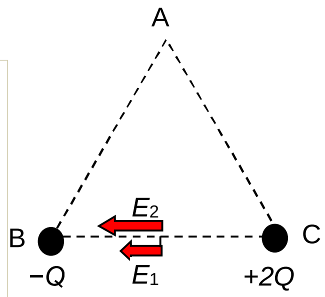
If the magnitude of $Q = 1.0 \times 10^{-7}$ C, calculate the magnitude and direction of the

(a) electric field strength at D, the midpoint of B and C,
(b) electric field strength at A,
(c) electric force exerted on a third charge $q = +3.0 \times 10^{-7}$ C at point D,
(d) electric force exerted on a third charge $q = +3.0 \times 10^{-7}$ C at point A.

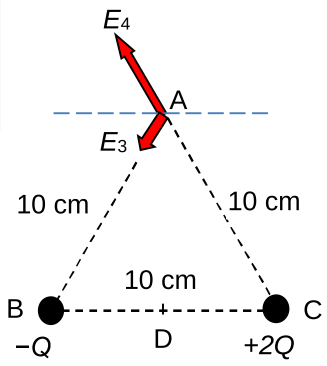
Solution

(a) Let E_1 and E_2 be the field strength acting at D due to the charges at B and C respectively. Both E_1 and E_2 are directed to the left.

$E_D = E_1 + E_2$



(b) Let E_3 and E_4 be the field strength acting at A due to the charges at B and C respectively. Both E_3 and E_4 are directed as shown in the diagram.



$E_3 =$

$E_4 =$

Let E_H be the resultant horizontal component and E_V be the resultant vertical component of the field at A.

$E_H =$

$E_V =$

The magnitude of the resultant E-field at A

$E_A =$

$=$

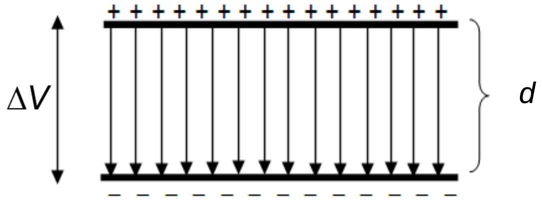
directed at an angle θ

(c) $F_D =$

(d) $F_A =$

(f) calculate the electric field strength of the uniform field between charged parallel plates in terms of potential difference and plate separation

3.2.2 Electric field strength of the uniform field between charged parallel plates



Consider a pair of charged parallel plates separated by a distance d, with a potential difference ΔV. The **uniform** electric field E set up in the space between the two plates would have a field strength with **magnitude** expressed as

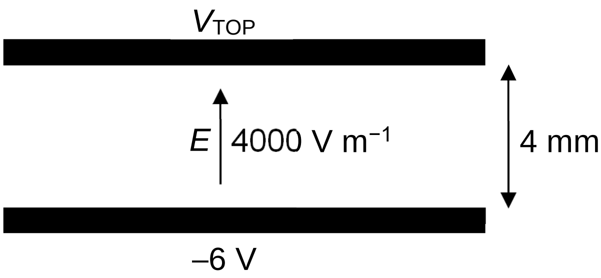
$$\boxed{E = \frac{|\Delta V|}{d}}$$

Note:

The separation between the charged parallel plates must be small compared to their length and width so as to minimise edge effect.

Example 7

Two large horizontal metal plates are separated by 4 mm. The lower plate is at a potential of –6 V. What potential V_{TOP} should be applied to the upper plate to create an electric field of strength 4000 V m^{-1} upwards in the space between the plates?



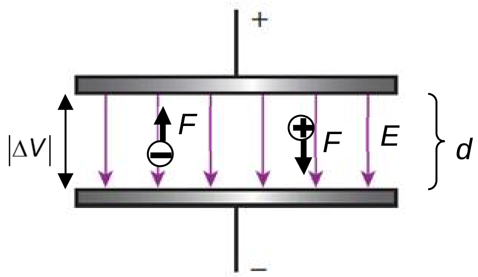
Solution

(g) calculate the forces on charges in uniform electric fields

3.2.2.1 Forces on charges in uniform electric fields

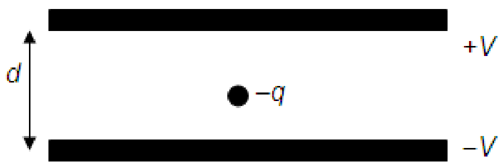
Electric force F on charge q in uniform electric field of field strength $E\left(=\frac{|\Delta V|}{d}\right)$:

$$F = qE = \frac{q|\Delta V|}{d}$$



Example 8

(Electric Fields with Static Equilibrium)



An oil droplet has a charge $-q$ and is situated between two charged parallel horizontal metal plates as shown in the diagram.
The separation of the plates is d. The droplet is observed to be stationary when the upper plate is at potential $+V$ and the lower at potential $-V$.
For this to occur, the weight of the droplet is equal in magnitude to

A $\dfrac{Vq}{d}$ **B** $\dfrac{2Vq}{d}$ **C** $\dfrac{Vd}{q}$ **D** $\dfrac{2Vd}{q}$

Solution (**Ans :**)

Example 9 **(N2007/1/24)**

(Electric Fields with Static Equilibrium)

The diagram shows two charged horizontal metal plates X and Z. The magnitudes of the charges on the plates are equal. Between the plates, suspended stationary in the uniform electric field, is a charged sphere Y. What could be the signs of the charges on the three components?

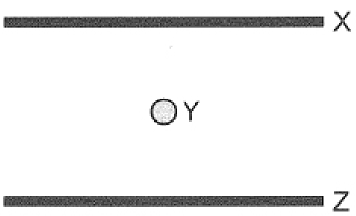
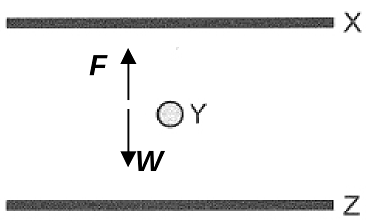


	plate X	sphere Y	plate Z
A	negative	positive	negative
B	negative	positive	positive
C	positive	positive	negative
D	positive	negative	positive

Solution **(Ans :** **)**



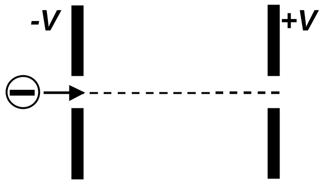
(h) describe the effect of a uniform electric field on the motion of charged particles

3.2.2.2 Motion of charged particles in uniform electric field

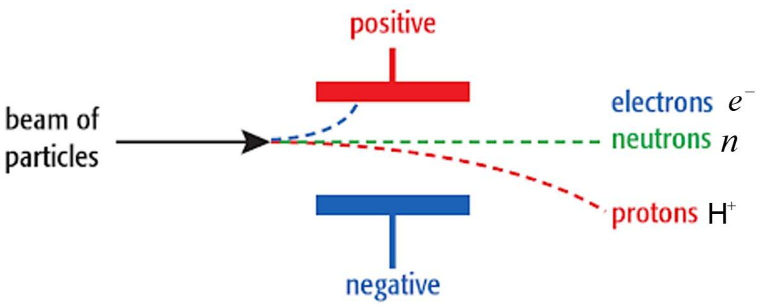
Within a uniform electric field, the field strength (vector) is the same (magnitude and direction) at all points. The force on a charged particle anywhere in the field is therefore a **constant** (magnitude and direction).

The motion of a charged particle in a uniform electric field is one of **constant acceleration**, depending on the relative direction of electric field to its motion, it could be in:

(a) linear motion



(b) or projectile motion (relate to topic of Kinematics).



Notes:

- For a charged particle with a very small mass, the effect of gravitational force is negligible compared to the electric force acting on it.
- In both cases, under the electric force, the charged particle of mass m will accelerate with an acceleration a. If the electric field is set up by the charged plates with potential difference $|\Delta V|$ and separated a distance d apart, then

$$\text{from } \sum F = ma \quad \Rightarrow \quad qE = ma \quad \Rightarrow \quad q\frac{|\Delta V|}{d} = ma \quad \Rightarrow \quad a = \frac{q|\Delta V|}{md}$$

Example 10

Two electrons are separated by 1.0 m. What is the ratio of the magnitude of the electric force on one electron and the gravitational force on that electron due to the other electron?

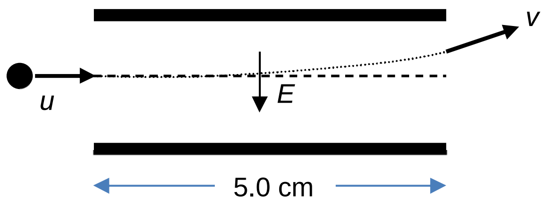
Solution

Example 11 (Electric Fields with Kinematics and Dynamics)

An electron is projected into the evacuated space between two charged plates of length 5.0 cm as shown in the diagram. The electric field strength E is 3.5×10^4 N C^{-1} and the velocity u of the electron is 3.2×10^7 m s^{-1}. Neglecting gravitational effects, determine

(a) the distance it deviates towards the positively-charged plate and

(b) its speed just as it leaves the electric field.



Solution

$s_y \uparrow$? $\underbrace{u_y = 0 \text{ m s}^{-1}}$ $v_y \uparrow$? $a_y \uparrow$?

Knowing these:

t = ? $\underbrace{a_x = 0 \text{ m s}^{-2} \quad 3.2 \times 10^7 \text{ m s}^{-1} \quad 3.2 \times 10^7 \text{ m s}^{-1} \quad 0.050 \text{ m}}$

$u_x \rightarrow$ $v_x \rightarrow$ $s_x \rightarrow$

For (a),

$[s_x = u_x t]$

$\left[s_y = u_y t + \tfrac{1}{2} a_y t^2 \right]$

For (b),

$\left[v_y = u_y + a_y t \right]$

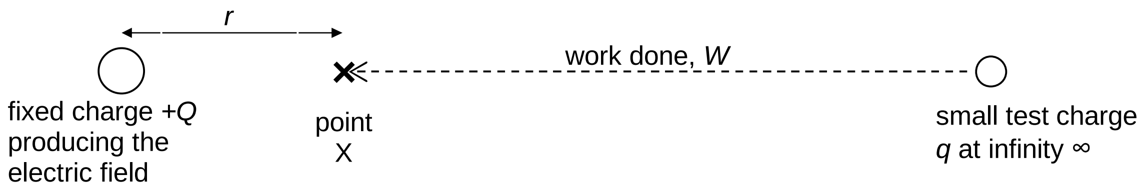
(i) define the electric potential at a point as the work done per unit positive charge in bringing a small test charge from infinity to that point

4.1 Electric potential

The **electric potential, V,** at a point in an electric field is defined as

in bringing a small test charge **from infinity to that point**.

(Infinity refers to any point where there is no electrical influence.)



Mathematically, electric potential V at a point can be expressed as

$$\boxed{V = \frac{W}{q}}$$

where W is the work done (by external agent) on the charge (unit: joule, symbol: J)
q is the electric charge (unit: coulomb, symbol: C).

Test charge is positive, by definition. Here, work is done (by an external agent) on test charge q as it has the same sign as Q (the electric force between them is repulsive). This work done is also the change in electric potential energy ΔEPE, or ΔU. Since EPE or U is taken to be zero when Q and q are infinitely far apart, $W = \Delta U = U_r - U_\infty = U_r - 0 = U_r$. So, V at a point is the electric potential energy U per unit charge associated with a test charge placed at that point.

Notes:

- Unit of electric potential V is J C^{-1} (or V).
- Electric potential V and electric potential energy U are scalar quantities.
- Electric potential at infinity is defined as zero, and at any other point it may be positive or negative depending on the polarity of charge Q which produces the field. Thus, the *sign* is not telling us about direction. The significance of the sign is simply to indicate if the potential is higher or lower than the zero at infinity. (In contrast, gravitational potential is always negative.)
- It doesn't have to be a charge at a given point for a potential V to exist there. (In the same way, an electric field can exist at a given point even if there is no charge there to respond to it.)

Example 12

A charge of 3 C is moved from infinity to a point X in an electric field. The work done in this process is 15 J. The electric potential at X is

A 45 V **B** 15 V **C** 5.0 V **D** 0.20 V

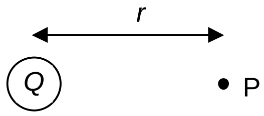
Solution (**Ans :**)

(k) use the equation $V = Q/4\pi\varepsilon_0 r$ for the electric potential in the field of a point charge, in free space or air.

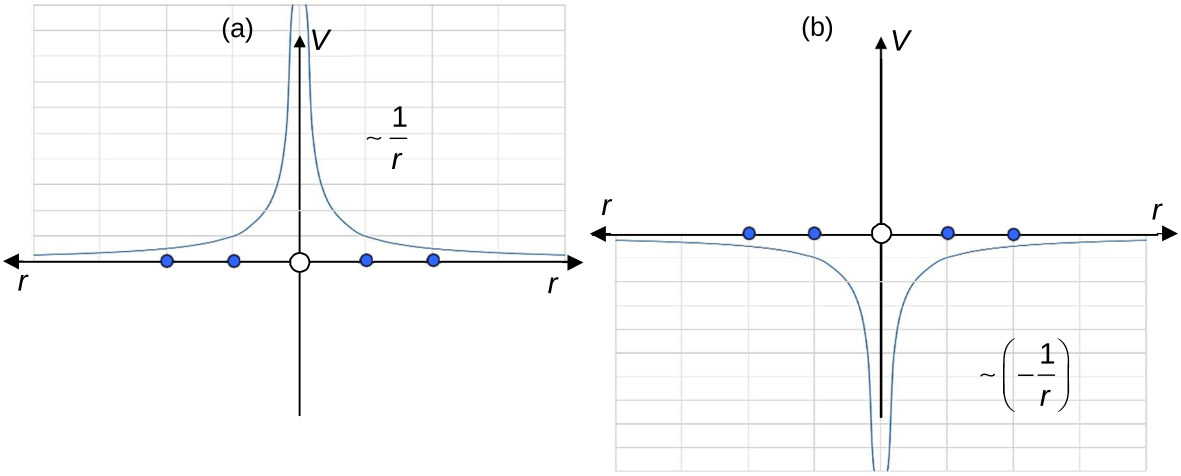
4.1.1 Electric potential due to a point charge

For an electric field generated by a point charge Q, the electric potential V at a distance r from the centre of Q is given by (*derivation is not required in syllabus*)

$$\boxed{V = \frac{Q}{4\pi\varepsilon_o r}}$$



Inquiry 03:
Sketch the variation of electric potential, V with distance r from (a) a positive point charge, and (b) a negative point charge:



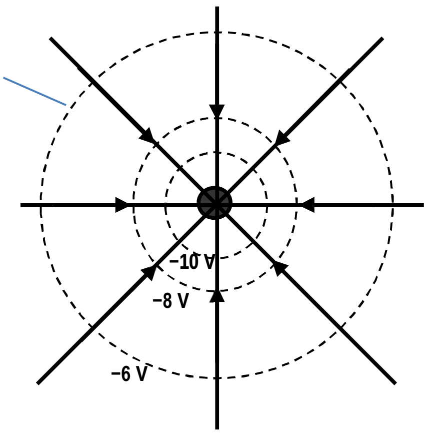
4.1.2 Equipotential lines in an electric field

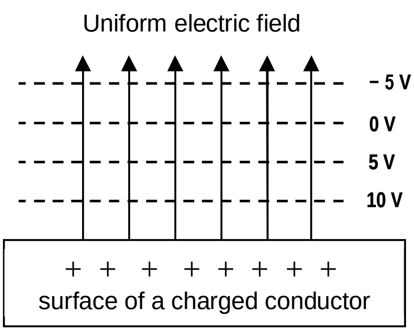
Electric potential in an electric field may be represented by *equipotential lines* indicated by dashed line: - - - - - - - - - - - - - -

Equipotential lines: Lines joining points in a field that have the same potential. They meet perpendicular to the electric field lines.

No work is done by an external force when a charge is moved *along* an equipotential. The electric force is always at 90° to the equipotential line. Work is only done if the charge has some component of its motion parallel to the electric force.

The equipotential lines of a negative point charge.
The dashed circles centred on the charge illustrate the scaling of voltage at equal increments (2V) - they get further apart with increasing *r*.





Example 13

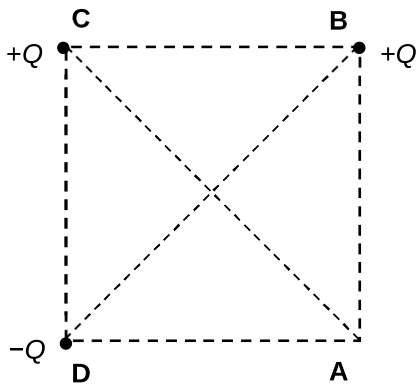
(Electric Fields with Measurements: Scalars and Vectors)

Three point charges are arranged at three corners of a square of side *r* as shown.

Derive an expression for

(a) the electric potential and
(b) the electric field strength

at corner A due to the three point charges in terms of *r* and *Q*.



Solution

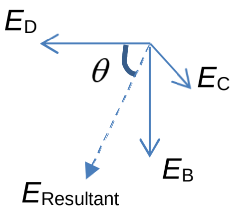
Note that electric potential *V* is a scalar while electric field strength *E* is a vector.

For (a),

$$\left[V = \frac{Q}{4\pi\varepsilon_0 r} \right]$$

For (b),

$$\left[E = \frac{Q}{4\pi\varepsilon_0 r^2} \right]$$



Magnitude of *x* - component of $E_{Resultant}$, $|E_x|$

Magnitude of *y* - component of $E_{Resultant}$, $|E_Y|$

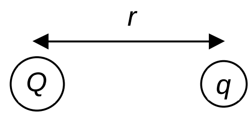
$|E_{Resultant}|$ =

$\theta =$

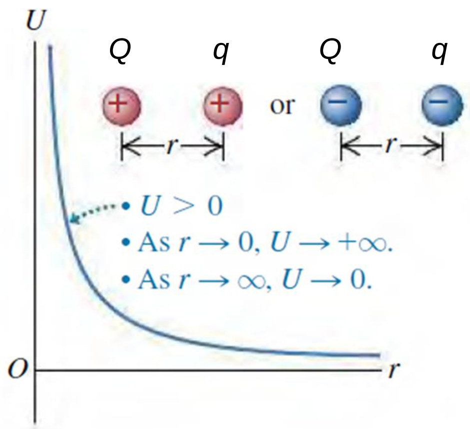
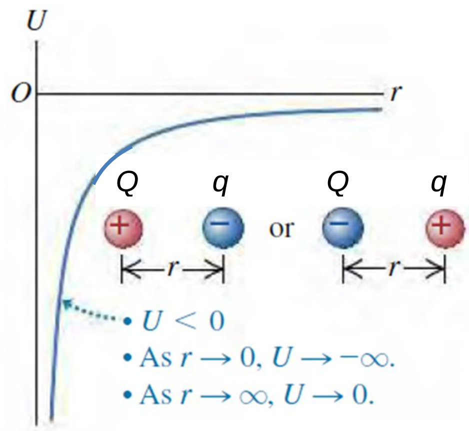
5. Electric potential energy of two point charges

When two point charges Q and q are at a distance r apart, the electric potential energy *EPE* or U of the system of two charges is given by

$$\boxed{U = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}}$$



This equation is valid for any combination of signs.

Q and q have the same sign	Q and q have opposite signs
• $U > 0$ • As $r \to 0$, $U \to +\infty$ • As $r \to \infty$, $U \to 0$ (U the smallest) • U increases as they come closer (work is done on the system)	• $U < 0$ • As $r \to 0$, $U \to -\infty$ • As $r \to \infty$, $U \to 0$ (U the largest) • U decreases as they come closer (work is done by the system)
	

Notes:

(a) Electric potential energy is a scalar quantity and its S.I. unit is joule (J).

(b) U is a shared property of the two charges Q and q: it is a consequence of the *interaction* between these two bodies. If the distance between the two charges is changed, the change in U is the same whether Q is held fixed and q is moved or q is held fixed and Q is moved. So, we never use the phrase "the electric potential energy of a point charge."

Example 14

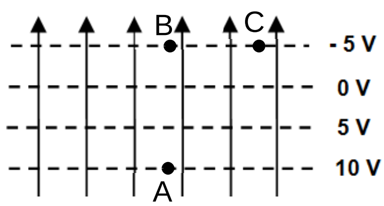
(Electric Fields with Conservation of Energy I)

A doubly-ionised negative charged ion (0^{2-}) travels from point A to point B and then to point C.

(i) Calculate the change in electric potential energy of this ion when it travels from point A to point B and its final kinetic energy when its initial kinetic energy is 6×10^{-18} J ;

(ii) Calculate the change in electric potential energy of this ion from point B to point C.

(iii) Calculate the change in electric potential energy of this ion from point A to point C.



Solution

(i)

By conservation of energy, gain in U = loss in E_K

(ii)

(iii)

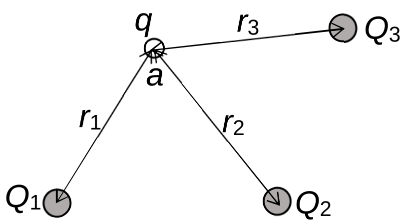
6. Electric Potential Energy with Several Point Charges

Suppose an electric field E in which charge q moves is caused by several point charges Q_1, Q_2, Q_3, … at distances r_1, r_2, r_3, … from q.

The total electric field at each point is the vector sum of the fields due to the individual charges, and the total work done on q during any displacement is the sum of the contributions from the individual charges.

From $U = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}$, we conclude that the potential energy associated with q at point a is the scalar sum (not a vector sum):

$$\boxed{U = \frac{q}{4\pi\varepsilon_0}\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3} + \dots\right)}$$



When q is at a different point b, the potential energy is given by the same expression, but r_1, r_2, r_3, … are the distances from Q_1, Q_2, Q_3, … to point b.

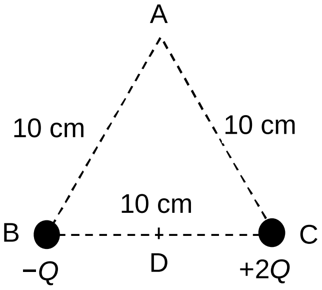
The work done by an external force on q when it moves from a to b along any path is equal to the change in U, $\Delta U = U_b - U_a$ between the potential energies when q is at b and at a.

The work done by the electric force on q when it moves from a to b along any path is equal to the negative of change in U, i.e., $-\Delta U$.

Example 15

(Electric Fields with Conservation of Energy II)

The figure shows 2 point *charges –Q and +2Q* fixed at the points B and C and at a distance of 10 cm apart.



If the magnitude of $Q = 1.0 \times 10^{-7}$ C, calculate the

(a) electric potential energy of the system,
(b) electric potential at D, the midpoint of B and C,
(c) electric potential at A,
(d) work done to bring a third charge $q = +3.0 \times 10^{-7}$ C from infinity to A,
(e) work done to move q from A to D.
(f) electric potential energy of the new 3 charge system, if q is placed at D.

Solution

(a)

(b) V at point D = (V at D due to charge at B) + (V at D due to charge at C)

(c) V at point A = (V at A due to charge at B) + (V at A due to charge at C)

(d) External work done to bring q from infinity to point A =

=

=

(e) External work done to move q from A to D =

=

=

(f) In bringing the charge q from infinity to point D,

the gain in EPE of the system =

=

Thus, the new EPE of the system is

(j) state that the field strength of the electric field at a point is numerically equal to the potential gradient at that point.

7. Relationship between field strength and potential

Similar to gravitational field strength, the electric field strength E at a point in an electric field is related to the electric potential V at that point by the equation

$$\boxed{E = -\frac{dV}{dr}} \qquad \text{compare to} \quad g = -\frac{d\phi}{dr}$$

The term $\frac{dV}{dr}$ is known as potential gradient (it is the gradient of the graph of potential V against distance r.).

Hence the field strength at a point in a field is <u>numerically equal to the potential gradient</u> at that point i.e. $E = \left|\frac{dV}{dr}\right|$. The negative sign indicates the direction of the electric field, which <u>points towards the decreasing electric potential</u>.

<u>Example 16 (1984/Jun/P1/Q6)</u>

The potential energy of a body when it is at a point P a distance x from a reference point O is given by $U = kx^2$, where k is a constant. What is the force acting on the body when it is at P?

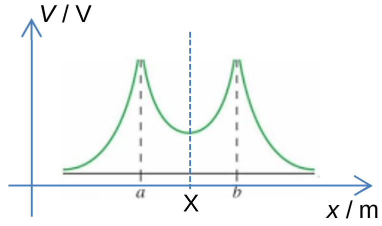
- **A** $2kx$ in the direction OP
- **B** kx in the direction OP
- **C** kx in the direction PO
- **D** $2kx$ in the direction PO

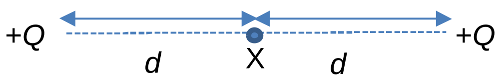
<u>Solution</u> (**Ans :**)

<u>Example 17 (1986/Nov/P3/Q6)</u>

Explain whether it is possible for the electric field strength to be zero at a point where the electric potential is *not* zero.

<u>Solution</u>

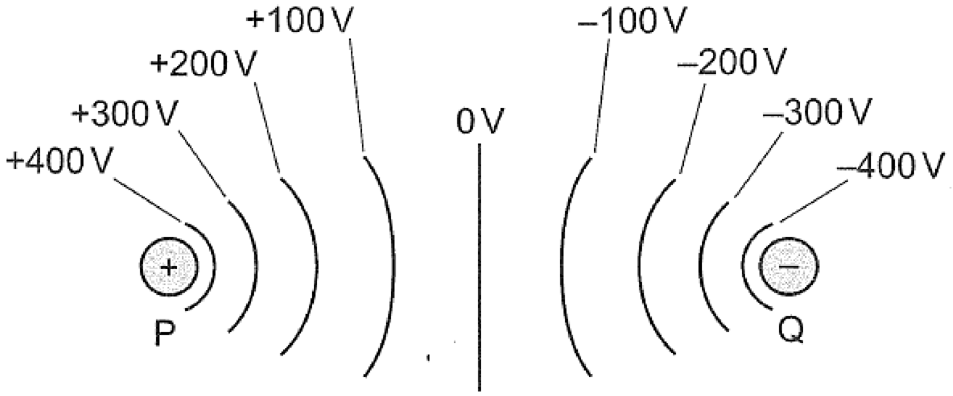




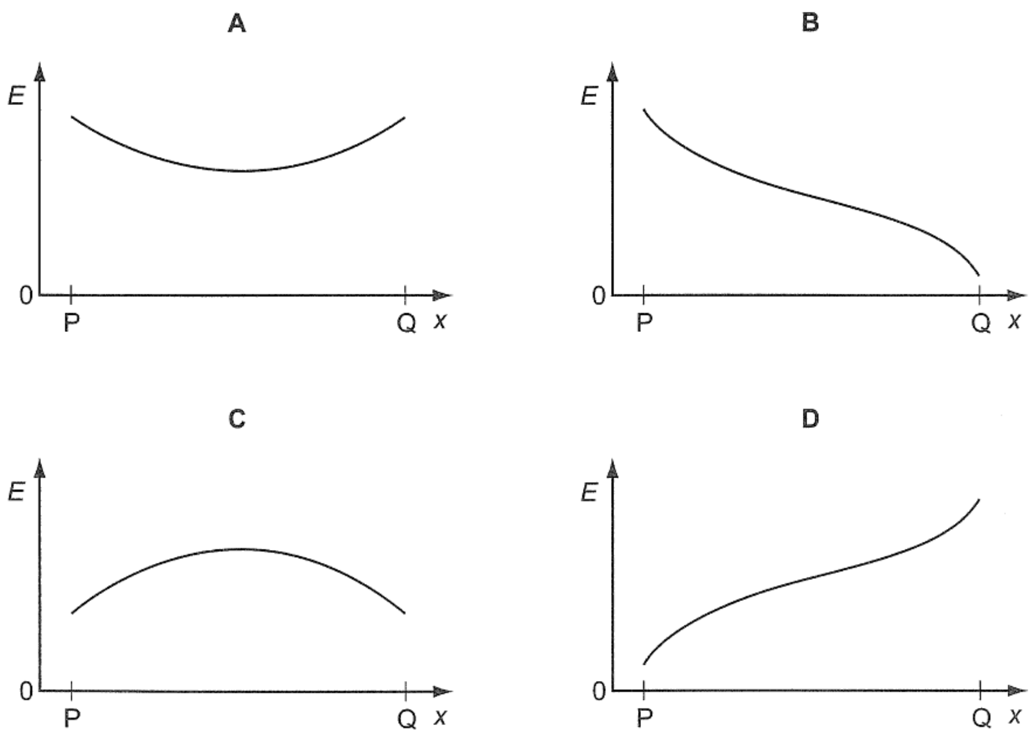
Example 18 (2007/H2/P1/Q25)

An object with a positive charge is placed at P and a similar object with a negative charge is placed at Q.

The diagram shows a number of lines along which the potential has a constant value.



Which graph shows the variation with distance *x* along line PQ of the electric field strength *E*?

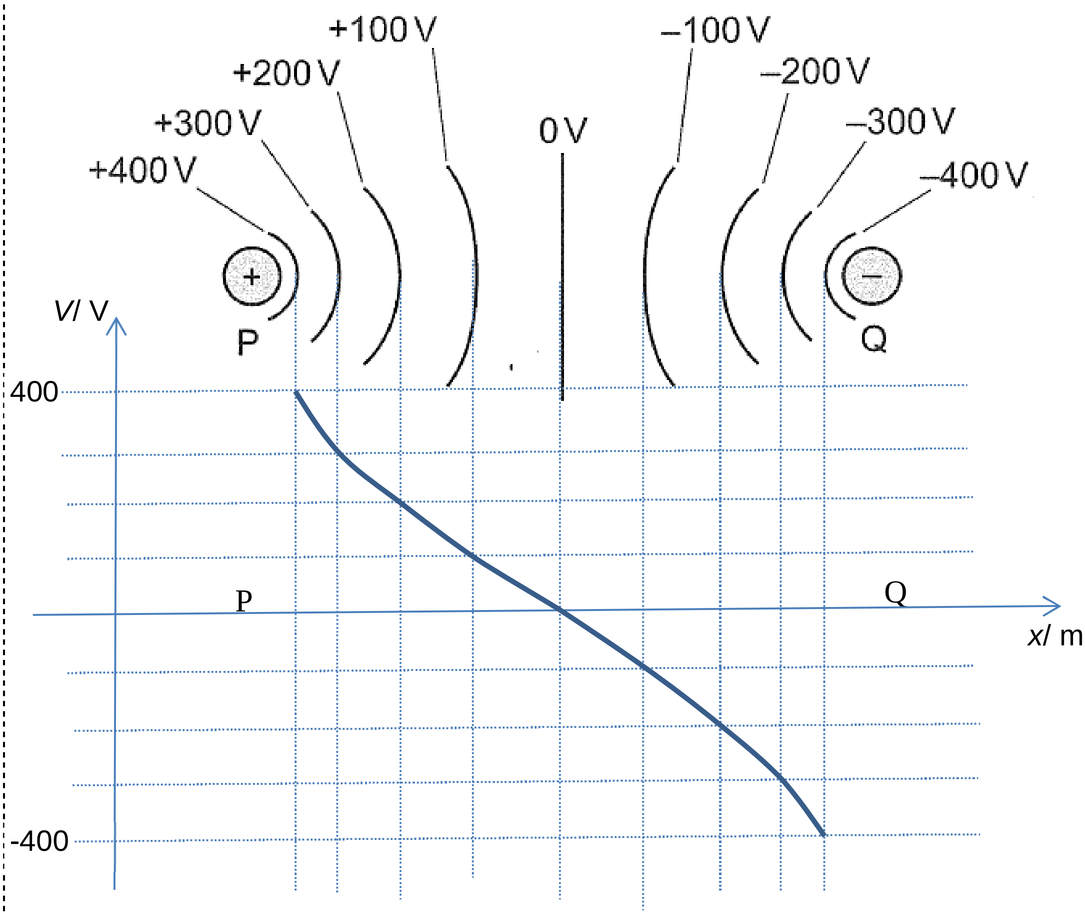


Solution (**Ans :**)

Method 1

By inspecting how the potential gradient $\left|\frac{dV}{dr}\right|$ changes along the line PQ.

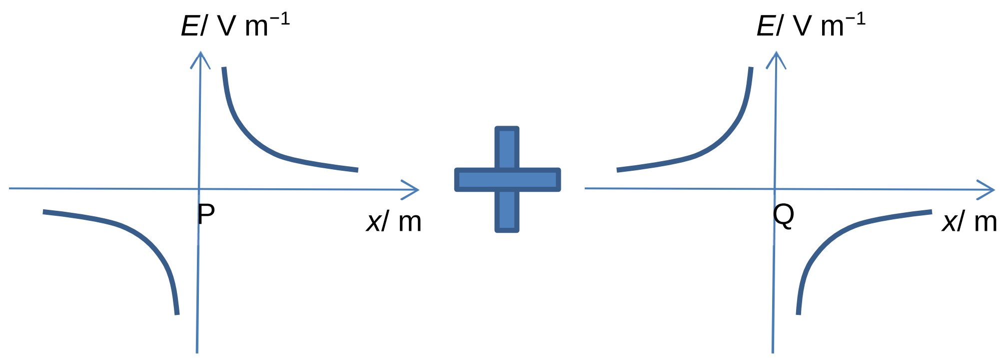
Method 2



Since $E=-\frac{dV}{dr}$, the negative gradient of the V against x graph will give the E against x graph.

Method 3

Superposition of individual E against x graph for P and Q, taking electric field strength to the right as positive.



Definition list

Electric field	Electric field is a region of space in which *a force acts on a stationary charge*. Direction of electric field is the direction of force on a *positive* charge.
Electric field strength	The electric field strength at a point is the electric force exerted *per unit* positive charge (on a test charge) placed at that point.
Coulomb's Law: Electrostatic force between point charges	Coulomb's law states that the electric force acting between any two point charges is directly proportional to the product of the charges and inversely proportional to the square of their separation. The direction of the force is along the line joining the two point charges.
Electric potential	The electric potential at a point is the work done per unit positive charge in bringing a small test charge from infinity to that point. Note: Infinity refers to any point where there is no electrical influence.
Electric potential gradient	The electric potential gradient at a point is numerically equal to the electric field strength at that point.

ANNEXES

(A) Charges

1) Electric charge is an important property of nature. There are 2 kinds of electric charges: **positive** and **negative** charges (Benjamin Franklin). Some particles, for example, the proton, have a positive charge. Some particles, like the electron, have a negative charge.
2) Objects that have electric charges exert **electrostatic** forces on each other.
3) Two **like** charges repel. Two **unlike** or **opposite** charges **attract**.
4) Electric charges may be transferred from one body to another via contact (rubbing using friction) or induction.
5) The very strong forces that exist between objects of opposite charge often cause a transfer of charge from one object to the other. This charge transfer discharges (neutralizes) the objects by bringing the opposite charges together. Examples:
   - discharges (often producing a spark) between the charged dome of a Van de Graaff generator and a second sphere brought close
   - lightning discharge between a thundercloud and ground
6) Amount of charge is measured in **Coulomb** (symbol: C) (Charles Coulomb). Physicists used to think that electrical charge could only be found in whole number multiples of a fundamental amount of charge, e = 1.602192 × 10^{-19} C.

   Charge on the proton = $+e$

   Charge on the electron = $-e$

   However, in the late 1960s, small particles called quarks were discovered with electric charges of either $+\frac{2}{3}e$ or $-\frac{1}{3}e$. These along with $-e$, seems to the basic quantity of electric charge.

(B) Formulae and relationship for electric fields

	Electric field described by		**Developed from**
Vectors	***Field strength*** ***E***	⟺ $F = qE$	***Electric force*** *F*
	due to a point charge: $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ ⇕ $E = -\frac{dV}{dr}$		between two point charges: $F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}$ ⇕ $F = -\frac{dU}{dr}$
Scalars	***Electric potential*** ***V***	⟺ $U = qV$	***Electric potential energy*** ***U***
	due to a point charge: $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$		between two point charges: $U = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r}$

(C) Derivation of electric potential of a point charge

From the definition of potential,

$$V = \frac{W}{q} = \frac{1}{q}\left(\int_{\infty}^{r} F_{ext} dr\right) = \frac{1}{q}\left(\int_{\infty}^{r} (-F)\ dr\right) = \frac{1}{q}\int_{\infty}^{r}\left(-\frac{qQ}{4\pi\varepsilon_0 r^2}\right)dr = \int_{\infty}^{r}\left(-\frac{Q}{4\pi\varepsilon_0 r^2}\right)dr$$

Hence, $V = \dfrac{Q}{4\pi\varepsilon_0 r}$

(D) Field strength and potential gradient

From the definition of potential,

$$V = \frac{W}{q} = \frac{1}{q}\left(\int_{\infty}^{r} F_{ext} dr\right) = \frac{1}{q}\left(\int_{\infty}^{r} (-F)\ dr\right)$$

Since the electric field is given by $E = \dfrac{F}{q}$, the expression becomes

$$V = \int_{\infty}^{r} (-E)\ dr$$

This implies that the electric field *E* can be obtained from the derivative of *V* with respect to *r*. Mathematically this is expressed as $E = -\dfrac{dV}{dr}$.

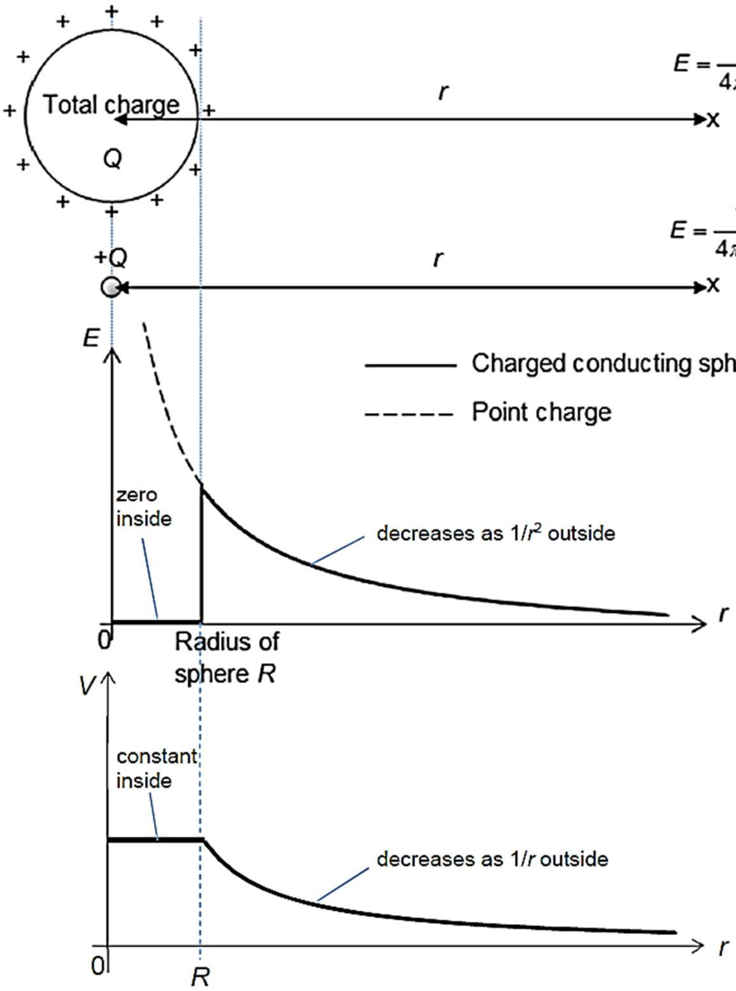
(E) Electric field strength and potential due to a charged conducting (hollow or solid) sphere

Every point inside a perfect conductor must have exactly the same potential. This is because the charge carriers are free to move about inside the material. If there were points of different potential the charge carriers would move in order to balance them out. A conductor forms an equipotential volume and has an equipotential surface. Conductors may be charged and yet still be an equipotential surface. Several interesting consequences of this:

- There can be no electric fields inside a perfect conductor. (Net field inside must be zero if all points in the conductor are at the same potential.)
- A charged conductor carries its charge in a thin layer on the surface. There can be no net charge in the volume of a conductor. (A charge inside the conductor would set up an *E* field and in response the charge carriers would move in order to cancel it.)
- Field lines touch conductors at 90°. (Field lines are always at 90° to equipotentials, so they must leave the surface of the conductor at 90°. If the field line was at an angle to the conductor's surface, then charge carriers would move about on the surface to cancel the effect.)

It can be shown that for a charged conducting sphere, the electric field **outside** the sphere behaves as though all its charges are concentrated at its centre. Therefore we can use the equation for point charges to determine the electric field strength outside the sphere.

Consider a charged conducting sphere with charge +*Q*, the graph showing the variation of *E* and *V* with distances *r*:



(F) Electric Permittivity

The permittivity of air at normal pressure is only about 1.0005 times that of a vacuum. Hence for most purposes we may assume the value of ε_0 for the permittivity of air.

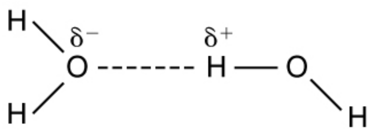
The permittivity of water is about eighty times that of a vacuum. When charges are placed in water, the force between them is very much reduced as compared to if they were situated the same distance apart in vacuum. This is the reason why common salt (sodium chloride) dissolves in water. The electric forces between the positive sodium ions and the negative chlorine ions, which keep the solid crystal structure in equilibrium are reduced considerably by the water and the solid structure collapses.

Permittivity values of some substances:
The permittivity of a substance is a characteristic which describes how it affects any electric field set up in it. A high permittivity tends to reduce any electric field present.

The permittivity of a material is usually given relative to that of free space, it is known as relative permittivity ε_r or dielectric constant κ. Permittivity ε is then calculated by multiplying ε_0 by this figure i.e., $\varepsilon = \varepsilon_0 \times \varepsilon_r$ or $\varepsilon = \varepsilon_0 \times \kappa$.

Substance	Dielectric constant κ or Relative Permittivity ε_r
vacuum	1.00000
air	1.00054
water	78
paper	3.5
porcelain	6.5
pyrex glass	4.5
polyethylene	2.3
titanium dioxide	100

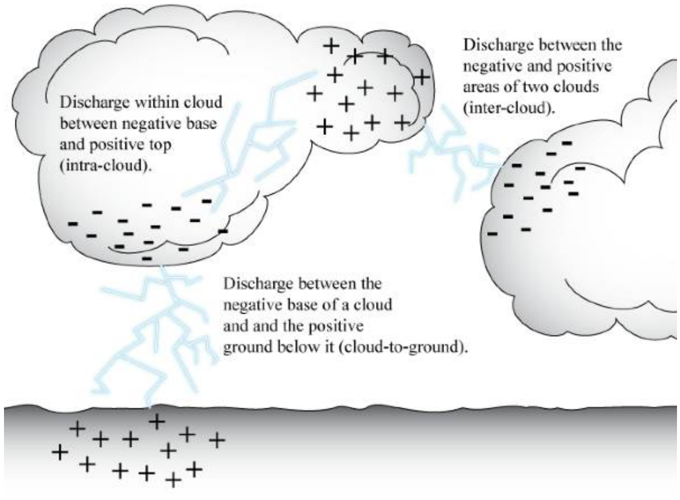
(G) Application 1: <u>Polar molecules</u>

	This schematic shows water (H_2O) as a polar molecule. Unequal sharing of electrons between the oxygen (O) and hydrogen (H) atoms leads to a net separation of positive and negative charge—forming a dipole. The symbols δ− and δ+ indicate that the oxygen side of the H_2O molecule tends to be more negative, while the hydrogen ends tend to be more positive. This leads to an attraction of opposite charges between molecules.

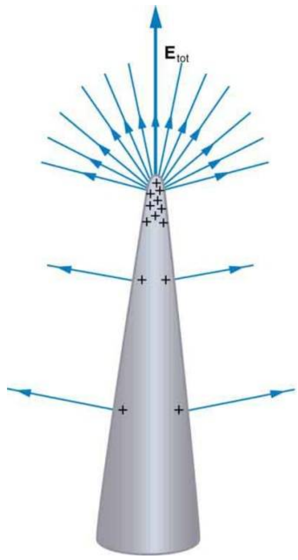
(H) Application 2: <u>Lightning rods</u>

According to NEA, Singapore has one of the highest occurrences of lightning activity in the world with 168 thunderstorm days per year.

Cause of lightning

Lightning is the result of the build-up of electrostatic charge in clouds. Positive and negative charges separate, negative usually towards the bottom of the cloud, while positive goes to the top. After a certain amount of time, the negative charge leaps, connecting with either another cloud or even the ground. Relating to electric fields, the stronger the field, the more likely lightning is attracted to the ground. If field lines are closer together, the field in that area is stronger and plausibility of a lightning strike is higher.	

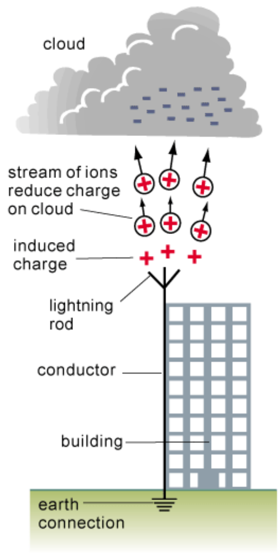
Lightning protection

<table>
<tr>
<td>
</td>
<td>A very pointed conductor has a large charge concentration at the point and is the prevalent shape of many existing lightning rods.

Lightning rods were originally developed by Benjamin Franklin.

Contrary to popular belief, the design of the lightning rod protects a building by

(a) attempting to prevent the lightning strike, and

(b) providing a conductive pathway to the earth.</td>
<td>
</td>
</tr>
<tr>
<td colspan="3">(a) If the structure that you are attempting to protect is out in an open, flat area, you often create a lightning protection system that uses a very tall lightning rod. This rod should be taller than the structure. If the area finds itself in a strong electric field, the tall rod can begin sending up positive streamers in an attempt to dissipate the electric field.

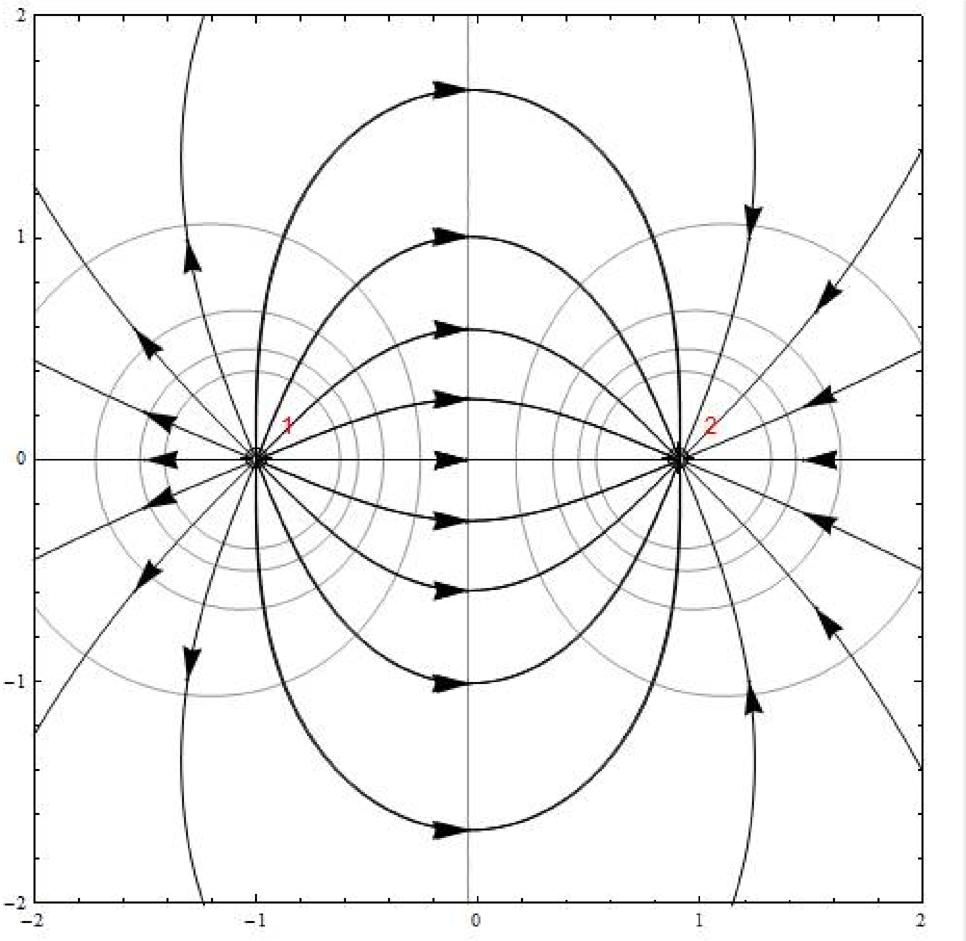
(b) While it is not a given that the rod will always conduct the lightning discharged in the immediate area, it does have a better possibility than the structure. This is because the lightning rod provides a low-resistance path to ground in an area that has the possibility to receive a lightning strike due to the strength of the electric field generated by the storm clouds.</td>
</tr>
</table>

~ THE END ~

Tutorial Questions

Concept of an electric field

1. The diagram below shows the electric field lines and equipotential lines of two electric charges of the same magnitude but opposite sign, namely +2.0 μC and –2.0 μC.



Sketch the electric field lines and equipotential lines if the negative charge is changed to (a) –1.0 μC, (b) –3.0 μC, (c) +1.0 μC and (b) +3.0 μC.

(You may use this demonstration located at http://demonstrations.wolfram.com/ElectricFieldLinesDueToACollectionOfPointCharges/ to help you.) [8]

2. Draw an analogy between certain qualitative and quantitative aspects of electric field and gravitational field. [5]

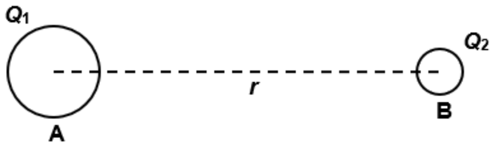
Point charges

3. (2012 P1 Q26)
If the distance from a charged particle is doubled, how do the new values of field strength and potential due to the charge compared to the original values?

	field strength	potential
A	1/4	1/4
B	1/4	1/2
C	1/2	1/4
D	1/2	1/2

4.

The diagram above shows two isolated charges Q_1 and Q_2 at points A and B separated by distance *r*.



(a) Write down the expression for the electric force between charges Q_1 and Q_2. [1]

(b) Write down the expression for the electric potential at point A due to charge Q_2. [1]

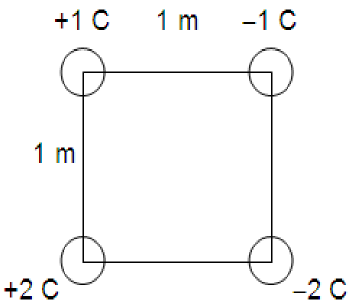
(c) Write down the expression for the electric potential energy of charge Q_1 and charge Q_2. [1]

(d) Write down the expression for the electric potential at point B due to charge Q_1. [1]

(e) Write down the expression for the electric potential at the mid-point between charges Q_1 and Q_2. [1]

(f) State an assumption you make in obtaining the above expressions. [1]

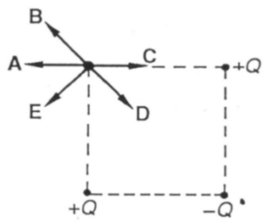
5. The diagram below shows four isolated charges placed at the corners of a square of side 1 m.



(a) Determine the electric field strength at the centre of the square.
[7.64×10^{10} N C^{-1}] [3]

(b) Determine the electric potential at the centre of the square.
[0 J C^{-1}] [1]

6. Point charges, each of magnitude Q, are placed at three corners of a square of side *r*.

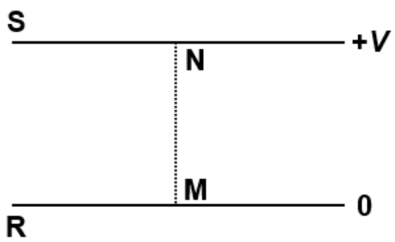


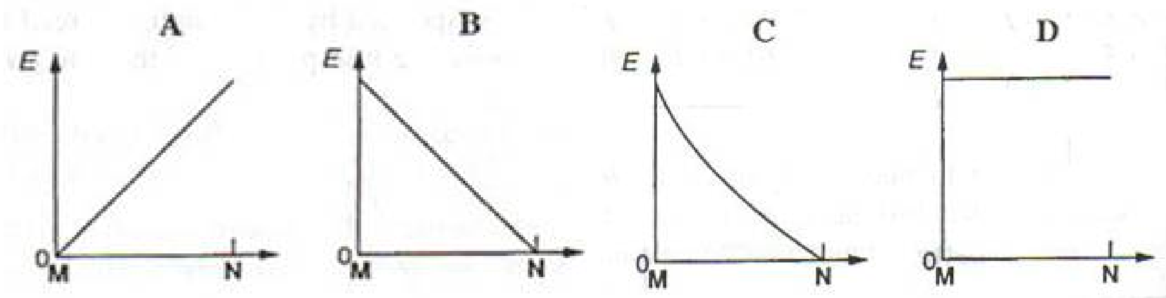
What is the magnitude and direction of the resultant electric field strength at the fourth corner if r = 1.00 m and Q = 1.00 C? For directions, choose from A to E. [3]
[8.22×10^9 N C^{-1}]

Uniform electric fields

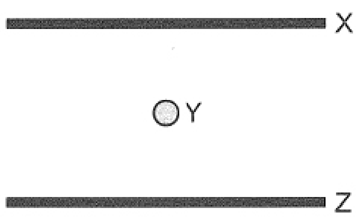
7. Two horizontal conducting plates **R** and **S** are a fixed distance apart. Plate **S** is at a potential +*V* with respect to plate **R**. **MN** is a line perpendicular to the plates.

Which graph shows how the magnitude *E* of the electric field strength varies along the line **MN**?





8. (2007 P1 Q24)
The diagram shows two charged horizontal metal plates X and Z. The magnitudes of the charged on the plates are equal. Between the plates, suspended stationary in the uniform electric field, is a charged sphere Y.



What could be the signs of the charges on the three components?

	plate X	sphere Y	plate Z
A	negative	positive	negative
B	negative	positive	positive
C	positive	positive	negative
D	positive	negative	positive

9. A pair of parallel plates is kept separated by 5.0 cm apart. A potential difference of 10 V is applied across the plates as shown in Fig 9.

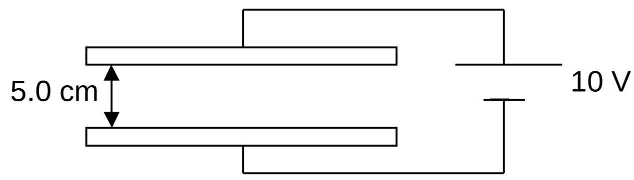


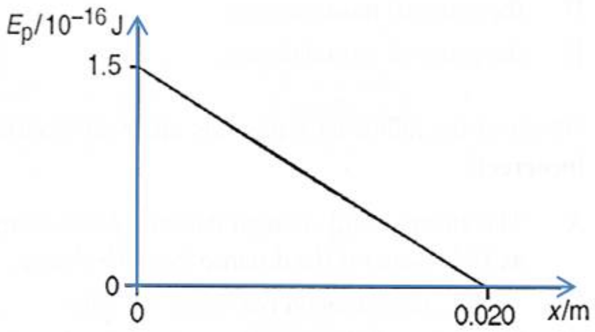
Fig. 9.

(a) Determine the electric field strength between the plates. [200 V m^{-1}] [2]

(b) Determine the electric force exerted on a – 2.0 μC placed in the electric field between the plates. [4.0×10^{-4} N] [2]

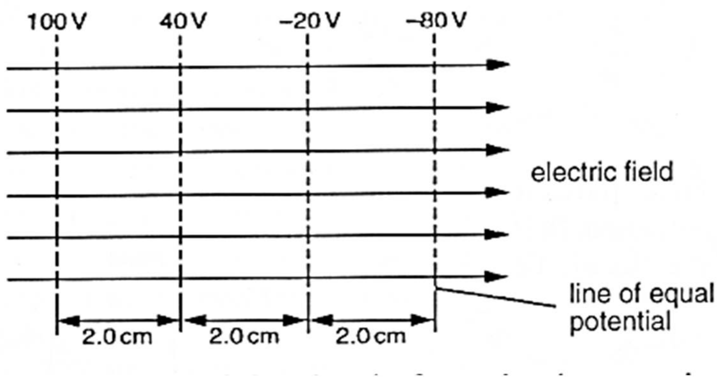
(c) In the Fig. 9., indicate the direction of the electric force acting on the charge. [1]

10. Two charged plates are 0.020 m apart, producing a uniform electric field. The potential energy E_p of an electron in the field varies with displacement x from one of the plates as shown.



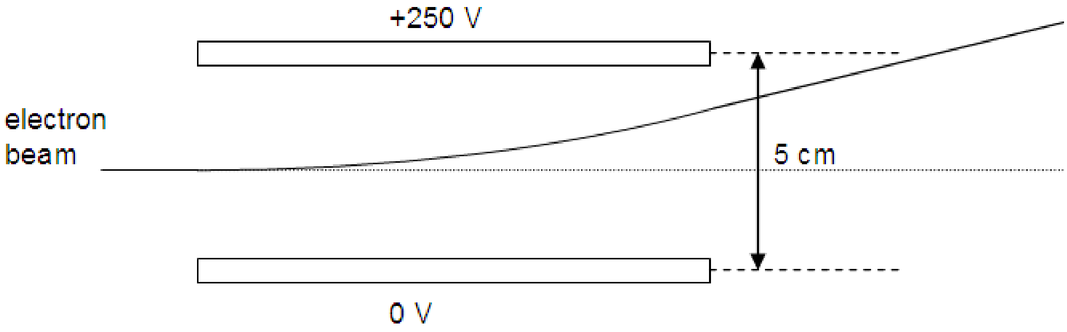
Determine the magnitude of the force on the electron. [7.5×10^{-15} N] [2]

11. The diagram shows a uniform electric field in which the lines of equal potential are spaced 2.0 cm apart.



Determine the magnitude and direction of the electric force that is exerted on a charge of +5 μC when placed in the field. [1.5×10^{-2} N] [2]

12.



An electron moving horizontally with an initial velocity of 2.0×10^7 m s^{-1} enters a vertical electric field set up by two parallel square metal plates, each of side 10 cm. The plate separation is 5.0 cm and a p.d. of 250 V is applied across the plates.

(a) Calculate the time interval that the electron is inside the electric field [5.0×10^{-9} s] [2]

(b) Calculate the acceleration experienced by the electron while travelling inside the electric field. [8.78×10^{14} m s^{-2}] [3]

(c) Calculate the vertical deflection of the electron from its original path as it leaves the electric field. [2]
[1.1 cm]

(d) Indicate on the diagram the direction of the electric field strength between the plates and the electric force acting on the electron when it is between the plates. [2]

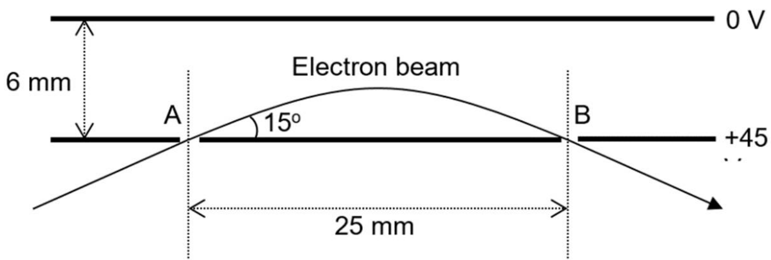
(e) Taking *x* as the displacement measured from the bottom plate to the top plate, draw graphs of

(i) force *F* against *x* [2]

(ii) potential *V* against *x* and [2]

(iii) field strength *E* against *x.* [2]

13. The figure below shows the principle of a type of velocity selector for electrons. Two parallel metal plates are arranged 6.0 mm apart in a vacuum. The lower plate is at a potential of +45 V relative to the upper and has two parallel slits A and B in it, 25 mm apart. A collimated beam containing electrons of different speeds is directed towards slit A at an angle of 15° with the plate. Electrons emerging from B have the same velocity.

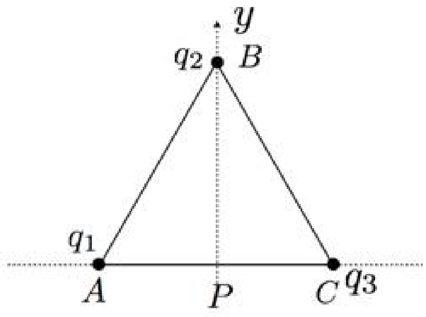


(a) What is the magnitude and direction of the acceleration of an electron while it is between the plates? [1.32×10^{15} m s^{-2} downwards] [4]

(b) Find the speed of the electrons which come out of slit B. [8.11×10^6 m s^{-1}] [4]

Electric potential energy

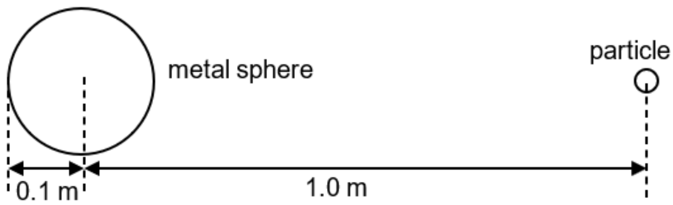
14. In the figure, a charge of $q_1 = -1.00 \times 10^{-6}$ C is located at corner **A** of an equilateral triangle. A charge $q_2 = -4.00 \times 10^{-6}$ C is located at corner **B** and a charge of $q_3 = -7.00 \times 10^{-6}$ C is located at corner **C**. The distance **AB** $= 44.0$ cm.



What is the electric potential energy of this system of three charges?

[0.798 J] [2]

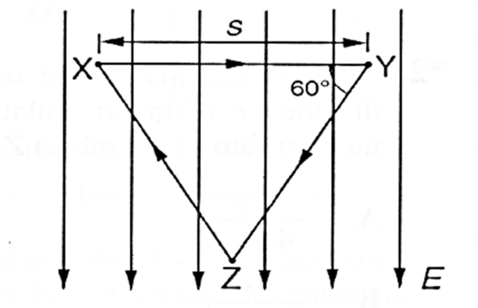
15. A metal sphere of radius 0.1 m carries a positive charge of 1.0×10^{-4} C. A particle of mass 2.0×10^{-5} kg carrying a negative charge of 1.5×10^{-10} C is released from rest at a distance of 1.0 m from the centre of the sphere.



Calculate the velocity of the particle when it strikes the surface of the sphere. Neglect gravitational effect.

[11 m s^{-1}] [3]

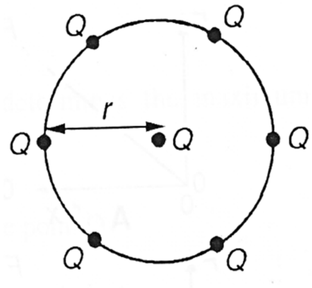
16. The diagram below shows three points X, Y and Z forming an equilateral triangle of side *s* in a uniform electric field of strength *E*. A unit positive test charge is moved from X to Y, Y to Z and Z back to X.



[3]

Calculate the work done **against** electric force in moving the charge along the paths XY, YZ and ZX. [0, −*Es* sin60°, *Es* sin60°]

17. A point charge is surrounded by six identical charges at distance *r* as shown in the diagram.



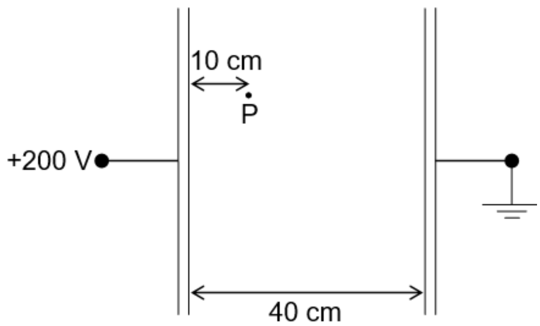
Determine the work done by the forces of electrostatic repulsion when the point charge at the centre is moved to infinity. $\left[\dfrac{6Q^2}{4\pi\varepsilon_0 r}\right]$ [2]

Sample Examination Questions

18. (2012 RJC P3 Q3)

(a) A student states that "*the electrostatic force acting on a charged particle placed in an electric field always acts in the direction of the field*". Comment on the statement. [2]

(b) Two parallel, infinitely-long vertical plates are placed 40 cm apart in a vacuum, with a potential difference of 200 V applied across them, as shown in the figure.



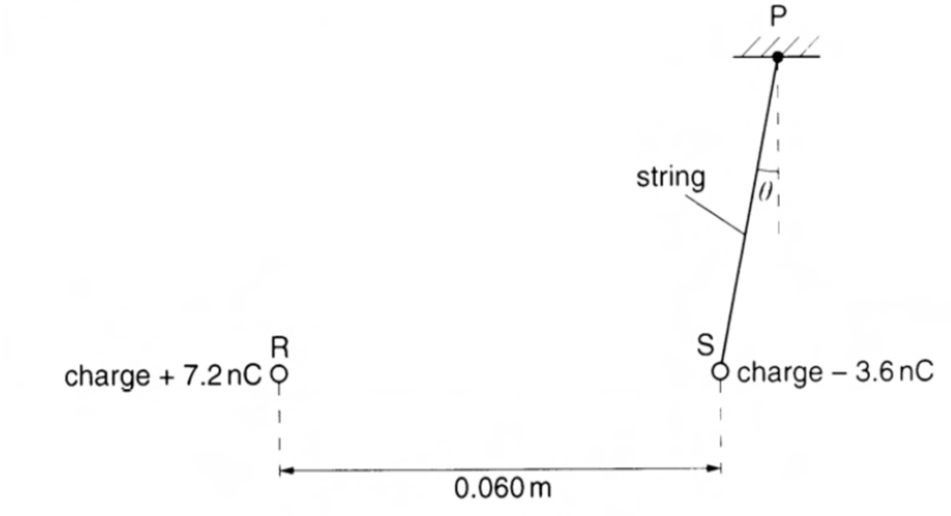
An oil drop of mass 1.4×10^{-15} kg is released from rest at P, 10 cm away from the plate of higher potential. The oil drop carries a charge of $+1.5\times10^{-17}$ C.

(i) Calculate the angle to the horizontal at which the oil drop will accelerate from P. [61.4º] [3]

(ii) The vertical distance travelled by the oil drop when it hits the plate is 55 cm. By considering the change in potential energies, calculate the kinetic energy of the oil drop when it hits the plate. [9.80 × 10^{-15} J] [3]

19. (2011 P2 Q4)

A small metal sphere S is supported by an insulating string that is fixed at point P, as shown in the figure.



Sphere S has a weight of 0.49×10^{-3} N and charge −3.6 nC. Another metal sphere R with a charge of +7.2 nC is placed 0.060 m from S. The diameter of the spheres is negligible compared to the distance between them.
A line joining R and S is horizontal. The string makes an angle θ with the vertical.

(a) **(i)** On the figure, draw electric field lines to represent the electric field in the region between R and S. [3]

(ii) Calculate the magnitude of the electric field strength at the mid-point between R and S. [1.1×10^5 N C^{-1}] [2]

(iii) Show that the electric force acting on S is 6.5×10^{-5} N. [2]

(b) Using resolution of forces or a vector triangle, determine the angle θ and the tension in the string. [7.5°, 4.9×10^{-4} N] [4]

(c) The horizontal distance from R towards S is x. Sketch the variation with x of the potential V. [2]

20. (2014 P3 Q6)
Two charged solid metal spheres A and B are situated in a vacuum. Their centres are separated by a distance of 30.0 cm, as illustrated in Fig. 20.1. The diagram is not to scale.

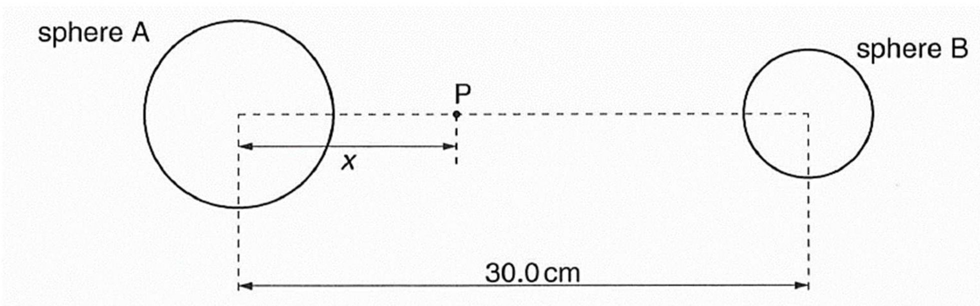


Fig. 20.1

Point P is a point on the line joining the centres of the two spheres. Point P is a distance *x* from the centre of sphere A.

The variation with distance *x* of the electric field strength *E* at point P is shown in Fig. 20.2.

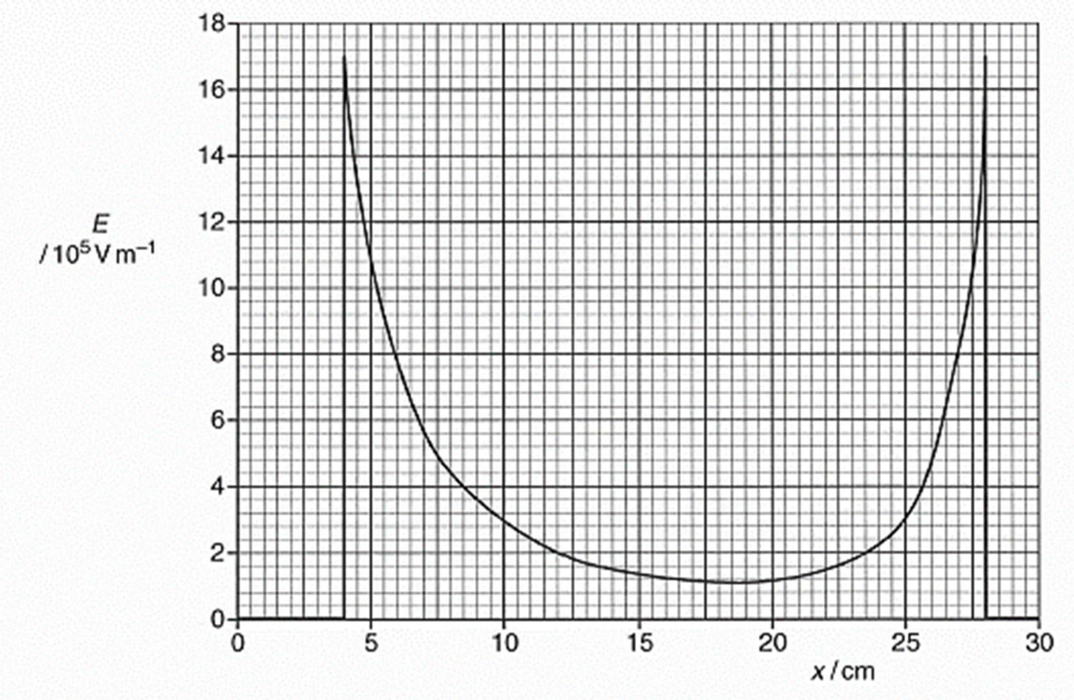


Fig. 20.2

(a) Suggest why the electric field strength is zero for two regions of *x*. [1]

(b) Use Fig. 20.2 to

(i) determine the radius of each sphere, [2]

(ii) state and explain whether the spheres have charges of the same, or opposite, sign. [2]

(c) A lithium-7 $\left({}^{7}_{3}\text{Li}\right)$ nucleus moves along the line joining the centres of the two spheres.

(i) Estimate the energy gained by this nucleus as it moves from point P where x = 16.0 cm to the point P where x = 21.0 cm.
[2.88×10^{-15} J][5]

(ii) Calculate the acceleration of the nucleus at point P where x = 25.0 cm. [1.24×10^{13} m s^{-2}] [2]

(iii) The nucleus is at rest at point P where x = 4.0 cm.
Describe qualitatively the variation with x of the acceleration of the nucleus for x = 4.0 cm to x = 28.0 cm. [3]

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