Topic: Logarithm

Question 1 [2008 EOY, Q7]	marks
Solve the following equations for *x*: $\log_2(x+6)+\log_2(x-1)=1+\log_2(3x)$ **<u>Solution:</u>** $\log_2(x+6)(x-1)=\log_2 2+\log_2(3x)$ $\log_2(x+6)(x-1)=\log_2 6x$ $(x+6)(x-1)=6x \Rightarrow x^2-x-6=0$ so $x=3$ $x=-2$ is rejected because $\log_2(3x)$ is undefined	[4]

Question 2 [2009 EOY, Q10]	marks
Solve the following equations: (a) $x^2-6=\lvert x\rvert$, **<u>Solution:</u>** $(x^2-6)^2=\left(\lvert x\rvert\right)^2$ $x^4-12x^2+36=x^2$ $x^4-13x^2+36=0$ $(x^2-9)(x^2-4)=0$ $x^2=9$ or $x^2=4$ $x=\pm 3$ or ± 2 (rej)	[4]

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(b)

$3^{2x-1}+3^{x-2}=3^x-\frac{4}{9}$,

<u>Solution:</u>

$$3^{2x}\cdot 3^{-1}+3^x\cdot 3^{-2}=3^x-\frac{4}{9}$$

Let $u = 3^x$

$$\frac{1}{3}u^2+\frac{1}{9}u=u-\frac{4}{9}$$

$$3u^2+u=9u-4$$

$$3u^2-8u+4=0$$

$$(3u-2)(u-2)=0$$

$$u=\frac{2}{3} \text{ or } 2$$

$3^x=2$

$x=\frac{\lg 2}{\lg 3}$ or 0.631

$3^x=\frac{2}{3}$

$x=\frac{\lg\frac{2}{3}}{\lg 3}$ or -0.369

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(c)

$1+\lg(x+1.6)=\lg 2+\lg(2x^2+1)$.

<u>Solution:</u>

$$\lg 10+\lg(x+1.6)=\lg 2+\lg(2x^2+1)$$

$$\lg[10(x+1.6)]=\lg[2(2x^2+1)]$$

$$10x+16=4x^2+2$$

$$4x^2-10x-14=0$$

$$2x^2-5x-7=0$$

$$(2x-7)(x+1)=0$$

$$x=-1 \text{ or } \frac{7}{2}$$

Optional Check:

$x=-1$
$\lg(0.6)\rightarrow$ defined
$\lg(3)\rightarrow$ defined

$x=3.5$
$\lg(5.1)\rightarrow$ defined
$\lg(25.5)\rightarrow$ defined

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Question 3 [2010 EOY, Q13]	marks
Solve the following equations: (a) $\log_8[2+\log_2(x-4)]=\log_{125}5$ **Solution:** $\log_8[2+\log_2(x-4)]=\log_{125}5 \Rightarrow \log_8[2+\log_2(x-4)]=\frac{1}{3}$ $\Rightarrow \sqrt[3]{8}=2+\log_2(x-4)$ $\Rightarrow \log_2(x-4)=0$ $\Rightarrow x-4=1$ $\Rightarrow x=5$	[3]
(b) $e^{2\ln y}=\lg 1.6$ **Solution:** $e^{2\ln y}=\lg 1.6 \Rightarrow 2\ln y=\ln(\lg 1.6)$ $\Rightarrow \ln y=\frac{\ln(\lg 1.6)}{2}$ $\Rightarrow y=e^{\frac{\ln(\lg 1.6)}{2}}$ $\Rightarrow y=0.452$ (3 s.f.)	[3]

Question 4 [2010 EOY, Q14]	marks
Solve the pair of simultaneous equations $e^x(e^2)=(e^2)^y$ $\lg(x-1)+\lg(x+1)=\lg y+\lg(2x-y)$ **Solution:** $e^x(e^2)=(e^2)^y \Rightarrow x+2=2y$, and $\lg(x-1)+\lg(x+1)=\lg y+\lg(2x-y) \Rightarrow (x-1)(x+1)=y(2x-y)$ Hence $(2y-2-1)(2y-2+1)=y(4y-4-y) \Rightarrow (2y-3)(2y-1)=y(3y-4)$ $\Rightarrow y^2-4y+3=0$ $\Rightarrow y=1$ or 3 But if $y=1$, $x=0$ and $\lg(x-1)$ will be undefined, hence solution rejected When $y=3$, $x=4$	[6]

Question 5 [2011 EOY, Q13]	marks
Solve the following equations (a) $$\log_2\left(\frac{x^2}{4}+12\right)=2\log_2(x+5)-2,$$ **Solution:** $$\log_2\left(\frac{x^2}{4}+12\right)=\log_2(x+5)^2-\log_2 4$$ $$\log_2\left(\frac{x^2}{4}+12\right)=\log_2\frac{(x+5)^2}{4}$$ $$\left(\frac{x^2}{4}+12\right)=\frac{(x+5)^2}{4}$$ $$(x^2+48)=(x+5)^2 \qquad x=2\frac{3}{10}$$	[4]
(b) $$\log_4\left(x^2\right)+3\log_{x^2}4=4$$ **Solution:** $\log_4 y+3\log_y 4=4$ $\log_4 y+3\left[\frac{\log_4 4}{\log_4 y}\right]=4$ $(\log_4 y)^2+3=4(\log_4 y)$ $Let \quad a=\log_4 y$ $a^2-4a+3=0$ $(a-3)(a-1)=0$ $a=3, a=1$ $\log_4 y=3, \log_4 y=1$ $y=4^3=64, y=4$ $x^2=64 \Rightarrow x=\pm 8 \;, x^2=4 \Rightarrow x=\pm 2$	[4]

Question 6 [2012 EOY, Q3]	marks
Solve the equation $\log_x(2x+1)-2\log_{x^2}(4x-1)+1=0$. **Solution:** $\log_x(2x+1)-\log_x(4x-1)=-1$ $\log_x\frac{2x+1}{4x-1}=-1$ $\frac{2x+1}{4x-1}=\frac{1}{x}$ $2x^2+x=4x-1$ $2x^2-3x+1=0$ $x=\frac{1}{2}$ or $x=1$ $(rej.\because x\neq 1)$	[4]

Question 7 [2013 EOY, Q1c]	marks
(iii) Solve the equation $\log_5 x + 1 = 6\log_x 5$. **Solution:** $\log_5 x + 1 = \frac{6}{\log_5 x}$ $(\log_5 x)^2 + \log_5 x - 6 = 0$ $(\log_5 x + 3)(\log_5 x - 2) = 0$ $\log_5 x = -3$ or $\log_5 x = 2$ $x = \frac{1}{125}$ or $x = 25$	[3]

Question 8 [2014 EOY, Q3]	marks
(i) Solve the equations $\log_{(x-1)}(3x^2-7x-2)=2$ **Solution:** $\log_{(x-1)}(3x^2-7x-2)=2$ $\log_{(x-1)}(3x^2-7x-2)=\log_{(x-1)}(x-1)^2$ $3x^2-7x-2=x^2-2x+1$ $2x^2-5x-3=0$ $(2x+1)(x-3)=0$ $x=-\frac{1}{2}$(rejected) or 3	[4]
(ii) $\ln\left\|\sqrt{x}-2\right\|+\frac{1}{2}\ln x=0,$ where $x>0$ **Solution:** $\ln\left\|\sqrt{x}-2\right\|+\frac{1}{2}\ln x=0,$ where $x>0$ $\ln\left\|\sqrt{x}-2\right\|+\ln\sqrt{x}=0$ $\ln\left\|x-2\sqrt{x}\right\|=0$ $\left\|x-2\sqrt{x}\right\|=1$ $x-2\sqrt{x}=1$ or -1 (Two cases) Case 1: $x-2\sqrt{x}=1$ $2\sqrt{x}=x-1$ $x^2-2x+1=4x$ $x^2-6x+1=0$ $x=3\pm2\sqrt{2}$ (reject $3-2\sqrt{2}$) OR $x=5.83$ or 0.172 (rejected, as it does not satisfy the Case 1 equation) [A1] Case 2: $x-2\sqrt{x}=-1$ or $x-2\sqrt{x}+1=0$ $2\sqrt{x}=x+1$ $\quad$ $(\sqrt{x}-1)^2=0$ $x^2+2x+1=4x$ $\quad$ $x=1$ $(x-1)^2=0$ $x=1$ Hence $x=3+2\sqrt{2}$ or 1	[5]

Question 9 [2015 EOY, Q6]	marks
Solve the following equations. (i) $\log_3 x = \ln 2$, **Solution:** $\log_3 x = \ln 2$ $x = 3^{\ln 2} \approx 2.14 \text{ (3 s.f.)}$	[2]
(ii) $1 + \log_{\sqrt{2}}(y-1) = \log_2(22-7y)$. **Solution:** $1 + \log_{\sqrt{2}}(y-1) = \log_2(22-7y)$ $\Rightarrow \log_2 2 + \log_2 (y-1)^2 = \log_2(22-7y)$ Hence $2(y-1)^2 = 22-7y \Rightarrow 2y^2 + 3y - 20 = 0$ Solving, $y = \frac{5}{2}$ or -4 (rej.)	[4]