2021 NYJC JC2 CT2 9649/2 Marking Guide

Qn		Key Points	Marks	Guidance
1	Source: NJC Promo 2025 Question 4 $\mathrm{f}(x,y)=x^2y+xy^3-3xy,\ \mathrm{f}(1,2)=4.$ $\mathrm{f}_x(x,y)=2xy+y^3-3y,\ \mathrm{f}_x(1,2)=6.$ $\mathrm{f}_y(x,y)=x^2+3xy^2-3x,\ \mathrm{f}_y(1,2)=10.$ $\mathrm{f}_{xx}(x,y)=2y,\ \mathrm{f}_{xx}(1,2)=4.$ $\mathrm{f}_{xy}(x,y)=2x+3y^2-3,\ \mathrm{f}_{xy}(1,2)=11.$ $\mathrm{f}_{yx}(x,y)=2x+3y^2-3,\ \mathrm{f}_{yx}(1,2)=11.$ $\mathrm{f}_{yy}(x,y)=6xy,\ \mathrm{f}_{yy}(1,2)=12.$ $Q(x,y)$ $=4+\mathrm{f}_x(1,2)(x-1)+\mathrm{f}_y(1,2)(y-2)$ $\quad+\frac{1}{2}\mathrm{f}_{xx}(1,2)(x-1)^2+\mathrm{f}_{xy}(1,2)(x-1)(y-2)+\frac{1}{2}\mathrm{f}_{yy}(1,2)(y-2)^2$ $=4+6(x-1)+10(y-2)$ $\quad+\frac{1}{2}(4)(x-1)^2+11(x-1)(y-2)+\frac{1}{2}(12)(y-2)^2$ $=4+\begin{pmatrix}6\\10\end{pmatrix}^T\begin{pmatrix}x-1\\y-2\end{pmatrix}+\frac{1}{2}\begin{pmatrix}x-1\\y-2\end{pmatrix}^T\begin{pmatrix}4&11\\11&12\end{pmatrix}\begin{pmatrix}x-1\\y-2\end{pmatrix}$ $=4+\begin{pmatrix}6\\10\end{pmatrix}^T\left[\begin{pmatrix}x\\y\end{pmatrix}-\begin{pmatrix}1\\2\end{pmatrix}\right]+\frac{1}{2}\left[\begin{pmatrix}x\\y\end{pmatrix}-\begin{pmatrix}1\\2\end{pmatrix}\right]^T\begin{pmatrix}4&11\\11&12\end{pmatrix}\left[\begin{pmatrix}x\\y\end{pmatrix}-\begin{pmatrix}1\\2\end{pmatrix}\right]$ where $k=4$, $\mathbf{w}=\begin{pmatrix}6\\10\end{pmatrix}$, $\mathbf{a}=\begin{pmatrix}1\\2\end{pmatrix}$ and $\mathbf{H}=\begin{pmatrix}4&11\\11&12\end{pmatrix}$.			

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	$\mathbf{w}=\begin{pmatrix}6\\10\end{pmatrix}=\begin{pmatrix}\mathrm{f}_x(1,2)\\\mathrm{f}_y(1,2)\end{pmatrix}=\nabla\mathrm{f}(1,2)=\nabla\mathrm{f}(\mathbf{a})$. $\therefore$ **w** is the gradient of f at **a**, i.e., $\nabla\mathrm{f}(\mathbf{a})$. Note that $\mathbf{a}\in\mathbb{R}^2$, not $\mathbb{R}^3$.			
2(i)	$\mathrm{f}(0.5)\approx 0.035533>0$ $\mathrm{f}(1)=-3<0$ $\therefore$ there is a root in $(0.5,1)$. $\mathrm{f}(13)\approx 3.2990>0$ $\therefore$ there is a root in $(1,13)$.			
2(ii)	$\mathrm{f}(12)=-0.2842$ $\therefore$ there is a root in $(12,13)$. $x_1=\dfrac{13\lvert 0.2842\rvert+12\lvert 3.2990\rvert}{\lvert 0.2842\rvert+\lvert 3.2990\rvert}\approx 12.0793$ $\mathrm{f}(12.0793)\approx -0.0157<0$ $\therefore$ there is a root in $(12.0793,13)$. $x_2=\dfrac{13\lvert 0.0157\rvert+12.0793\lvert 3.2990\rvert}{\lvert 0.0157\rvert+\lvert 3.2990\rvert}\approx 12.0836$ Note that $\mathrm{f}(12.0793)\approx -0.0157<0$ and $\mathrm{f}(12.085)\approx 0.0036>0$ $\therefore$ there is a root in $(12.0793,12.085)$. $\therefore$ root is 12.08, correct to 2 decimal places.			
2(iii)	$\mathrm{f}'(x)=3x^{\frac{1}{2}}-7-x^{-\frac{3}{2}}$ $x_{n+1}=x_n-\dfrac{2x_n^{\frac{3}{2}}-7x_n+2x_n^{-\frac{1}{2}}}{3x_n^{\frac{1}{2}}-7-x_n^{-\frac{3}{2}}}$ Using GC,			

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<tr><th>Qn</th><th></th><th>Key Points</th><th>Marks</th><th>Guidance</th></tr>
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n	x_n
1	0.1
2	0.2509
3	0.4358
4	0.5010
5	0.5046

$\mathrm{f}\left(0.495\right) \approx 0.0742 > 0$
$\mathrm{f}\left(0.505\right) = -0.0028 < 0$
$\therefore$ there is a root in $\left(0.495, 0.505\right)$.
$\therefore$ root is 0.50, correct to 2 decimal places.
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<tr><td>2(iv)</td><td>
For the recurrence to work, $x_{n+1} > 0$

$$x_n - \frac{2x_n^{\frac{3}{2}} - 7x_n + 2x_n^{-\frac{1}{2}}}{3x_n^{\frac{1}{2}} - 7 - x_n^{-\frac{3}{2}}} > 0$$

$$\frac{x^{\frac{3}{2}} - 3x^{-\frac{1}{2}}}{3x_n^{\frac{1}{2}} - 7 - x_n^{-\frac{3}{2}}} > 0$$

Using GC, $0 < x < \sqrt{3}$ or $x > 5.5636$

To solve for a, $x^{\frac{3}{2}} - 3x^{-\frac{1}{2}} = 0$

$$x = \sqrt{3} \approx 1.7320 = 1.73 \text{ (3sf)}$$

To solve for b,
$\mathrm{f}'\left(x\right) = 3x^{\frac{1}{2}} - 7 - x^{-\frac{3}{2}} = 0$
Using GC $x \approx 5.5636 = 5.56$ (3sf)
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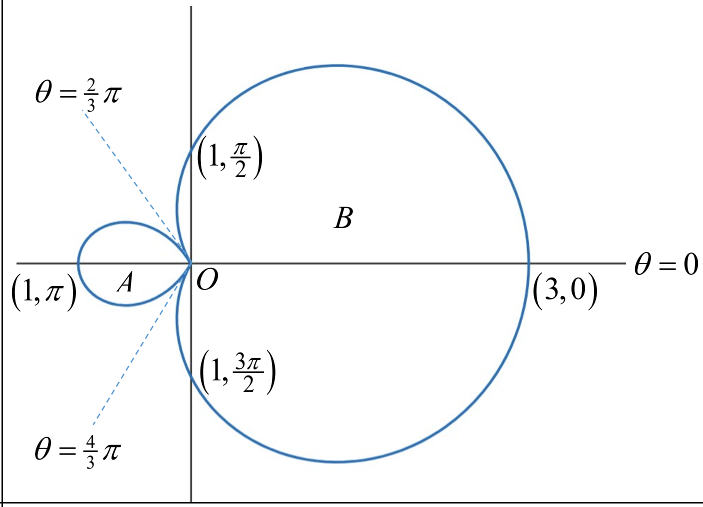
Qn		Key Points	Marks	Guidance
Q3 (a) [3]	Source: RI Promo 2025 Qn 10 			
(b) [4]	By symmetry, $k\int_{2\pi/3}^{\pi}\sqrt{(1+2\cos\theta)^2+(2\sin\theta)^2}\ \mathrm{d}\theta=\int_0^{2\pi/3}\sqrt{(1+2\cos\theta)^2+(2\sin\theta)^2}\ \mathrm{d}\theta$ $\therefore k=\dfrac{\int_0^{2\pi/3}\sqrt{(1+2\cos\theta)^2+(2\sin\theta)^2}\ \mathrm{d}\theta}{\int_{2\pi/3}^{\pi}\sqrt{(1+2\cos\theta)^2+(2\sin\theta)^2}\ \mathrm{d}\theta}$ $=3.98$ (3 sf)			
(c) [5]	Since the flower bed is symmetrical about $\theta=0$, the optimal position lies on $\theta=0$. The mid-point of $(1,\pi)$ and $(3,0)$ is $Q(1,0)$, where the sprinkler should be positioned. For any point P on curve C, by cosine rule,			

Qn		Key Points	Marks	Guidance
	$QP^2 = r^2 + 1^2 - 2r\cos\theta$ $= (1+2\cos\theta)^2 + 1 - 2\cos\theta\lvert 1+2\cos\theta\rvert$ $= \begin{cases} (1+2\cos\theta)^2 + 1 + 2\cos\theta(1+2\cos\theta), & \text{if } P \text{ lies on small loop} \\ (1+2\cos\theta)^2 + 1 - 2\cos\theta(1+2\cos\theta), & \text{if } P \text{ lies on big loop} \end{cases}$ $= \begin{cases} 2+8\cos^2\theta + 6\cos\theta, & \frac{2}{3}\pi \le \theta \le \frac{4}{3}\pi \\ 2+2\cos\theta, & \text{otherwise} \end{cases}$ $= \begin{cases} 8\left(\cos\theta + \frac{3}{8}\right)^2 + \frac{7}{8}, & \frac{2}{3}\pi \le \theta \le \frac{4}{3}\pi \\ 2+2\cos\theta, & \text{otherwise} \end{cases}$ If P lies on the small loop, QP is largest when $\theta = \pi$, and $AP^2 \le 8\left(-1+\frac{3}{8}\right)^2 + \frac{7}{8} = 4$. If P lies on the big loop, QP is largest when $\theta = 0$, and $AP^2 \le 2+2 = 4$ In both cases, $QP \le 2$. Thus minimum $a = 2$.			
4	$(1+z)^{2n} + (1-z)^{2n} = 0$ $\left(\frac{1+z}{1-z}\right)^{2n} = -1 = \mathrm{e}^{\mathrm{i}(2k+1)\pi}$ $\frac{1+z}{1-z} = \mathrm{e}^{\mathrm{i}\frac{(2k+1)\pi}{2n}}$ where $k = 0,1,\ldots,2n-1$ $1+z = \mathrm{e}^{\mathrm{i}\frac{(2k+1)\pi}{n}} - z\mathrm{e}^{\mathrm{i}\frac{(2k+1)\pi}{2n}}$ $z = \frac{\mathrm{e}^{\mathrm{i}\frac{(2k+1)\pi}{2n}} - 1}{\mathrm{e}^{\mathrm{i}\frac{(2k+1)\pi}{2n}} + 1}$ $= \frac{\mathrm{e}^{\mathrm{i}\frac{(2k+1)\pi}{4n}}\left(\mathrm{e}^{\mathrm{i}\frac{(2k+1)\pi}{4n}} - \mathrm{e}^{-\mathrm{i}\frac{(2k+1)\pi}{4n}}\right)}{\mathrm{e}^{\mathrm{i}\frac{(2k+1)\pi}{4n}}\left(\mathrm{e}^{\mathrm{i}\frac{(2k+1)\pi}{4n}} + \mathrm{e}^{-\mathrm{i}\frac{(2k+1)\pi}{4n}}\right)}$			

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	$=\dfrac{2\mathrm{i}\sin\dfrac{(2k+1)\pi}{4n}}{2\cos\dfrac{(2k+1)\pi}{4n}}$ $=\mathrm{i}\tan\dfrac{(2k+1)\pi}{4n}, k=0,1,2,\ldots,2n-1$			
	$\tan\dfrac{\left[2(2n-1-k)+1\right]\pi}{4n}=\tan\left[\pi-\dfrac{(2k+1)\pi}{4n}\right]=-\tan\dfrac{(2k+1)\pi}{4n}$. Thus $\tan\dfrac{(2k+1)\pi}{4n}=-\tan\dfrac{\left[2(2n-1-k)+1\right]\pi}{4n}$.			
	By above result and the factor theorem, $(1+z)^{2n}+(1-z)^{2n}$ $=2\prod_{k=0}^{2n-1}\left(z-\mathrm{i}\tan\dfrac{(2k+1)\pi}{4n}\right)$ $=2\left[\prod_{k=0}^{n-1}\left(z-\mathrm{i}\tan\dfrac{(2k+1)\pi}{4n}\right)\right]\left[\prod_{k=n}^{2n-1}\left(z-\mathrm{i}\tan\dfrac{(2k+1)\pi}{4n}\right)\right]$ $=2\left[\prod_{k=0}^{n-1}\left(z-\mathrm{i}\tan\dfrac{(2k+1)\pi}{4n}\right)\right]\left[\prod_{k=n}^{2n-1}\left(z+\mathrm{i}\tan\dfrac{\left[2(2n-1-k)+1\right]\pi}{4n}\right)\right]$ $=2\left[\prod_{k=0}^{n-1}\left(z-\mathrm{i}\tan\dfrac{(2k+1)\pi}{4n}\right)\right]\left[\prod_{k=0}^{n-1}\left(z+\mathrm{i}\tan\dfrac{(2k+1)\pi}{4n}\right)\right]$ $=2\prod_{k=0}^{n-1}\left\{\left(z-\mathrm{i}\tan\dfrac{(2k+1)\pi}{4n}\right)\left(z+\mathrm{i}\tan\dfrac{(2k+1)\pi}{4n}\right)\right\}$ $=2\prod_{k=0}^{n-1}\left\{z^2+\tan^2\left[\dfrac{(2k+1)\pi}{4n}\right]\right\}$			

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	$(1+z)^{2n}+(1-z)^{2n}$ $=1+\binom{2n}{1}z+\binom{2n}{2}z^2+\binom{2n}{3}z^3+\binom{2n}{4}z^4+\ldots+\binom{2n}{2n-1}z^{2n-1}+z^{2n}$ $+1-\binom{2n}{1}z+\binom{2n}{2}z^2-\binom{2n}{3}z^3+\binom{2n}{4}z^4-\ldots-\binom{2n}{2n-1}z^{2n-1}+z^{2n}$ $=2+2\binom{2n}{2}z^2+2\binom{2n}{4}z^4+\ldots+2\binom{2n}{2n-2}z^{2n-2}+2z^{2n}$			
	On the other hand, $2\prod_{k=0}^{n-1}\left\{z^2+\tan^2\left[\frac{(2k+1)\pi}{4n}\right]\right\}$ $=2\left(z^2+\tan^2\frac{\pi}{4n}\right)\left(z^2+\tan^2\frac{3\pi}{4n}\right)\left(z^2+\tan^2\frac{5\pi}{4n}\right)\cdots\left(z^2+\tan^2\frac{(2n-1)\pi}{4n}\right).$ Equating coefficients of z^{2n-2}, $2\binom{2n}{2n-2}=2\left(\tan^2\frac{\pi}{4n}+\tan^2\frac{3\pi}{4n}+\tan^2\frac{5\pi}{4n}+\ldots+\tan^2\frac{(2n-1)\pi}{4n}\right)$ $\binom{2n}{2}=\sum_{k=1}^{n}\tan^2\frac{(2k-1)\pi}{4n}$ $\frac{(2n)(2n-1)}{2!}=\sum_{k=1}^{n}\left[\sec^2\frac{(2k-1)\pi}{4n}-1\right]$ $\sum_{k=1}^{n}\left[\sec^2\frac{(2k-1)\pi}{4n}\right]-n=n(2n-1)$ $\sum_{k=1}^{n}\left[\sec^2\frac{(2k-1)\pi}{4n}\right]=2n^2$			

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<tr><td>5</td><td>(ai)</td><td>Let μ mm be the population mean distance.

A 99% confidence interval for μ is $\left(\overline{x} \pm 2.5758\left(\frac{0.43}{\sqrt{n}}\right)\right)$

We require $2 \times 2.5758\left(\frac{0.43}{\sqrt{n}}\right) \le 0.60$

$n \ge 13.631$
Least value of $n = 14$</td><td></td><td></td><td></td></tr>
<tr><td></td><td>(aii)</td><td>Since population standard deviation is known and sample size is small, we need to assume that the distance between pairs of holes drilled by machine is normally distributed.</td><td></td><td></td><td></td></tr>
<tr><td></td><td>(b)</td><td>Let p be the population proportion of parked cars fitted with an anti-theft device.
Let $\hat{p} = \frac{72}{248} \approx 0.29032$ be the unbiased estimate for p
A 95% confidence interval for p is
$\left(0.29032 \pm 1.9600\sqrt{\frac{(0.29032)(1-0.29032)}{248}}\right)$
$\Rightarrow (0.23383,\ 0.34682)$
$\Rightarrow (0.234,\ 0.347)$ (3 d.p)
Only cars in the car park were sampled which may not constitute a random sample.</td><td></td><td></td><td></td></tr>
<tr><td>6</td><td colspan="2">$\mathrm{f}(x) = \frac{a}{2^x} = a2^{-x} = a\mathrm{e}^{\ln 2^{-x}} = a\mathrm{e}^{-x\ln 2} = \ln 2\mathrm{e}^{-x\ln 2}$.
Thus $a = \ln 2$.
$Y = 2^X$
$x \ge 0 \Rightarrow y = 2^x \ge 1$</td><td></td><td></td><td></td></tr>
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	$P(Y \le y) = P(2^X \le y)$ $= P(X \ln 2 \le \ln y)$ $= P(X \le \frac{\ln y}{\ln 2})$ $= 1 - e^{-\ln 2(\frac{\ln y}{\ln 2})} = 1 - e^{-\ln y} = 1 - \frac{1}{y}$ For $y \ge 1$ $\mathrm{f}_Y(y) = \frac{d}{dy}(1 - \frac{1}{y}) = \frac{1}{y^2}$ $\mathrm{f}(y) = \begin{cases} \frac{1}{y^2}, & y \ge 1 \\ 0, & \text{o.w} \end{cases}$			
7(a) (i)	$\int_0^a x^{i-1}\,\mathrm{d}x = \left[\frac{x^i}{i}\right]_0^a = \frac{a^i}{i}$			
7(a) (ii)	$\mathrm{E}\left(\frac{1}{X}\right) = \sum_{i=1}^{\infty} \frac{1}{i}\mathrm{P}(X = i) = \sum_{i=1}^{\infty} \frac{1}{i} q^{i-1} p$ $= \sum_{i=1}^{\infty} \frac{p}{q}\frac{q^i}{i}$ $= \frac{p}{q}\sum_{i=1}^{\infty}\left(\int_0^q y^{i-1}\,\mathrm{d}y\right)$ $= \frac{p}{q}\int_0^q \left(\sum_{i=1}^{\infty} y^{i-1}\right)\mathrm{d}y$ $= \frac{p}{q}\int_0^q \frac{1}{1-y}\,\mathrm{d}y$ sum to infinity of GP $= \frac{p}{q}\left[-\ln\lvert 1-y\rvert\right]_0^q$ $= \frac{-p\ln(1-q)}{q} = \frac{-p\ln(p)}{1-p}$	$\int_0^q \sum_{i=1}^{\infty} y^{i-1}\,\mathrm{d}y$ $= \int_0^q y^0 + y^1 + \ldots\,\mathrm{d}y$ $= \int_0^q y^0\,\mathrm{d}y + \int_0^q y^1\,\mathrm{d}y + \ldots$ $= \sum_{i=1}^{\infty}\left(\int_0^q y^{i-1}\,\mathrm{d}y\right)$ Note that since q is a probability, $0 < \frac{q^i}{i} < 1$ for all i. Thus $0 < \int_0^q y^{i-1}\,\mathrm{d}y < 1$. Hence $\lvert y\rvert < 1$ as otherwise the integral >1 as i goes to infinity.		

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7(b)	Let $M = \min\left(X_1, X_2, \ldots, X_n\right)$. $F_M(m) = \mathrm{P}(M \le m)$ $= 1 - \mathrm{P}(M > m)$ $= 1 - \mathrm{P}(X_1 > m, X_2 > m, \ldots, X_n > m)$ $= 1 - \mathrm{P}(X_1 > m)\mathrm{P}(X_2 > m)\ldots\mathrm{P}(X_n > m)$ $= 1 - \left(q^m\right)^n$ $= 1 - q^{mn}$ $= 1 - \left((1-p)^n\right)^m$ Thus M follows a geometric distribution with parameter p_n where $p_n = 1-(1-p)^n$.			
7(c)	Let X be the number of tosses needed to get a head. Let M be the number of tosses needed to stop the game. Note that $M = \min\left(X_1, X_2, \ldots, X_n\right)$. Thus $E(M) = \dfrac{1}{1-(1-p)^n}$.			
8(i)	Let $N(t)$ be the number of customers joining the queue in t minutes. $N(t) \sim Po(\lambda t)$. Let T be the waiting time for the first customer to enter the queue. $\mathrm{P}(T > t) = P(N(t) = 0) = \mathrm{e}^{-\lambda t}$ Hence $F_T(t) = \mathrm{P}(T \le t) = 1 - \mathrm{e}^{-\lambda t}$. $f_T(t) = \dfrac{\mathrm{d}}{\mathrm{d}t}\mathrm{P}(T \le t) = \lambda \mathrm{e}^{-\lambda t}$ Thus T follows an exponential distribution with parameter λ.			
8(ii)	Let $N(60)$ be the number of customers joining the queue from 8am to 9am. $N(60) \sim Po(60\lambda)$ Let T_i be the waiting time for the i-th customer to enter the queue. $T_i \sim \exp(\lambda)$			

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	Thus $T_1+\cdots+T_{n+1}$ will be the waiting time for the queue to have $n+1$ customers and it has gamma distribution with density function $\frac{1}{((n+1)-1)!}\lambda^{n+1}x^{(n+1)-1}\mathrm{e}^{-\lambda x}$ $\mathrm{P}(N(60)\le n)=\mathrm{P}(T_1+\cdots+T_{n+1}\ge 60)$ $=\int_{60}^{\infty}\frac{1}{n!}\lambda^{n+1}x^{n}\mathrm{e}^{-\lambda x}\,\mathrm{d}x$	Note that we cannot do $\mathrm{P}(T_1+\cdots+T_n<60)$. This may include unwanted cases such as the first n customers came in by for e.g. 8.40am and the n+1 customer came in between 8.40am and 9am.		
8(iii)	Let S be the serving time of one customer at the counter. $S\sim\exp(\mu)$. The waiting time for the n th customer in the queue is the total serving time of the first n-1 customers at the counter. $\mathrm{E}(S_1+S_2+\ldots+S_{n-1})=\mathrm{E}(S_1)+\mathrm{E}(S_2)+\ldots+\mathrm{E}(S_{n-1})$ $=(n-1)\mathrm{E}(S)$ $=\frac{n-1}{\mu}$			
8(iv)	$\text{Prob. Req'd}=\mathrm{P}(N(t)=1 \text{ and } S>t)$ $=\mathrm{P}(N(t)=1)\mathrm{P}(S>t)$ by independence $=\left(\lambda t\mathrm{e}^{-\lambda t}\right)\left(\mathrm{e}^{-\mu t}\right)$ $=\lambda te^{-(\lambda+\mu)t}$	Serving time of the first customer must be more than t mins and in that t minutes, exactly 1 customer join the queue.		
8(v)	$P_n=\frac{\lambda}{\mu}P_{n-1}$ Thus the sequence is geometric with common ratio $\frac{\lambda}{\mu}$. Hence $P_k=P_0\left(\frac{\lambda}{\mu}\right)^k$. For steady state, the distribution is independent of t and $\sum_{k=0}^{\infty}P_k=1$. For the geometric sum to converge, $\frac{\lambda}{\mu}<1$.	$P_n=\frac{\lambda}{\mu}P_{n-1}$ $=\left(\frac{\lambda}{\mu}\right)^2P_{n-2}$ $=\ldots$ $=\left(\frac{\lambda}{\mu}\right)^nP_0$		

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	$\sum_{k=0}^{\infty} P_k = 1$ $\Rightarrow \dfrac{P_0}{1-\lambda/\mu} = 1$ $\Rightarrow P_0 = 1 - \dfrac{\lambda}{\mu}$ Thus $P_k = \left(1-\dfrac{\lambda}{\mu}\right)\left(\dfrac{\lambda}{\mu}\right)^k$, $n = 0, 1, 2, \ldots$.			

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9(a)	Let X be the the height of a randomly chosen plant of this species in the plantation.			

Height (cm)	52.5	57.5	62.5	67.5	72.5	77.5	82.5	87.5
Frequency	7	14	15	25	17	10	8	4

Using GC, $\bar{x} = 68.15$, $s^2 = 9.0636^2$.

H_0: X follows a normal distribution

H_1: X does not follow a normal distribution

Level of significance = 5%

Estimate the mean and variance of the distribution by $\bar{x}$ and s^2.

Height (cm)	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75	75 – 80	80 – 85	85 – 90
O_i	7	14	15	25	17	10	8	4
E_i	7.3410	11.0863	17.9819	21.6776	19.4239	12.9360	6.4026	3.1508

Combining the last two columns, we have

Height (cm)	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75	75 – 80	80 – 90
O_i	7	14	15	25	17	10	12
E_i	7.3410	11.0863	17.9819	21.6776	19.4239	12.9360	9.5534

There are seven classes and 3 restrictions.

Under H_0, $X^2 = \sum_{i=1}^{7} \dfrac{(O_i - E_i)^2}{E_i} \sim \chi^2(4)$

Reject H_0 if p-value ≤ 0.05.

	Calculations: $\chi^2_{calc} = 3.3807$, p-value $= 0.496$ Since p-value > 0.05, we do not reject H_0 and conclude that there is insufficient evidence at 5% level of significance to say that the height of the plants do not follow a normal distribution.			
9(b)	From earlier result, we can assume that X follows a normal distribution. Under H_0, since the population variance is unknown, $T = \dfrac{\overline{X} - 70}{s/\sqrt{20}} \sim t(19)$. Reject H_0 if $T \geq 1.729$ Since H_0 is rejected, $\dfrac{72.5-70}{s/\sqrt{20}} \geq 1.729$ $\Rightarrow s \leq 6.4659$ Thus the range of value of the sample standard deviation is $\hat{\sigma} \leq 6.4659\sqrt{\dfrac{19}{20}}$, i.e., $\hat{\sigma} \leq 6.30$.			
9(c)	For the combined sample, $\overline{x}$ remains the same. Let s_1^2 be the unbiased estimate based on the combined sample. Thus $s_1^2 = \dfrac{1}{39}\left[\sum_{i=1}^{40}(x_i - \overline{x})^2\right]$ $= \dfrac{38}{39}s^2$ Hence $t_{calc} = \dfrac{\overline{x}-70}{s_1/\sqrt{39}} = \dfrac{97.5}{s\sqrt{38}}$. Since $s \leq 6.4659$, thus $t_{calc} \geq 2.415 > t_{0.05}(39)$. We will still reject H_0.			